

# CONJECTURE ON A PRIME NUMBER GENERATOR

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## Abstract-

I have developed a mathematical formula to generate prime numbers with the help of prime numbers. If  $2^n - 1 = P$  is a prime number and  $n$  is a positive integer, then  $n$  must be prime number. But the reverse is not always true. This conjecture is evident from numerous examples.

## Introduction-

Prime numbers and their properties were first studied extensively by the ancient Greek mathematicians.

The mathematicians of Pythagorus's school (500 BC to 300 BC) were interested in numbers for their mystical and numerological properties. They understood the idea of primality and were interested in *perfect* and *amicable* numbers.

A *perfect number* is one whose proper divisors sum to the number itself. e.g. The number 6 has proper divisors 1, 2 and 3 and  $1 + 2 + 3 = 6$ , 28 has divisors 1, 2, 4, 7 and 14 and  $1 + 2 + 4 + 7 + 14 = 28$ .

A *pair of amicable numbers* is a pair like 220 and 284 such that the proper divisors of one number sum to the other and vice versa

By the time Euclid's *Elements* appeared in about 300 BC, several important results about primes had been proved. In Book IX of the *Elements*, Euclid proves that there are infinitely many prime numbers. This is one of the first proofs known which uses the method of contradiction to establish a result. Euclid also gives a proof of the Fundamental Theorem of Arithmetic: Every integer can be written as a product of primes in an essentially unique way.

Number of the form  $2^n - 1$  also attracted attention because it is easy to show that if unless  $n$  is prime these number must be composite. These are often called *Mersenne numbers* because Mersenne studied them.

Not all numbers of the form  $2^n - 1$  with  $n$  prime are prime. For example  $2^{11} - 1 = 2047 = 23 \times 89$  is composite, though this was first noted as late as 1536

Conjecture on a prime-to-prime generator

If  $2^n - 1 = P$  is a prime number and  $n$  is a positive integer, then  $n$  must be a prime number. But the reverse is not always true.

Actually, for all prime values of  $n$ ,  $P$  is either prime or composite odd numbers. This is why, my conjecture is if  $P$  is a prime and  $n$  is a positive integer, then  $n$  must be a prime number. This is obvious from numerous numerical examples.

Again, it can be mathematically written as,  $n = \log(p+1)/\log 2$  is a prime when  $p$  is a prime number and  $n$  is a positive integer.

That is, when  $p=3$ (prime),  $n=2$  is an integer and therefore, it is a prime number. Again, when  $p=5$ (prime),  $n = \log 6/\log 3$  is not an integer and therefore, it must not be a prime.

So, my formula holds true when both the conditions are applied, i.e.,  $p$  is a prime and  $n$  is a positive integer. Numerical examples—

When,  $p=7$ (prime),  $n=3$  is a prime. When,  $p=31$ (prime),  $n=5$  is a prime. When,  $p=8191$ (prime),  $n=13$  is a prime.

When,  $p=131071$ (prime),  $n=17$  is a prime, When,  $p=524287$ (prime),  $n=19$  is a prime, When,  $p=2147483647$ (prime),  $n=31$  is a prime. There will be many more such examples.

Even, if  $p$  is taken as a very large prime 170141183460469231731687303715884105727, then  $n=127$  is a prime.

Pierre de Fermat stated an important theorem (now known as Fermat's little theorem) which states that given a prime  $p$  and a coprime base  $b$ ,

the congruence  $b^{p-1} \equiv 1 \pmod p$  holds true. Fermat also conjectured incorrectly

that  $2^{2^n+1}$  is always prime (the Fermat primes) while Marin Mersenne incorrectly conjectured that in  $2^p-1$  the primes 67 and 257 give Mersenne primes.

Nevertheless, these numbers were still named after them.

### Application of prime numbers

Beside cryptography is coding theory. Random number generators, error correcting codes, and hashes often involve primes: either directly or indirectly. Another not so obvious (indirect) application: many libraries which perform arithmetic on large integers, or polynomials involve reductions modulo primes (as stated in Hensel's lemma) for computational complexity reason.

Cicadas are insects which hibernate underground and emerge every 13 or 17 years to mate and die (while the newborn cicadas head underground to repeat the process). Some people have speculated that the 13/17-year hibernation is the result of evolutionary pressures. If cicadas hibernated for  $X$  years and had a predator which underwent similar multi-year hibernations, say for  $Y$  years, then the cicadas would get eaten if  $Y$  divided  $X$ . So by "choosing" prime numbers, they made their predators much less likely to wake up at the right

Prime numbers are used in developing machine tools. Utilizing primes we can avoid setting up harmonics which "eat" your very expensive tools. Tools chatter, (bounce up and down), as they are being used. Allowing those vibrations to propagate intensifying the chatter and the wear.

Quadratic Reciprocity is stated in terms of the residues modulo primes. This "Golden Theorem" as called by Gauss, is one of the main threads leading up to Langlands program and eventually to the geometric Langlands Program. This later area of research has been shown to have ties with S-duality in string theory. String theory is just now being proven useful in understanding phenomena in condensed matter physics.

Also, it is the techniques that are used to prove results about prime numbers that have applications rather than a particular theorem about specific families of primes. Prime numbers are often test beds for more general results used in other areas of mathematics.

### Conclusion

Prime number generator is a mathematical formula to generate primes. Here, I have developed a mathematical formula to generate primes with primes. Fermat and Mersenne tried in this field. I have also made a conjecture in this field of mathematical generation of primes.

There are many unsolved questions about old conjectures on prime number generation. Some of them are--

1. The *Twin Primes Conjecture* that there are infinitely many pairs of primes only 2 apart.
2. *Goldbach's Conjecture* (made in a letter by Goldbach to Euler in 1742) that every even integer greater than 2 can be written as the sum of two primes.
3. Are there infinitely many primes of the form  $(n^2+1)$  ?
4. Is there always a prime between  $n^2$  and  $(n+1)^2$ ?
5. Are there infinitely many prime Fermat numbers? Indeed, are there *any* prime Fermat numbers after the fourth one?
6. Is there an arithmetic progression of consecutive primes for any given (finite) length? e.g. 251, 257, 263, 269 has length 4. The largest example known has length 10.

So, prime numbers and generating prime numbers are still an important field of research.

I have just made another conjecture to create further interest in this field.

After several attempts at a formula for the prime counting function, the prime number theorem was proven in the 19th Century. The famous Riemann hypothesis was also stated and to this day remains unproven despite much evidence supporting it.

### References

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