Common Fixed Point Theorem for Six Self-Mappings Satisfying CLR Property via A - Class Functions in Fuzzy Metric Space

Dheeraj Mali[1], Mala Hakwadiya[2]

[1]Assistant Professor, Department of Mathematics, Govt Adarsh College Jhabua MP INDIA
[2]Assistant Professor, Department of Mathematics, Govt College Daloda, Mandsour MP INDIA

Abstract

The main objective of this paper is to prove common fixed point theorems for two pairs of weakly compatible mappings satisfying CLR property in the framework of KM and GV fuzzy metric space by using a new theory of A – class functions. some examples are also given in support of the result. in this paper we prove some theorems without using the condition of completeness and closedness of range of space. We will generalize the result of Kamal Wadhawa and Arvind Kumar Gupta.

keywords: A-class function, E.A. property, K.M. & G.V. fuzzy metric space, weakly compatible mappings, (CLR_{SM}), (CLR_{TD})($CLR_{(SM)(TD)}$).

1. INTRODUCTION :

Fixed point theory has been fructuous and generative area of research and it is one of the most powerful tool of modern mathematical analysis. It is a beautiful combination of analysis, topology, and geometry. The concept of fuzzy sets was given by Zadeh [5] in 1965. The notion of fuzzy metric space was introduced by Kramosil and Michelek [6] in 1975. After a decade in 1994 George and Veeramani [2] defined continuous t-norms and modified the concept of fuzzy metric space of KM. In 1986 Jungck [3] proposed a generalization of the commuting mapping concept and introduced concept and introduced compatible function. After then Jungck and Rhoadas [4] introduced the concept of weak compatibility, which is new general form of compatibility. Many researchers used this concept to prove various common fixed point theorems. In 2002Amri and E.I. Moutawakil [1] established a new property (E.A.) which generalized the concept of non-compatible mappings. After some time in 2005, Liu et al [13] gives the notion of common property (E.A) for a hybrid common (E.A) property requires special condition that is closed-ness of ranges for the existence of fixed points. In 2011 W. Sintunavarat and P. Kuman [12] give the idea of "common limit in the range property" (CLR) which does not require the closed-ness of the subspace for the existence of fixed point. Further in the same order Imdad et al.[10] expanded the concept of common limit in the range property to two pair of self-mapping, called(CLRst) with respect mappings S & T .

Objective of this paper is to prove some common fixed point theorems for six self –mappings using CLR_{SM}, CLR_{T} and $CLR_{(SM)(TD)}$ property in fuzzy metric space with the help of new theory of A-class function. Our result modify the result of Kamal Wadhawa and Arvind Kumar Gupta [14.]

2. PRELIMINARIES

Defination2.1: W. sintunavarat, P.kumam [12]. A binary operation* :[0,1] × [0, 1] → [0,1] is continuous t-norm if for all a, b, c, d, ϵ [0, 1] the following conditions are satisfied

1. (a*b) = (b*a) commutative.
2. (a*b)*c = a* (b*c) associative
3. a*1 = a identity element.
4. * is continuous.
5. a*b ≤ c*d whenever a ≤ c and b ≤ d for all a, b, c, d ϵ [0, 1].

Example 2.2: [12] there are some examples of t- norms.

1. (The product t- norm) T.P.: A mapping Tp :[0,1]×[0, 1]→[0, 1] is defined by

$$Tp(a, b) = a . b$$

2. (The minimum t- norm) T.M.: A mapping Tm: [0,1] ×[0, 1]→[0, 1] is defined by

$$Tm (a, b) = \min \{a, b\}$$

Definition 2.3: (Kramosil and Michelek (1975) [6]) : Let X be an arbitrary set ,* is a continuous t-norm, then the 3-tuple (X,M,*) is called a KM fuzzy metric space if M is a fuzzy set on X×X×[0,∞) such that the following axioms hold.

For all x, y, z ϵ X and s, t > 0,

(KM-1) M(x, y, 0) = 0 for all x, y, ϵ X

(KM-2) M(x , y, t) = 1 ⇔ x = y for all x, y, ϵ X where t > 0,

(KM-3) M(x, y, t) = M(y, x, t) for all x, y, ϵ X

(KM-4) M(x, y, t) * M(y, z, s) ≤ M(x, z, t + s)

(KM-6) $lim_{t\to\infty}$ M (x, y, t) = 1 for all x, y, ϵ X

Definition 2.4: (George and Veeramani (1994) [2]): A 3- tuple (X, M, *) is called a GV fuzzy metric space where X is nonempty set,* is a continuous t-norm, and if M is a fuzzy set on X× X×[0,∞) is satisfy the following conditions.

For all x, y, z ϵ X and s, t > 0,

(GV-1) M(x, y, t) > 0 for all x, y, ϵ X

(GV-2) M(x, y, t) = 1 ⇔ x = y for all x, y, ϵ X where t > 0,

(GV-3) M(x, y, t) = M (y, x, t) for all x, y, ϵ X

(GV-4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

(GV-5) $M(x, y, .):[0,\infty)\rightarrow[0, 1]$ is continuous. For all x, y, z ϵ X and s, t > 0

where M(x, y, t) is a degree of nearness between x and y with respect to t. and axiom (GV-1) and (GV-2) we see that if $x \neq y$ and ∀ t > 0

$$0 < M(x, y, t) < 1$$

Example 2.5: [12] Let (X, d) be a metric space, a * b = Tm (a, b) and for all x, y, ϵ X

Where t > 0,

$$M(x, y, t) = \frac{t}{t+d(x,y)}$$

Then (X, M,*) is a GV – fuzzy metric space.

Definition 2.6: [7] 1. Let (X, M,*) be fuzzy metric space. Then a sequence $\{x_n\}$ in X is called convergent to x ϵ X if

$$lim_{n\to\infty} M(xn , x, t) = 1$$

2. Let (X, M,*) be fuzzy metric space. Then a sequence $\{x_n\}$ in X is called Cauchy sequence if

$$\lim_{n\to\infty} M(x_n, x_{n+m}, t) = 1 \quad \forall\, t > 0 \text{ and } m \in N$$

3. A fuzzy metric space (X, M,*) is called complete if every Cauchy sequence convrgent to a point in X.

Definition 2.7: Jungck and Rhoades [4] Suppose that f and g are two self – mappings which are defined on nonempty set X. I. e. f, g : X → X, Then they are said to be weakly compatible If they are commute at their coincidence point.

$$Fx = gx \Rightarrow fgx = gfx .$$

Definition 2.8: Aamri and EI Moutawakli [1] Let f and g be two self – mappings of a metric space (X, d). Then we say that f and g satisfy E.A. property if there exist a sequence $\{x_n\}$ in X. such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t \text{ For some } t \in X$$

Definition 2.9: M. Abbas et al. [9] Let (X, M,*) be a fuzzy metric space, and A, B, S, T are four self – mappings on X. Then A, B, S, T, are said to satisfy common (E.A.) property if there exists sequence $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = x \text{ For some } x \in X$$

For more information about common (E.A.) properties we refer to [11] and [13].

Definition 2.10: W. sintunavarat, P. Kumam [12] Suppose that (X, M,*) is a fuzzy metric space and f, g : X → X. Then two mappings f and g are said to satisfy the common limit in the range g property if

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = gx \qquad \text{For some } x \in X$$

The common limit in the range of g property will be denoted by the (CLRg) property.

First of all we show that two pairs (P, S) and (Q, T) of four self – mappings satisfying the (E.A.) property along with the closed-ness of the subspace S(X) and T(X) always enjoy the (CLRs) and (CLR_T) properties with respect to S and T respectively with the help of following example.

Example 2.11: Wadhwa & Gupta [8] Let $\mathcal{F}(x, y)$ be any function of class A , X = [0 , 2] and d : X × X →[0,∞) be usual metric defined by d(x , y) = │x - y│ , For * is minium t- norm and for all x, y ϵ X,

$$M(x, y, t) = \frac{t}{t + d(x, y)}$$

Then (X, M,*) is a GV – fuzzy metric space,

Suppose that P, Q, S and T are four self-mappings of X , defined as.

$$P(x) = \begin{cases} \frac{3}{4} \; for \; 0 \le x \le 1 \\ \frac{\frac{x}{x+2}}{4} for \; 1 < x \le 2 \end{cases} \quad \text{and} \quad Q(x) = \begin{cases} x \; for \; 0 \le x < 1 \\ \frac{1}{2} \; for \; x = 1 \\ \frac{2x}{5} \; for \; 1 < x \le 2 \end{cases}$$

$$S(x) = \begin{cases} x \; for \; 0 \le x \le 1 \\ \frac{3x}{4} \; for \; 1 < x \le 2 \end{cases} \quad \text{and} \quad T(x) = \begin{cases} \frac{3x}{2} \; for \; 0 \le x < 1 \\ \frac{x+1}{5} for \; 1 \le x \le 2 \end{cases}$$

Clearly P(X), Q(X), S(X), and T(X) are closed , and P(X) ⊆ T(X), & Q(X) ⊆ S(X). The seq. $\{x_n\}$ and $\{y_n\}$ are defined by $x_n = 1+ \frac{2}{n+3}$, and $y_n = 1+ \frac{1}{2n}$, in X such that

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Sx_n = \frac{3}{4} \qquad \text{and} \qquad \lim_{n\to\infty} Qy_n = \lim_{n\to\infty} Ty_n = \frac{2}{5}$$

So the pairs (P, S) and (Q,T) satisfy the E.A. property

Here

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Sx_n = \frac{3}{4} = S\left(\frac{3}{4}\right),$$

That is $\frac{3}{4} \in S(X)$, hence pair (P, S) satisfy the (CLRs) property. Again

$$\lim_{n\to\infty} Qy_n = \lim_{n\to\infty} Ty_n = \frac{2}{5} = T(1)$$

That is $\frac{2}{5} \in T(X)$, hence pair (Q, T) satisfy the (CLR_T) property. Thus both pairs (P, S) and (Q, T) enjoys the (CLRs) and (CLR_T) properties with respect to S and T.

Definition 2.12: Imdad et. [10] Two pairs (P, S) and (Q,T) of self-mappings of a menger space (X, M,*) are said to be satisfy the (CLR_{ST}) property with respect to mappings S and T, if there exist two ssequences {x_n} and {y_n} in X such that

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Qy_n = \lim_{n\to\infty} Ty_n = z$$

Where z ϵ S(X) ∩ T(X).

Definition 2.13: Wadhwa & Gupta [8] A – class functions. A mapping $\mathcal{F}$: (0 , 1] × (0 , 1] → R^+ is called A – class function, if it is continuous and satisfies following conditions.

1. $\mathcal{F}$ (x, y) ≥ x
2. $\mathcal{F}$ (x, y) = x ⇒ either x = 1 or y = 1 for all x, y, ϵ (0, 1]
3. $\mathcal{F}$ (1, 1) = 1
4. We denote A – class functions by $\mathcal{F}$.

Definition 2.14: [8] Let φ be a class of all mappings ∅ : [0, 1] → [0, 1] satisfying the condition

($\emptyset_1$) ∅ is continuous and non-decreasing on [0, 1].

($\emptyset_2$) ∅ (x) > x for all x ϵ (o, 1).

($\emptyset_1$) ∅ (0) = 0 and ∅ (1) = 1.

Definition 2.15: [8] Let : [0, 1] → [0, 1] is monotonic increasing function with, ▯(t) > t and

ψ (0) = 0 and (1) = 1.

MAIN THEOREM

Theorem 3.1: Let (X, M, *) be a fuzzy metric space and P, Q, D, M, S, T :X→ are six self mapping with P(X) ⊆ TD(x) and Q(X) ⊆ SM(x)such that

ψ{M(Px ,Qy, t) ≥ $\mathcal{F}${ ψ {M_a (x, y, t), ∅{M_a (x ,y, t)} for all x, y ϵ X where t > 0,

where

M_a(x.y,t) ≥ ∅{min {M(Px, SMx, t) M(Qy, TDy, t),M(Px, TDy, t), M(SMx, Qy, 2t)} --------(1)

Suppose that the pairs (P, SM) and (Q, TD) satisfy (CLR_{SM}), (CLR_{TD}) properties with respect to SM and TD respectively and so (CLR_{SMTD}). Then the pairs (P, SM) and (Q, TD) have a point coincidence in X. And if pairs (P, SM) and (Q, TD) are weakly compatible then P, Q, D, S, M, T have a unique common fixed point.

Proof: Suppose that (X, M, *) is a fuzzy metric space and and P, Q, D, M, S, T:X→X are six self mapping with P(X) ⊆ TD (X) and Q(X) ⊆ SM (X) such that.

Case I : If the pair (P, SM) satisfy the (CLR_{SM}) property, then there exists a sequence {x_n} in X such that

$$\mathrm{Lim}_{n\to\infty} Px_n = \lim_{n\to\infty} SMx_n = z$$

Where $z \in SM(X)$

Since $P(X) \subseteq TD(X)$, then there exists a sequence $\{y_n\} \subset X$ such that $Px_n = TDy_n$, on taking limit

$$\mathrm{Lim}_{n\to\infty} Px_n = \lim_{n\to\infty} TDy_n = z$$

Now we show that $\lim_{n\to\infty} Qy_n = z$

Put $x = x_n$ And $y = y_n$ in (1)

$\psi\, M(Px_n, QY_n,t) \geq \mathcal{F}\{\, \psi\, \{M_a(x_n, y_n , t) , \emptyset\, \{M_a(x_n, y_n, t)\}$

Where

$M_a\,(x_n, y_n, t) \geq \emptyset\, \{\min\{M(Px_n, SMx_n, t) , M(QY_n, TDY_n, t), M(Px_n,SMx_n,t) , M(SMx_n, QY_n, 2t)\}\}$

$> \emptyset\{\min\{M(Px_n, SMx_n, t) , M(QY_n , TDY_n, t), M(Px_n, TDY_n, t) ,$

$M(SMx_n, Px_n, t) * M(Px_n, QY_n, t)\}\}$

$\lim_{n\to\infty} M_a\,(x_n, y_n , t) > \emptyset\{\min\{M(z, z, t), M(QY_n, z, t) , M(z, z, t), M(z, z, t) * M(z, QY_n, t)\}\}$

$= \emptyset\, \{\min\{1, M(QY_n, z, t), 1 , 1 * M(z, QY_n, t)\}\}$

$= \emptyset\, \{M(QY_n, z, t)\}$

$> M(QY_n, z, t)$

Now taking limit in equation (1)

$\psi\, \{lim_{n\to\infty} M(Px_n, QY_n, t)\} \geq \mathcal{F}\{\, \psi\, \{lim_{n\to\infty} M_a\,(x_n, y_n , t) , \emptyset\, \{lim_{n\to\infty} M_a(x_n, y_n, t)\}$

$\psi\, \{M(z, QY_n, t)\} > \mathcal{F}\{\ \{M(QY_n, z, t)\} , \emptyset\{M(QY_n, z, t)\}\} \geq \psi\, \{M(z, QY_n, t)\}$

Thus $\mathcal{F}\{\, \psi\, \{M(z, QY_n, t)\} , \emptyset\{M(z, QY_n, t)\}\} = \psi\, \{M(z, QY_n, t)\}$

Implies that

Either $\psi\, \{M(QY_n, z, t)\} = 1$ or $\emptyset\{M(z, QY_n, t)\} = 1$

$\Rightarrow M(z, QY_n, t) = 1$

Hence $\lim_{n\to\infty} QY_n = z$

Case II : If the pair (Q, TD) satisfy the (CLR_{TD}) property , then we can prove it in the same way.

So the pair (P, SM) and (Q, TD) satisfying the $(CLR_{(SM)(TD)})$, then there exists sequence $\{x_n\}$and $\{Y_n\}$ in X such that

$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} SMx_n = \lim_{n\to\infty} QY_n = \lim_{n\to\infty} TDY_n = z$

where $z \in SM(X) \cap TD(X)$, therefore there exists two points $p, q \in X$ such that

$$z = SM(p) \text{ and } z = TD(q)$$

we have to show that

$P(p) = SM(p) = z$ and $Q(q) = TD(q) = z$

Put $x = x_n$ and $y = q$ in (1)

$\psi\{M(Px_n, Qq, t)\} = \mathcal{F}\{\psi\{M_a(x_n, q, t)\}, = \emptyset\{M_a(x_n, q, t)\}\}$

Where

$M_a(x_n, q, t) \geq \emptyset\{\min\{M(Px_n, SMx_n, t), M(Qq, TDq, t), M(Px_n, TDq, t), M(SMx_n, Qq, 2t)\}\}$

$> \emptyset\{\min\{M(Px_n, SMx_n, t), M(Qq, TDq, t), M(Px_n, TDq, t), M(SMx_n, TDq, t)^* M(TDq, Qq, t)\}\}$

$\lim_{n\to\infty} M_a(x_n, q, t) > \emptyset\{\min\{M(z, z, t), M(Qq, z, t), M(z, z, t), M(z, z, t)^* M(z, Qq, t)\}\}$

$= \emptyset\{\min\{1, M(Qq, z, t), 1, 1^* M(z, Qq, t)\}\}$

$= \emptyset\{M(Qq, z, t)\}$

$> M(Qq, z, t)$

Again taking limit in equation (1)

$\psi\{\lim_{n\to\infty} M(Px_n, Qq, t)\} \geq \mathcal{F}\{\psi\{\lim_{n\to\infty} M_a(x_n, q, t)\}, \emptyset\{\lim_{n\to\infty} M_a(x_n, q, t)\}\}$

$\psi\{M(z, Qq, t)\} > \mathcal{F}\{\psi\{M(Qq, z, t)\}, \emptyset\{M(Qq, z, t)\}\} \geq \psi\{M(z, Qq, t)\}$

Thus, $\mathcal{F}\{\psi\{M(Qq, z, t), \emptyset\{M(Qq, z, t)\}\} = \psi\{M(z, Qq, t)\}$

Implies that

Either $\psi\{M(z, Qq, t)\} = 1$ or $= \emptyset\{M(Qq, z, t)\} = 1$

$$\Rightarrow M(z, Qq, t) = 1$$

Hence $Qq = z$ or $Qq = Tq = z$

So q is a point of coincidence of the pair (Q, TD)

Again we put x = p and y = q in (1)

$\{M(p, q, t)\} \geq \mathcal{F}\{\psi\{M_a(p, q, t)\}, = \emptyset\{M_a(p, q, t)\}\}$

where

$M_a(p, q, t) \geq \emptyset\{\min\{M(Pp, SMp, t), M(Qq, TDq, t), M(Pp, Tq, t), M(SMp, Qq, 2t)\}\}$

$\geq \emptyset$ {min {M(Pp, SMp, t), M(Qq, TDq, t), M(Pp, Tq, t), M(SMp, TDq, t)* M(TDq, Qq, t)}}

$> \emptyset$ {min {M(Pp, z, t), M(z, z, t), M(Pp, z, t), M(z, z, t) * M(z, z, t)}}

$= \emptyset$ {min {M(Pp, z, t), 1 , M(Pp, z, t) , 1*1 }}

$= \emptyset$ {M(Pp, z, t) }

>M(Pp, z, t)

Thus ψ {M (Pp, z, t) } > $\mathcal{F}$ { ψ {M (Pp, z, t) } , $\emptyset${M(Pp, z, t)} $\geq$ ψ {M(Pp, z, t)}

So $\mathcal{F}$ { ψ {M(Pp, z, t)} , $\emptyset$ {M(Pp, z, t)}} = ψ {M(Pp, z, t)}

Implies that

either ψ {M(Pp, z, t)} = 1 or $\emptyset$ {M(Pp, z, t)} =1

$\Rightarrow$ M (Pp, z, t) = 1

$\Rightarrow$ Pp = z or Pp = SMp = z

Hence p is a point of coincidence of the pair (P, SM)

So Pp = SMp = Qq = TDq = z

Since pairs (P, SM) and (Q, TD) are wekley compatible. Then

Pz = SMz = Qz = TDz

Now we show that z is common fixed point of mappings P, Q, D, M, S and T.

We have ψ (M(Pz, z, t) = ψ {M(Pz, Qq, t)} $\geq$ $\mathcal{F}$ { ψ {M_a(z, q, t)}, $\emptyset${ M_a(z, q, t)}}

Where

M_a(z, q, t) $\geq \emptyset${min {M (Pz, SMz, t), M (Qq, TDq, t), M(Pz, TDq, t), M(SMz, Qq, 2t)}}

$> \emptyset${min{M(Pz, SMz, t), M(Qq, TDq, t), M(Pz, TDq, t), M(SMz, TDq, t) * M(TDq, Qq, t)}}

$= \emptyset${min {M(Pz, Pz, t), M(z, z, t), M(Pz, z, t), M(Pz, z, t) * M(z, z, t)}}

$= \emptyset${min {1, 1, M(Pz, z, t), M(Pz, z, t) * 1 }}

$= \emptyset${M(Pz, z, t)}

> M{Pz, z, t)

Thus ψ {M(Pz, z, t)} > $\mathcal{F}$ { ψ {M (Pz, z, t)} , $\emptyset${M(Pz, z, t)}} $\geq$ ψ {M(Pz, z, t)}

So $\mathcal{F}$ { ψ {M (Pz,z,t)} , $\emptyset${M(Pz, z, t)}} = ψ {M(Pz, z, t)}

Implies that

Either ψ {M(Pz, z, t)} =1 or = Ø{M(Pz, z, t)} = 1

⇒ M(Pz, z, t) = 1

Thus Pz = z ⇒ Pz = SMz = z

In the same way

Qz = TDz = z

So Pz = SMz = Qz = TDz = z

Now, by putting x = Mz and y = z in (1)

ψ {M(PMz, Qz, t)≥ $\mathcal{F}$ { ψ (M_a(Mz, z, t) , Ø {M_a(Mz , z , t)}

Where

M_a(Mz, z , t) ≥ ϕ{min{M(PMz, SM(Mz), t) , M(Qz, TDz , t), M(PMz, TDz, t),

M(SM(Mz), Qz, 2t)}}

≥ ϕ{min (Mz, Mz, t) , M (z, z, t) , M (Mz, z, t) . M (Mz, z, t) * M (Mz, Mz, t)

⇒ ϕ{min {M(Mz, Mz, t), 1, M (Mz, z, t), M(Mz, z, t) * 1 }}

= ϕ{min{ 1, 1, M (Mz, z, t) , M (Mz, z, t) *1}}

= ϕ{M(Mz, z, t)}

> M (Mz, z, t)

Thus ψ {M (Mz, z, t)} ≥ $\mathcal{F}$ { { M (Mz, z, t), Ø{M(Mz, z, t)}} ≥ ψ {M(Mz, z, t)

So

$\mathcal{F}$ { ψ {M(Mz, z, t), Ø{M (Mz, z, t)} = ψ {M(Mz, z, t)}

Implies that

Either ψ {M(Mz, z, t)} = 1 or Ø{M (Mz, z, t)} = 1

⇒ M (Mz, z, t) = 1

Thus

Mz = z ⇒ Pz = SMz = Sz = z

Now by putting y = Dz and x = z in (1) , then

ψ {M(Pz, QDz, t) ≥ $\mathcal{F}$ { ψ {M_a (z, Dz, t)}, ∅ {M_a(z, Dz, t)}}

Where

Ma (z, Dz, t) ≥ ∅{min {M(Pz, SMz, t), M(QDz, TD(Dz), t), M(Pz, TD(Dz), t),

M(SMz, QD(Dz), 2t)}}

≥ ∅ {min {M(z, z, t), M(Dz, Dz, t), M(z, Dz, t), M(z,Dz, t) * (Dz, Dz, t)}}

≥ ∅ {min{1, 1, M(z, Dz, t), M(z, Dz, t) * 1}}

= ∅ {M(z, Dz, t)}

> M(z, Dz, t)

Thus

ψ {M (z, Dz, t)} ≥ $\mathcal{F}$ { ψ {M(z, Dz, t)} , ∅{M (z, Dz, t)}} ≥ ψ {M(z, Dz, t)}

So $\mathcal{F}$ { ψ {M (z, Dz, t)}, ∅{M(z, Dz, t)}} = ψ {M(z, Dz, t)}

Implies that

ψ {M (z, Dz, t)} = 1 or = ∅ {M(z, Dz, t)} = 1

⇒ M(z, Dz, t) = 1

Thus Dz = z .

Since TDz = z ⇒ Tz = z .

Therefore Qz = TDz = z ⇒ Qz = z .

By combining the above results , Pz = Sz = Mz = Qz = Tz = z

Thus z is common fixed point of P, Q, D, M, S and T .

Hence z is common fixed point of P, Q, D, M, S and T.

Suppose that w is another fixed point P, Q, D, M, S and T then by eq. (1)

ψ { M(z, w, t) } = ψ { M (Pz, Qw, t)} ≥ $\mathcal{F}${ ψ {M_a (z, w, t)}, ∅{M_a (z, w, t)}}

Where

M_a (z, w, t) ≥ =∅{min {M(Pz, SMz, t) , M(Qw, TDw, t) , M{Pz, TDw, t) ,M(SMz, Qw, 2t)}

>∅{min {M(Pz, SMz, t), M(Qw, TDw, t), M{Pz, TDw, t), M{SMz, Pz, t) * M (Pz, Qw, t)}}

= ∅{min{M(z, z, t), M(w, w, t) , M(z, w, t), M(z, z, t)*, M(z, w, t)}

= Ø{min { 1, 1, M(z, w, t), 1* M(z, w, t)}

= Ø {M(z, w, t)}

> M(z, w, t)

Thus

ψ { M (z,w,t)} = ψ { M(Pz, Qw, t)} > $\mathcal{F}$ { ψ { M(z,w,t)}, Ø{M(z, w, t)}}≥ ψ { M (z, w, t)}

So $\mathcal{F}$ { ψ { M(z, w, t)} , Ø{M(z, w, t)} = ψ {M(z, w, t)

⇒ either ψ { M (z,w,t)} = 1 or Ø {M(z ,w, t)} = 1

⇒ M (z, w, t) = 1

Hence z = w. So common fixed point is unique.

Corollary 3.2 : Let (X, M,*) be a fuzzy metric space and P, Q, D, M, S, T : X⇢X are six self-meppings with P(X) ⊆ TD(X) and Q(X) ⊆ SM(X) such that.

M (Px, Qy, t) ≥ $\mathcal{F}$ { ψ{M_a(x, y, t)}, M_a(x, y, t)} for all x, y, ϵ X where t > 0,

Where

$$M_a(x, y, t) \geq \emptyset \{\min\{M(Px, SMx, t), M(Qy, TDy, t), M(SMx, Qy, t), \frac{M(Px,Qy,t)+M(SMx,TDy,t)}{M(Px,SMx,t)+M(Qy,TDy,t)}\}\}$$

…………..(2)

Suppose that the pairs (P, SM) and (Q, TD) satisfy (CLR_{SM}) , (CLR_{TD}) properties with respect to SM and TD respectively and so ($CLR_{(SM)(TD)}$). Then the pairs (P, SM) and (Q, TD) have a point of coincidence in X. and if pairs (P, SM) and (Q, TD) are weakly compatible. Then P, Q, D, M, S, T have a unique common fixed point.

Corollary 3.3: Let (X, M,*) be a fuzzy metric space and P, S, M X→X are three self-mappings with following condition.

M(Px, Py, t) ≥ $\mathcal{F}$ {M_a(x, y, t) , Ø {M_a(x, y, t)}} for all x, y, ϵ X where t > 0,

Where

M_a(x, y, t) ≥ Ø{{min M(Px, SMx, t), M(Py, SMy, t) , M(Px, SMy, t), M(SMx, Py, 2t)}}
……………….(3)

Suppose that the pair (P, SM) satisfy (CLR_{SM}) property. Then the pair (P, SM) has a point of coincidence in X. and if pair (P, SM) is weakly compatible . Then P, S and M have a unique common fixed point.

References:

1. Aamri,M.andMoutawakil, D."Some new common fixed point theorems under strict Contractive conditions",J. Math. Anal. Appl., 270(2002), 181-188.
2. George A. and Veeramani P., " On some results in fuzzy metric spaces and fuzzy sets and system" vol. 64 (1994) 395-399.
3. Jungck G., "Compatible mappings and common fixed points."Journal of mathematics & Maths Sci. vol.9 no. 4(1986) 771-779.
4. Jungck G. and Rhoades B.E.,"fixed points for set valued functions without continuity,"Indian Journal of pureand applied mathematics, Vol.29,pp.227- 238,1998.
5. Zadeh I.A., fuzzy sets, Inform.and control,, 8(1965) 338-353.
6. Kromosil I., and Michelek J., "fuzzy metric space and statistical metric space ", kybernetica Vol. 11, pp.326-334, 1975.
7. Wadhwa K. & Dubey A., "some common fixed point theorems in fuzzy metric space by using CLRg property"Global journal of pure and applied mathematics (2017),pp 6327-6342
8. Wadhwa K. and Gupta A. K.,"Common fixed point theorems for four self-mappings satisfying (E.A.) property via A-class functions in fuzzy metric space." Ref. No. IJMA-12043.(2021) ISSN 2229-5046.
9. Abbas M., Ishak Altun and Gopal D., "Common fixed point theorems for noncompatible mappings in fuzzy metric spaces" Bull. of Mathematical analysis and applications. ISSN: 1821-1291, VOL. 1 Issue2,(2009) 47-56.
10. Imdad M., Pant B.D. and Chauhan S., " Fixed point theorems in menger spaces using the (CLR_{ST}) property and applications"Journal of Nonlinear Analysis and Optimization vol.3, no. 2(2012), 225-237.
11. Imdad M., Ali J., and Bahuguna D., " Fixed point theorems in menger spaces with common property (E.A.)" computer and Mathematics with applications 60(2010) 3152-3159.
12. Sintunavarat W . and Kumam P., "Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric space." Hindwani publishing corporation journal of applied mathematics, Vol. 2011,1-14.
13. Liu Y., et al." Common fixed points of single-valued and multivalued maps." Int.J.Math. Sci., 19(2005). 3045-3055.DOI:10.1155/IJMMS.
14. Wadhwa K. and Gupta A. K., "Common fixed point theorems for four self-mappings satisfying (CLR_{ST}) property via A-class functions in fuzzy metric space". Global Journal of Pure and Applied Mathematics, ISSN 0973 – 1768 Volume 17, Number 2(2021), pp. 319-332.