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Blockwise Robust Singular Value Decomposition for MRI Brain Image Denoising

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Abstract

MRI is one of the high-dimensional high-throughput technologies that are playing a major role for diagnosing brain tumor. With the help of MRI images, brain tumor is diagnosed at advanced stages. Correct identification of brain tumor or abnormality, image processing in MRI of brain is highly essential that can reduce the chance of fatal stage. Occasionally, these MR images are introduced with noise during acquisition which reduces the image quality and limits the accuracy in diagnosis. Therefore, preliminary diagnosis of MRI brain images from the hospital may not be always reliable for further analysis because of the presence of noise. Reduction/Elimination of noise in medical images is an important task in preprocessing which is one of the previous crucial parts for further image analysis. Although, several denoising techniques are available in the literature including median filter, box filter, low pass filter and high pass filter. In this paper, we have measured the influence of the above methods for denoising. We have also proposed a new blockwise robust singular value decomposition technique for denoising image. The results of our analysis showed that blockwise robust singular value decomposition technique gives the better performance compared to the other methods. Therefore, our recommendation is to use the Blockwise robust singular value decomposition technique for brain image denoising.

Keywords: MRI imaging, Image filtering, Dimension reduction, Singular value decomposition, Image denoising.

1. Introduction

MRI is known as nuclear magnetic resonance imaging which is a scanning technique for creating detailed images of the human body. MRI scan is a common procedure around the world. MRI scanning is a painless and confined procedure. MRI scan represents a huge milestone in the medical science. Doctors, scientists, and researchers are now able to get the inside images of the human body in high details using magnetic resonance imaging (MRI). The scan uses a strong magnetic field and radio waves to generate images of parts of the body that can't be seen as well with X-rays, CT scans or ultrasound. MRI scan produces different slices of an image. However, MRI scan image often contain outliers [1] and the presence of noise/outliers hampers the disease diagnosis [2]. Therefore, image preprocessing as well as image filtering can help minimize the effects of these degradations [3]. A fundamental problem in the image processing is the improvement of their quality through the reduction of the noise. Noise removal is essential in medical imaging applications in order to enhance and recover anatomical details that may be hidden in the data. Nowadays, MR image de-noising has become an important purpose in medical imaging particularly the Magnetic Resonance Imaging (MRI). Many de-noising and enhancement techniques are using on MRI images [4–5]. However, the performance of the de-noising techniques depends on the type and amount of noise present in the MR image. Researcher may also consider the other factors like performance in de-noising the MR image, computational time, and computational cost [6–7]. In recent years, a variety of non-linear filters like median filter, adaptive median filter, min filter, max filter have been developed to overcome the consequences of linear filter. Non-linear filters give better performance than linear filters [8]. The important property of a good image-denoising model is that it should completely remove noise as far as possible as well as preserve edges. Recently, Low rank approximation in singular value decomposition are using for image de-noising [9]. However, singular value decomposition itself is sensitive to outliers because it is based on the squared error loss function. Thus, here, we have proposed a block-wise robust singular value decomposition method that can reduce noise comparative to other filtering methods. Experimental results showed that our proposed methods provide comparatively better performance than other four filtering methods.

2. Methods and Materials

There are many filtering techniques are found in the literature for MRI brain image data. Here, the discussion of median filter, high-pass filter, low-pass filter, box filter low rank approximation in singular value decomposition and our proposed block-wise robust singular value decomposition techniques are given below.

2.1. Median filter

The median filter is the most common type of nonlinear filter. It is a well-known order-statistic filter due to its better performance for some specific types of noise such as 'Gaussian', 'random' and 'salt and pepper' noises. It is very effective to remove impulse noise in the image when the noise amplitude probability density has periodic patterns and large tails. The median filtering process is performed by sliding a window over the image. The filtered image is obtained by putting the median of the values in the input window, at the location of the center of that window, and at the output image. The median is the MLE of location in the case of the Laplacian noise distribution. The drawback of median filters is that in the presence of small signal-to-noise ratios, they tend to break up image edges and may produce false noise edges, and they cannot suppress medium-tailed (Gaussian) noise distributions. In the literature, there are both recursive and non-recursive types of Median filters are found. Recursive median filters are more efficient than those of the non-recursive type. The median filter replaces the gray level of each pixel with the median of the gray levels in a neighborhood of the pixels, instead of using the average operation.

2.2. High-Pass Filter

A high-pass filter is used for passing high frequencies but the strength of the frequency is lower as compared to the cut-off frequency. High-pass filters enhance the rapidly varying spatial components within a digital image - in other words, they enhance the high spatial frequencies. Many high-pass filters are also referred to as edge enhancement filters, since they make it easier to detect edges in imagery. Perhaps the simplest way to develop a high-pass filter is to run a low-pass pass filter on a digital image, then subtract the output values of the low-pass filter from the input values in the original image. The resultant image has enhanced high spatial frequency information. The first derivative filter (another form of high pass filter) calculates the gradient of pixels values. If DN values are constant, the output of a first derivative filter is zero and it only returns non-zero values where DN values change from one pixel to the next. Larger first derivative values indicate areas of rapid change in the image.

2.3. Low-Pass Filter

A low-pass filter is applied to pass low-frequency signals. The strength of the signal is reduced and the frequencies that are passed are higher than the cut-off frequency. The volume of strength concise for each frequency depends on the design of the filter. In the frequency domain smoothing is a low-pass operation. There are some low-pass filters like ideal low-pass filter, Butterworth low-pass filter, Gaussian low-pass filter etc.

2.4. Box Filter

Box filter is one of the most commonly used filters in graphics. It is a spatial domain linear filter in which each pixel in the resulting image has a value equal to the average value of its neighboring pixels in the input image. The box filter equally weights all the samples within a square region of the image.

2.5. Classical and Robust Singular Value Decomposition

Singular value decomposition (SVD) was originally developed by Eugenio Beltrami and Camille Jordan in the mid-to-late 1800's. Further developments of the SVD were made by many mathematicians, including James Joseph Sylvester, Erhard Schmidt and Hermann Weyl who studied the SVD into the mid-1900's. Although SVD is an old technique in mathematics, however, recently it is widely used in computer science, bioinformatics, statistics etc. The most important property of SVD is the low rank approximation of a matrix [10]. SVD has a capacity of data reduction in both row and column wise. Moreover, SVD is the extension of the eigenvalue decomposition for the case of nonsquare matrices. Both variables (columns) as well as observations (rows) can be reduced by SVD. Therefore, taking only two or three singular vectors we can easily draw a two or three dimensional plot that can show the pattern of the data. The importance of SVD is that it is used in many of the multivariate data analytic techniques including principal component analysis, correspondence analysis, cluster analysis, factor analysis and discriminant analysis [11].

If X is a $m \times n$ data matrix with rank $k \leq \min(m, n)$ then using SVD we can write $X = U \Lambda V^T$; where U is a column orthonormal matrix, V is the row orthogonal matrix and Λ is a diagonal matrix that contains the singular values. U and V are the column matrices of eigen vectors of XX^T and $X^T X$, respectively. X can be approximated as $\tilde{X}$ with rank $r \leq k$, $\tilde{X} = \lambda_1 u_1 v_1^T + \lambda_2 u_2 v_2^T + \cdots + \lambda_r u_r v_r^T$ and in matrix form $\tilde{X} = U_r \Lambda_r V_r^T$, that imply $\tilde{X} V_r = U_r \Lambda_r$, its first column represents the first principal component (PC), the second column represents the second PC and so on. If the variables are correlated then the first few PC's contains most of the variation of data. Therefore, we can see the pattern or structure of the data by considering two or three PC's. However, SVD is sensitive to outliers as well as its calculation procedure cannot be performed if the data matrix X has any missing element [12]. For this reason, "Robust Singular Value Decomposition" (RSVD) was developed by Hawkins, Liu and Young on the basis of the alternating L1 regression approach that can be used to solve the outlier problem as well as handle missing elements [13].

2.6. Proposed Method

Let X_{ij} be a data matrix which can be written as, $X_{ij} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{bmatrix}$

Where $i = 1,2,\ldots,n$ and $j = 1,2,\ldots,p$; X data matrix may contain noise at any single point. When RSVD is applied on the whole data matrix then sometimes it can't remove this noise properly. Therefore, firstly we divide the image into some blocks (k x l) row and column wise according to the structure of the data. Then we get the sub data matrices from original image data matrix. Secondly we apply RSVD in each sub data matrix to remove noise. Finally we set the entire reconstructed block according to the original image and get the new image data. In this research we have taken 3×3 block for all cases.

2.7. Sources of Data

To remove the noise from brain image, we have collected the MRI brain image data from the url- https://radiopaedia.org/cases/normal-brain-mri-5?lang=us. The image sizes were 630×630 pixel. We also artificially added different percentage (1%, 5% and 10%) of Gaussian noise in the original image to produce modified image for measuring the performance of the proposed method.

3. Results and Discussion

Here, we have measured the performance of our proposed method compared to the four filtering methods (Median Filter, Box Filter, High-Pass Filter, Low-pass Filter) on MRI brain images. To measure the performance of filtering methods, we have calculated the accuracy of four filtering methods and our proposed technique by using one of the performance measurement criteria RMSE in both absence and presence of different percentages of noise.

In absence of outliers (original MRI image), if we apply the filtering methods including our proposed method then the different filtering images are given in figure-1.

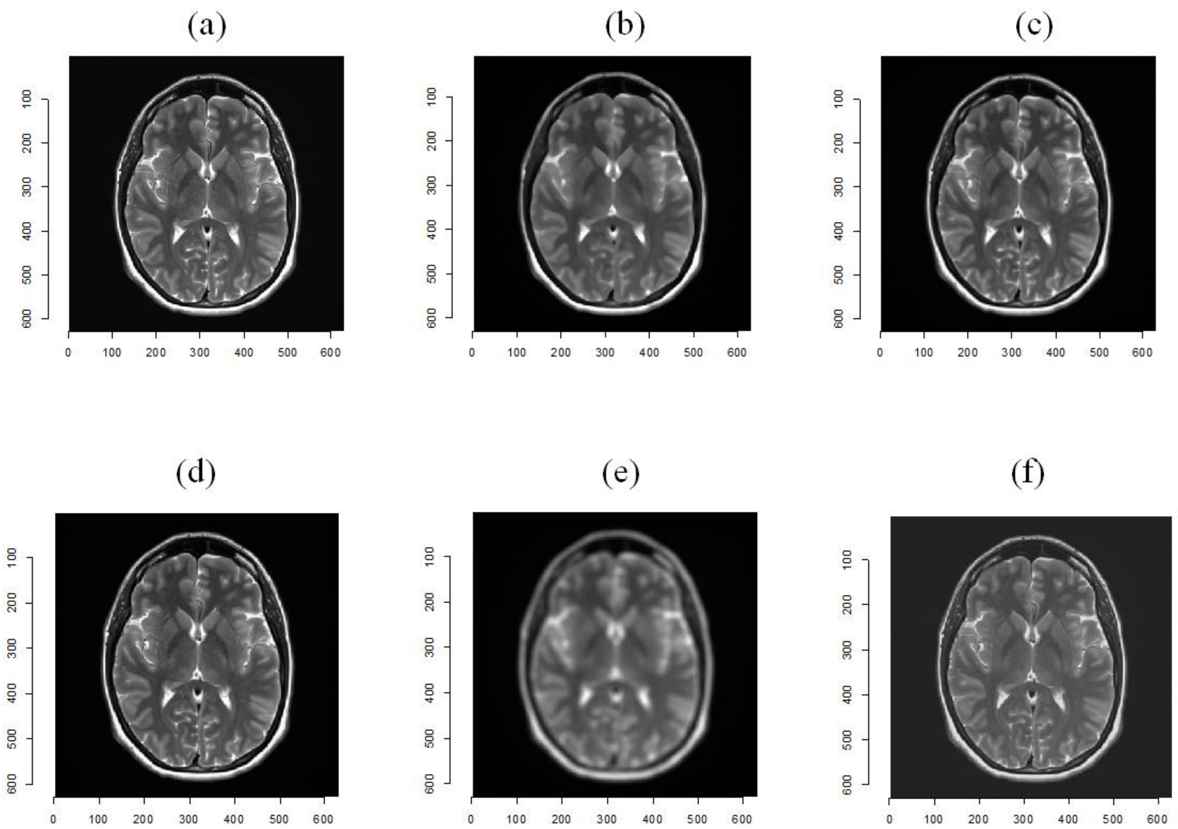

Figure 1: Filtering Images (without noise) (a) Original image (b) median filtering image (c) Box filtering image (d) High-Pass filtering image (e) Low-Pass filtering image (f) Proposed filtering image for 3×3 block

From figure-1 it is seen that our proposed filtering method image is more similar to the original image. We also measure the RMSE for each filtering method that are given in table 1. Our proposed filtering method shows the lower RMSE compared to the other filtering methods.

After Artificially adding 1% Gaussian noise in the original MRI brain image data, If we apply the different filtering methods then we get filtering images that are given in figure-2. From figure-2 it is seen that our proposed filtering method image is more similar to the original image. We also measure the RMSE for each filtering method that are given in table 1. Our proposed filtering method shows the lower RMSE compared to the other filtering methods in presence of 1% Gaussian noise.

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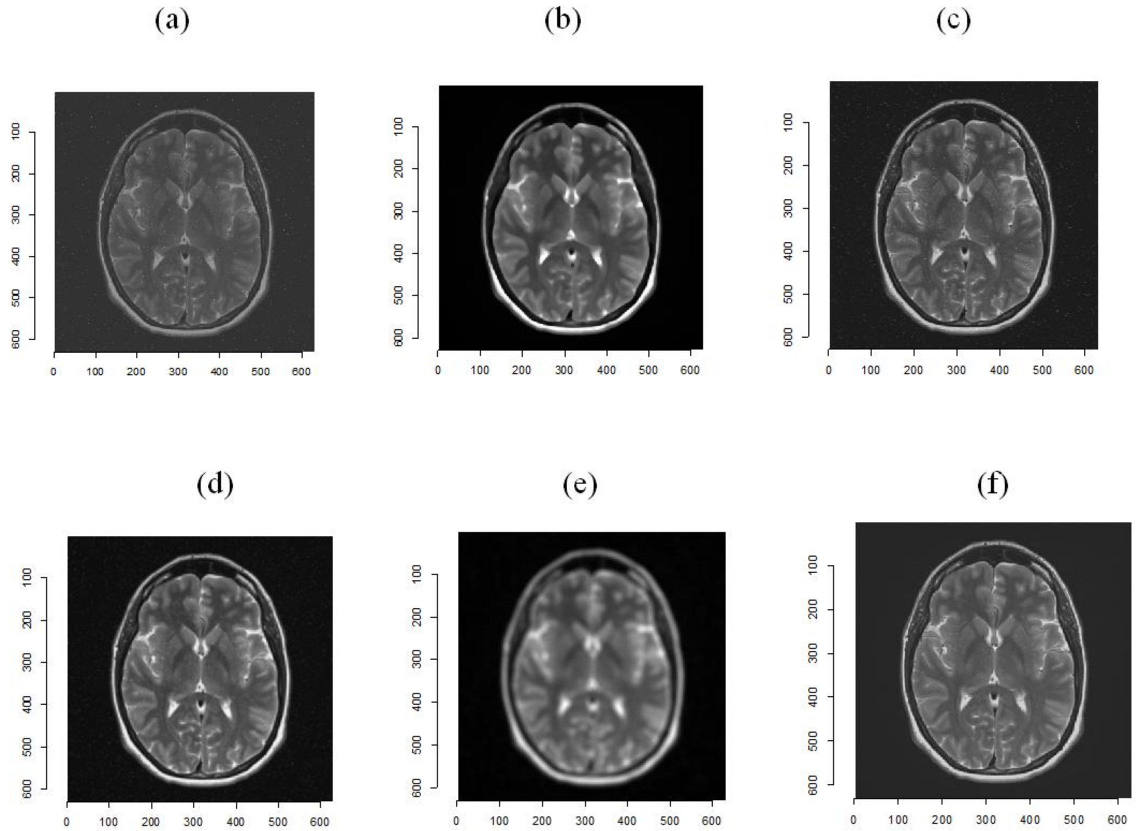

Figure 2: Filtering Images (Adding 1% noise) (a) Contaminated image (b) median filtering image (c) Box filtering image (d) High-Pass filtering image (e) Low-Pass filtering image (f) proposed filtering image for 3 × 3 block

Similarly for 5% and 10% contaminated image, we have applied the different filtering methods and calculated the RMSE for all filtering image. The values of RMSE are given in table 1.

From table 1, we observed that the RMSE values of our proposed technique are lower compared to the other four filtering techniques in both absence and presence of different percentage of noise. Since our proposed filtering technique gave lower RMSE, therefore, we can say that our proposed filtering technique shows better performance compared to the four well known filtering techniques. The RMSE values of table-1 are plotted on figure 3.

Table 1 Performance measurement of different filtering techniques for MRI brain image data using RMSE with different rates of noise

Filtering Techniques	0% Noise	1%Noise	5% Noise	10% Noise
	RMSE	RMSE	RMSE	RMSE
Median Filter	36.505	37.389	44.062	46.053
Box Filter	36.256	41.773	61.224	90.104
High-Pass Filter	313.056	325.86	377.03	438.634
Low-Pass Filter	121.255	126.073	143.84	171.147
Proposed Method (3 × 3)	**13.917**	**15.435**	**23.5**	**43.960**

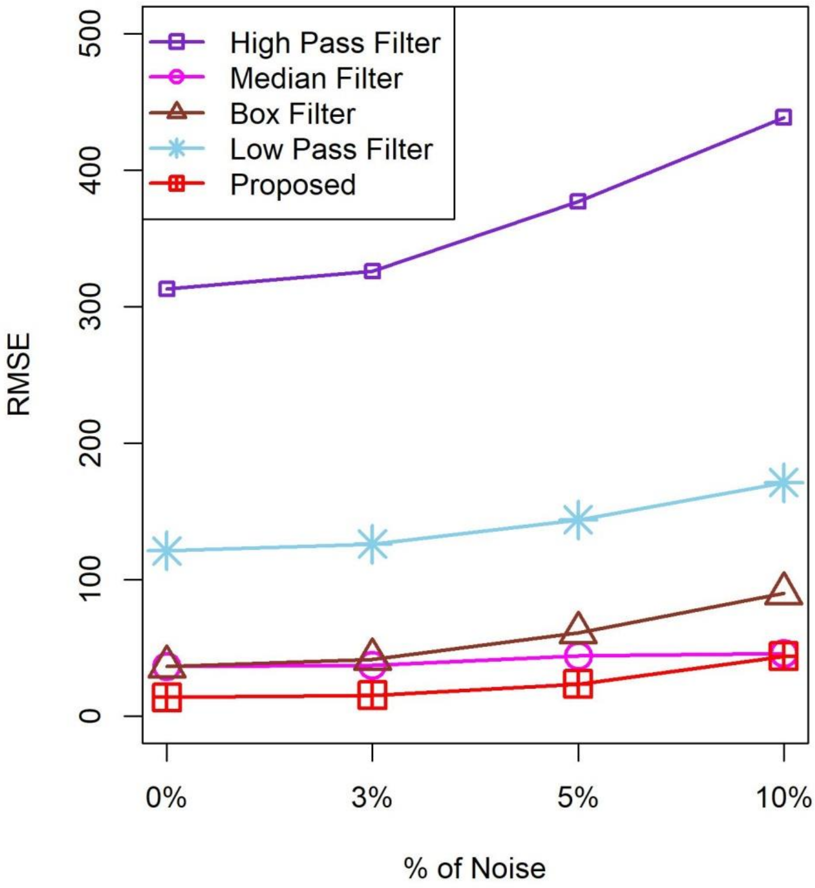

Figure 3 Performance investigation of different techniquesby using RMSE for MRI brain image data

4. Conclusion

Filtering algorithm is the most common method used to remove the noise as preprocessing. De-noising should be used to improve the image quality for more precise diagnosis. In this research, we have discussed about four filtering methods and our proposed method and also measure the performance of this four filtering methods and our proposed method in terms of one of the performance measurement criteria named RMSE. Our proposed method provides better performance than other four filtering methods. Therefore, our recommendation is to use our proposed method for removing noise on MRI brain image data in both absence and presence of different rates of noise.

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