

Evaluation of Reliability Parameters of a Sugar Manufacturing Plant

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Abstract:

In this paper, Investigations have been carried out for the evaluation of reliability in three different cases i.e., if the reliability of each component of the complex system is R , if failure rates follow Weibull time distribution & Exponential distribution and MTTF of a sugar manufacturing plant by using Boolean function technique. The sugar plant under consideration is a complex system which consists of various subsystems viz., washing, sugar cane cutter, crushing system, Defecation, Filtration, Evaporation, Crystallization, centrifugal, refining. The units such as Defecation, Filtration, Evaporation, centrifugal have two states.

Keywords: Algebra of logic, Boolean function technique, Exponential time distribution, Weibull time distribution, Reliability, Mean time to failure.

1. Introduction:

The considered sugar manufacturing plant consists of nine subsystems. The first subsystem is washing and it clearly wash the cane sugar without any dust. The second subsystem is sugar cane cutter, where cutters start cutting of cane in fine small pieces and then the pieces go to the third subsystem namely crushing subsystem, the sugar cane juice will be extracted. This juice goes into the subsystems like defecation, filtration, evaporation, crystallization, centrifugal. At the end the crystals of sugar enter into refining unit. Finally, we get crystal clear sugar as an output.

2. Assumptions:

1. The reliabilities of all constituent components of the system are known.
2. The states of all components are independent.
3. The state of each component and of the whole system is either good or bad
4. The failure times of all components are arbitrary.
5. All components are always operating (no stand by or switched redundancy)
6. There is no repair facility.

3. Notations:

x_1 : State of Washing

x_2 : State of sugar cane cutter

x_3 : State of crushing system

x_4, x_5 : State of Defecation

x_6, x_7 : State of Filtration

x_8, x_9 : State of Evaporation

x_{10} : State of crystallization

x_{11}, x_{12} : State of centrifugal

x_{13} : State of refining

\wedge : Conjunction

$x_i = \begin{cases} 0 & \text{in bad state} \\ 1 & \text{in good state} \end{cases}$

x_i^1 Negation of x_i

R_i Reliability of component corresponding to system state x_i

$Q_i = 1 - R_i$ Unreliability of component corresponding to system state x_i

R_s Reliability of the system as a whole

$R_{SW}(t) / R_{SE}(t)$ Reliability function when failure follow Weibull /Exponential time distribution.

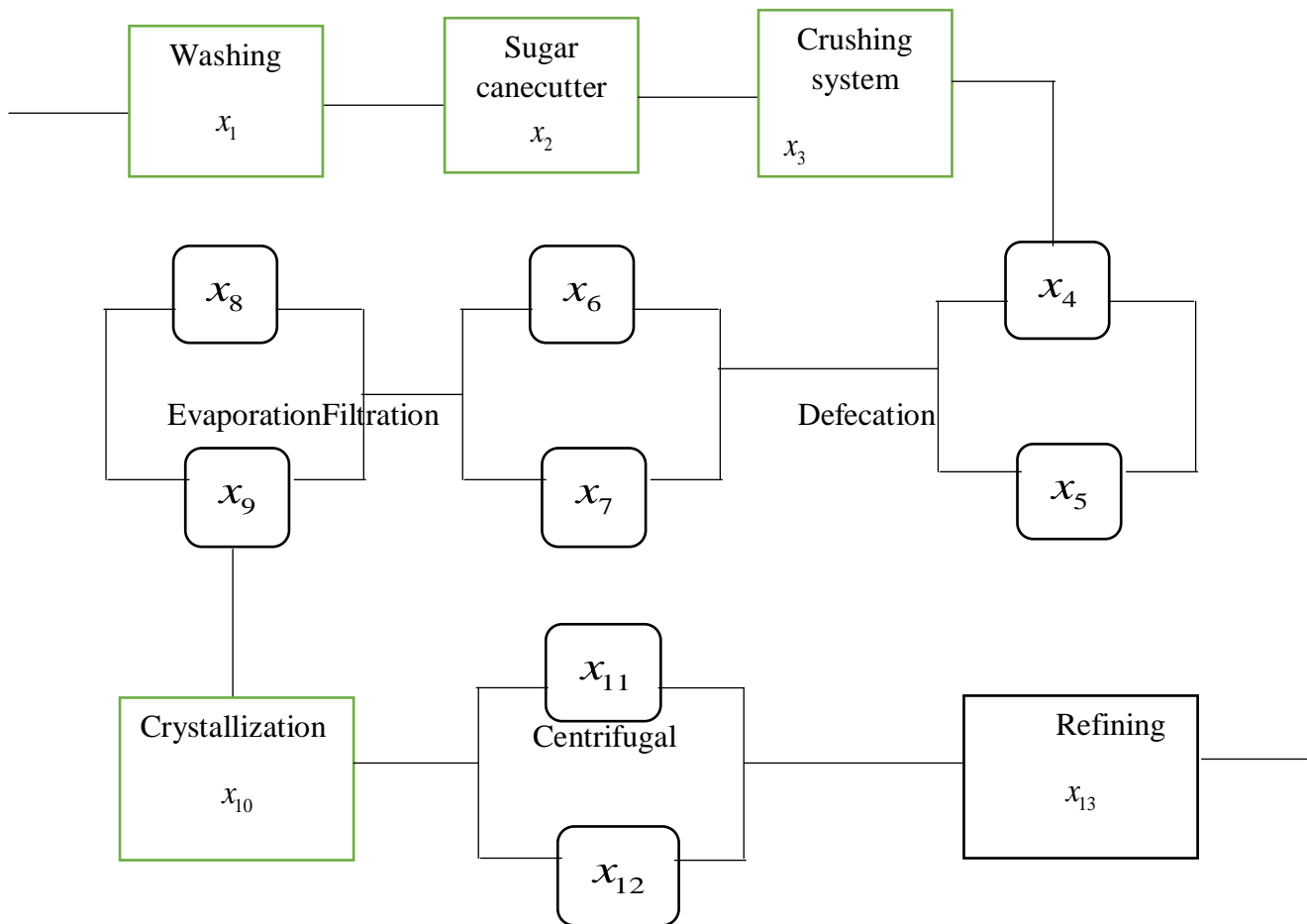


Fig.1

4. FORMATION OF MATHEMATICAL MODEL:

By using Boolean Function Technique, the conditions of capability for the successful operation of the system in terms of logical matrix are expressed as:

$$F(x_1, x_2, x_3, x_4, \dots, x_{13}) = \begin{array}{|cccccccc|} \hline x_1 & x_2 & x_3 & x_4 & x_6 & x_8 & x_{10} & x_{11} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_4 & x_6 & x_8 & x_{10} & x_{12} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_5 & x_6 & x_8 & x_{10} & x_{11} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_5 & x_6 & x_8 & x_{10} & x_{12} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_4 & x_7 & x_8 & x_{10} & x_{11} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_4 & x_7 & x_8 & x_{10} & x_{12} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_5 & x_7 & x_8 & x_{10} & x_{11} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_5 & x_7 & x_8 & x_{10} & x_{12} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_4 & x_6 & x_9 & x_{10} & x_{11} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_4 & x_6 & x_9 & x_{10} & x_{12} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_5 & x_6 & x_9 & x_{10} & x_{11} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_5 & x_6 & x_9 & x_{10} & x_{12} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_4 & x_7 & x_9 & x_{10} & x_{11} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_4 & x_7 & x_9 & x_{10} & x_{12} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_5 & x_7 & x_9 & x_{10} & x_{11} & x_{13} \\ \hline x_1 & x_2 & x_3 & x_5 & x_7 & x_9 & x_{10} & x_{12} & x_{13} \\ \hline \end{array} \quad (1)$$

5. SOLUTION OF THE MODEL:

By the application of algebra of logic, equation (1) may be written as

$$F(x_1, x_2, x_3, x_4, \dots, x_{13}) = [x_1 x_2 x_3 x_7 x_{13} \wedge f(x_4, x_5, x_6, x_8, x_9, x_{10}, x_{11}, x_{12})] \quad (2)$$

Where

$$f(x_4, x_5, x_6, x_8, x_9, x_{10}, x_{11}, x_{12}) = \begin{vmatrix} x_4 & x_6 & x_8 & x_{11} \\ x_4 & x_6 & x_8 & x_{12} \\ x_5 & x_6 & x_8 & x_{11} \\ x_5 & x_6 & x_8 & x_{12} \\ x_4 & x_7 & x_8 & x_{11} \\ x_4 & x_7 & x_8 & x_{12} \\ x_5 & x_7 & x_8 & x_{11} \\ x_5 & x_7 & x_8 & x_{12} \\ x_4 & x_6 & x_9 & x_{11} \\ x_4 & x_6 & x_9 & x_{12} \\ x_5 & x_6 & x_9 & x_{11} \\ x_5 & x_6 & x_9 & x_{12} \\ x_4 & x_7 & x_9 & x_{11} \\ x_4 & x_7 & x_9 & x_{12} \\ x_5 & x_7 & x_9 & x_{11} \\ x_5 & x_7 & x_9 & x_{12} \end{vmatrix} = \begin{vmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \\ K_5 \\ K_6 \\ K_7 \\ K_8 \\ K_9 \\ K_{10} \\ K_{11} \\ K_{12} \\ K_{13} \\ K_{14} \\ K_{15} \\ K_{16} \end{vmatrix} \quad (3)$$

Where,

$$K_1 = |x_4 \ x_6 \ x_8 \ x_{11}| \quad (4) \quad K_9 = |x_4 \ x_6 \ x_9 \ x_{11}| \quad (12)$$

$$K_2 = |x_4 \ x_6 \ x_8 \ x_{12}| \quad (5) \quad K_{10} = |x_4 \ x_6 \ x_9 \ x_{12}| \quad (13)$$

$$K_3 = |x_5 \ x_6 \ x_8 \ x_{11}| \quad (6) \quad K_{11} = |x_5 \ x_6 \ x_9 \ x_{11}| \quad (14)$$

$$K_4 = |x_5 \ x_6 \ x_8 \ x_{12}| \quad (7) \quad K_{12} = |x_5 \ x_6 \ x_9 \ x_{12}| \quad (15)$$

$$K_5 = |x_4 \ x_7 \ x_8 \ x_{11}| \quad (8) \quad K_{13} = |x_4 \ x_7 \ x_9 \ x_{11}| \quad (16)$$

$$K_6 = |x_4 \ x_7 \ x_8 \ x_{12}| \quad (9) \quad K_{14} = |x_4 \ x_7 \ x_9 \ x_{12}| \quad (17)$$

$$K_7 = |x_5 \ x_7 \ x_8 \ x_{11}| \quad (10) \quad K_{15} = |x_5 \ x_7 \ x_9 \ x_{11}| \quad (18)$$

$$K_8 = |x_5 \ x_7 \ x_8 \ x_{12}| \quad (11) \quad K_{16} = |x_5 \ x_7 \ x_9 \ x_{12}| \quad (19)$$

Using orthogonalization algorithm equation (3) may be written as:

$$f(x_4, x_5, x_6, x_8, x_9, x_{10}, x_{11}, x_{12}) = \begin{vmatrix} K_1 \\ K_1^1 & K_2 \\ K_1^1 & K_2^1 & K_3 \\ K_1^1 & K_2^1 & K_3^1 & K_4 \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5 \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6 \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6^1 & K_7 \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6^1 & K_7^1 & K_8 \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6^1 & K_7^1 & K_8^1 & K_9 \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6^1 & K_7^1 & K_8^1 & K_9^1 & K_{10} \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6^1 & K_7^1 & K_8^1 & K_9^1 & K_{10}^1 & K_{11} \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6^1 & K_7^1 & K_8^1 & K_9^1 & K_{10}^1 & K_{11}^1 & K_{12} \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6^1 & K_7^1 & K_8^1 & K_9^1 & K_{10}^1 & K_{11}^1 & K_{12}^1 & K_{13} \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6^1 & K_7^1 & K_8^1 & K_9^1 & K_{10}^1 & K_{11}^1 & K_{12}^1 & K_{13}^1 & K_{14} \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6^1 & K_7^1 & K_8^1 & K_9^1 & K_{10}^1 & K_{11}^1 & K_{12}^1 & K_{13}^1 & K_{14}^1 & K_{15} \\ K_1^1 & K_2^1 & K_3^1 & K_4^1 & K_5^1 & K_6^1 & K_7^1 & K_8^1 & K_9^1 & K_{10}^1 & K_{11}^1 & K_{12}^1 & K_{13}^1 & K_{14}^1 & K_{15}^1 & K_{16} \end{vmatrix} \quad (20)$$

Now using algebra of logic, one can get

$$K_1^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_6^1 \\ x_4 & x_6 & x_8^1 \\ x_4 & x_6 & x_8 & x_{11}^1 \end{vmatrix} \quad (21) \quad K_2^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_6^1 \\ x_4 & x_6 & x_8^1 \\ x_4 & x_6 & x_8 & x_{12}^1 \end{vmatrix} \quad (22)$$

$$K_3^1 = \begin{vmatrix} x_5^1 \\ x_5 & x_6^1 \\ x_5 & x_6 & x_8^1 \\ x_5 & x_6 & x_8 & x_{11}^1 \end{vmatrix} \quad (23) \quad K_4^1 = \begin{vmatrix} x_5^1 \\ x_5 & x_6^1 \\ x_5 & x_6 & x_8^1 \\ x_5 & x_6 & x_8 & x_{12}^1 \end{vmatrix} \quad (24)$$

$$K_5^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_7^1 \\ x_4 & x_7 & x_8^1 \\ x_4 & x_7 & x_8 & x_{11}^1 \end{vmatrix} \quad (25) \quad K_6^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_7^1 \\ x_4 & x_7 & x_8^1 \\ x_4 & x_7 & x_8 & x_{12}^1 \end{vmatrix} \quad (26)$$

$$K_7^1 = \begin{vmatrix} x_5^1 \\ x_5 & x_7^1 \\ x_5 & x_7 & x_8^1 \\ x_5 & x_7 & x_8 & x_{11}^1 \end{vmatrix} \quad (27) \quad K_8^1 = \begin{vmatrix} x_5^1 \\ x_5 & x_7^1 \\ x_5 & x_7 & x_8^1 \\ x_5 & x_7 & x_8 & x_{12}^1 \end{vmatrix} \quad (28)$$

$$K_9^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_6^1 \\ x_4 & x_6 & x_9^1 \\ x_4 & x_6 & x_9 & x_{11}^1 \end{vmatrix} \quad (29) \quad K_{10}^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_6^1 \\ x_4 & x_6 & x_9^1 \\ x_4 & x_6 & x_9 & x_{12}^1 \end{vmatrix} \quad (30)$$

$$K_{11}^1 = \begin{vmatrix} x_5^1 \\ x_5 & x_6^1 \\ x_5 & x_6 & x_9^1 \\ x_5 & x_6 & x_9 & x_{11}^1 \end{vmatrix} \quad (31) \quad K_{12}^1 = \begin{vmatrix} x_5^1 \\ x_5 & x_6^1 \\ x_5 & x_6 & x_9^1 \\ x_5 & x_6 & x_9 & x_{12}^1 \end{vmatrix} \quad (32)$$

$$K_{13}^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_7^1 \\ x_4 & x_7 & x_9^1 \\ x_4 & x_7 & x_9 & x_{11}^1 \end{vmatrix} \quad (33) \quad K_{14}^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_7^1 \\ x_4 & x_7 & x_9^1 \\ x_4 & x_7 & x_9 & x_{12}^1 \end{vmatrix} \quad (34)$$

$$K_{15}^1 = \begin{vmatrix} x_5^1 \\ x_5 & x_7^1 \\ x_5 & x_7 & x_9^1 \\ x_5 & x_7 & x_9 & x_{11}^1 \end{vmatrix} \quad (35) \quad K_{16}^1 = \begin{vmatrix} x_5^1 \\ x_5 & x_7^1 \\ x_5 & x_7 & x_9^1 \\ x_5 & x_7 & x_9 & x_{12}^1 \end{vmatrix} \quad (36)$$

$$K_1^1 K_2^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_6^1 \\ x_4 & x_6 & x_8^1 \\ x_4 & x_6 & x_8 & x_{11}^1 \end{vmatrix} \wedge \begin{vmatrix} x_4 & x_6 & x_8 & x_{12} \end{vmatrix} = \begin{vmatrix} x_4 & x_6 & x_8 & x_{11} & x_{12} \end{vmatrix} \quad (37)$$

$$K_1^1 K_2^1 K_3^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_6^1 \\ x_4 & x_6 & x_8^1 \\ x_4 & x_6 & x_8 & x_{11}^1 \end{vmatrix} \wedge \begin{vmatrix} x_4^1 & x_5 & x_6 & x_8 & x_{11} \\ x_4 & x_5 & x_6 & x_8 & x_{11} & x_{12}^1 \end{vmatrix}$$

$$= \begin{vmatrix} x_4^1 & x_5 & x_6 & x_8 & x_{11} \end{vmatrix} \quad (38)$$

$$K_1^1 K_2^1 K_3^1 K_4^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_6^1 \\ x_4 & x_6 & x_8^1 \\ x_4 & x_6 & x_8 & x_{11}^1 \end{vmatrix} \wedge \begin{vmatrix} x_4^1 & x_5 & x_6 & x_8 & x_{11} & x_{12} \end{vmatrix}$$

$$= \begin{vmatrix} x_4^1 & x_5 & x_6 & x_8 & x_{11} & x_{12} \end{vmatrix} \quad (39)$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_6^1 \\ x_4 & x_6 & x_8^1 \\ x_4 & x_6 & x_8 & x_{11}^1 \end{vmatrix} \wedge \begin{vmatrix} x_4 & x_5^1 & x_6^1 & x_7 & x_8 & x_{11} \\ x_4 & x_5 & x_6^1 & x_7 & x_8 & x_{11} \\ x_4 & x_5^1 & x_6 & x_7 & x_8 & x_{11} & x_{12}^1 \end{vmatrix}$$

$$= \begin{vmatrix} x_4 & x_5^1 & x_6^1 & x_7 & x_8 & x_{11} \\ x_4 & x_5 & x_6^1 & x_7 & x_8 & x_{11} \end{vmatrix} \quad (40)$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_6^1 \\ x_4 & x_6 & x_8^1 \\ x_4 & x_6 & x_8 & x_{11}^1 \end{vmatrix} \wedge \begin{vmatrix} x_4 & x_5^1 & x_6^1 & x_7 & x_8 & x_{11}^1 & x_{12} \\ x_4 & x_5 & x_6^1 & x_7 & x_8 & x_{11}^1 & x_{12} \end{vmatrix}$$

$$= \begin{vmatrix} x_4 & x_5^1 & x_6^1 & x_7 & x_8 & x_{11}^1 & x_{12} \\ x_4 & x_5 & x_6^1 & x_7 & x_8 & x_{11}^1 & x_{12} \end{vmatrix} \quad (41)$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 = \begin{vmatrix} x_4^1 \\ x_4 & x_6^1 \\ x_4 & x_6 & x_8^1 \\ x_4 & x_6 & x_8 & x_{11}^1 \end{vmatrix} \wedge \begin{vmatrix} x_4^1 & x_5 & x_6^1 & x_7 & x_8 & x_{11} \end{vmatrix}$$
$$= \begin{vmatrix} x_4^1 & x_5 & x_6^1 & x_7 & x_8 & x_{11} \end{vmatrix} \quad (42)$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1$$
$$= \begin{vmatrix} x_4^1 \\ x_4 & x_6^1 \\ x_4 & x_6 & x_8^1 \\ x_4 & x_6 & x_8 & x_{11}^1 \end{vmatrix} \wedge \begin{vmatrix} x_4^1 & x_5 & x_6^1 & x_7 & x_8 & x_{11}^1 & x_{12} \end{vmatrix}$$
$$= \begin{vmatrix} x_4^1 & x_5 & x_6^1 & x_7 & x_8 & x_{11}^1 & x_{12} \end{vmatrix} \quad (43)$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9^1$$
$$= \begin{vmatrix} x_4^1 \\ x_4 & x_6^1 \\ x_4 & x_6 & x_8^1 \\ x_4 & x_6 & x_8 & x_{11}^1 \end{vmatrix} \wedge \begin{vmatrix} x_4 & x_5^1 & x_6 & x_7^1 & x_8^1 & x_9 & x_{11} \\ x_4 & x_5^1 & x_6 & x_7 & x_8^1 & x_9 & x_{11} \\ x_4 & x_5 & x_6 & x_7^1 & x_8^1 & x_9 & x_{11} \\ x_4 & x_5 & x_6 & x_7 & x_8^1 & x_9 & x_{11} \\ x_4 & x_5^1 & x_6 & x_7^1 & x_8 & x_9 & x_{11} & x_{12}^1 \end{vmatrix}$$
$$= \begin{vmatrix} x_4 & x_5^1 & x_6 & x_7^1 & x_8^1 & x_9 & x_{11} \\ x_4 & x_5^1 & x_6 & x_7 & x_8^1 & x_9 & x_{11} \\ x_4 & x_5 & x_6 & x_7^1 & x_8^1 & x_9 & x_{11} \\ x_4 & x_5 & x_6 & x_7 & x_8^1 & x_9 & x_{11} \end{vmatrix} \quad (44)$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9^1 K_{10}^1$$

$$\begin{aligned}
 &= \left| \begin{array}{cccc} x_4^1 & & & \\ x_4 & x_6^1 & & \\ x_4 & x_6 & x_8^1 & \\ x_4 & x_6 & x_8 & x_{11}^1 \end{array} \right| \wedge \left| \begin{array}{cccccccc} x_4 & x_5^1 & x_6 & x_7^1 & x_8^1 & x_9 & x_{11}^1 & x_{12} \\ x_4 & x_5^1 & x_6 & x_7 & x_8^1 & x_9 & x_{11}^1 & x_{12} \\ x_4 & x_5 & x_6 & x_7^1 & x_8^1 & x_9 & x_{11}^1 & x_{12} \\ x_4 & x_5 & x_6 & x_7 & x_8^1 & x_9 & x_{11}^1 & x_{12} \end{array} \right| \\
 &= \left| \begin{array}{cccccccc} x_4 & x_5^1 & x_6 & x_7^1 & x_8^1 & x_9 & x_{11}^1 & x_{12} \\ x_4 & x_5^1 & x_6 & x_7 & x_8^1 & x_9 & x_{11}^1 & x_{12} \\ x_4 & x_5 & x_6 & x_7^1 & x_8^1 & x_9 & x_{11}^1 & x_{12} \\ x_4 & x_5 & x_6 & x_7 & x_8^1 & x_9 & x_{11}^1 & x_{12} \end{array} \right| \quad (45)
 \end{aligned}$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9^1 K_{10}^1 K_{11}^1$$

$$\begin{aligned}
 &= \left| \begin{array}{cccc} x_4^1 & & & \\ x_4 & x_6^1 & & \\ x_4 & x_6 & x_8^1 & \\ x_4 & x_6 & x_8 & x_{11}^1 \end{array} \right| \wedge \left| \begin{array}{cccccc} x_4^1 & x_5 & x_6 & x_7^1 & x_8^1 & x_9 & x_{11} \\ x_4^1 & x_5 & x_6 & x_7 & x_8^1 & x_9 & x_{11} \end{array} \right| \\
 &= \left| \begin{array}{cccccc} x_4^1 & x_5 & x_6 & x_7^1 & x_8^1 & x_9 & x_{11} \\ x_4^1 & x_5 & x_6 & x_7 & x_8^1 & x_9 & x_{11} \end{array} \right| \quad (46)
 \end{aligned}$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9^1 K_{10}^1 K_{11}^1 K_{12}^1$$

$$\begin{aligned}
 &= \left| \begin{array}{cccc} x_4^1 & & & \\ x_4 & x_6^1 & & \\ x_4 & x_6 & x_8^1 & \\ x_4 & x_6 & x_8 & x_{11}^1 \end{array} \right| \wedge \left| \begin{array}{cccccccc} x_4^1 & x_5 & x_6 & x_7^1 & x_8^1 & x_9 & x_{11}^1 & x_{12} \\ x_4^1 & x_5 & x_6 & x_7 & x_8^1 & x_9 & x_{11}^1 & x_{12} \end{array} \right| \\
 &= \left| \begin{array}{cccccccc} x_4^1 & x_5 & x_6 & x_7^1 & x_8^1 & x_9 & x_{11}^1 & x_{12} \\ x_4^1 & x_5 & x_6 & x_7 & x_8^1 & x_9 & x_{11}^1 & x_{12} \end{array} \right| \quad (47)
 \end{aligned}$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9^1 K_{10}^1 K_{11}^1 K_{12}^1 K_{13}^1$$

$$\begin{aligned}
 &= \left| \begin{array}{cccc} x_4^1 & & & \\ x_4 & x_6^1 & & \\ x_4 & x_6 & x_8^1 & \\ x_4 & x_6 & x_8 & x_{11}^1 \end{array} \right| \wedge \left| \begin{array}{cccccc} x_4 & x_5^1 & x_6^1 & x_7 & x_8^1 & x_9 & x_{11} \\ x_4 & x_5 & x_6^1 & x_7 & x_8^1 & x_9 & x_{11} \end{array} \right| \\
 &= \left| \begin{array}{cccccc} x_4 & x_5^1 & x_6^1 & x_7 & x_8^1 & x_9 & x_{11} \\ x_4 & x_5 & x_6^1 & x_7 & x_8^1 & x_9 & x_{11} \end{array} \right| \quad (48)
 \end{aligned}$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9^1 K_{10}^1 K_{11}^1 K_{12}^1 K_{13}^1 K_{14}^1$$

$$\begin{aligned}
 &= \left| \begin{array}{cccc} x_4^1 & & & \\ x_4 & x_6^1 & & \\ x_4 & x_6 & x_8^1 & \\ x_4 & x_6 & x_8 & x_{11}^1 \end{array} \right| \wedge \left| \begin{array}{cccccc} x_4 & x_5^1 & x_6^1 & x_7 & x_8^1 & x_9 & x_{11}^1 & x_{12} \\ x_4 & x_5 & x_6^1 & x_7 & x_8^1 & x_9 & x_{11}^1 & x_{12} \end{array} \right| \\
 &= \left| \begin{array}{cccccc} x_4 & x_5^1 & x_6^1 & x_7 & x_8^1 & x_9 & x_{11}^1 & x_{12} \\ x_4 & x_5 & x_6^1 & x_7 & x_8^1 & x_9 & x_{11}^1 & x_{12} \end{array} \right| \quad (49)
 \end{aligned}$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9^1 K_{10}^1 K_{11}^1 K_{12}^1 K_{13}^1 K_{14}^1 K_{15}^1$$

$$\begin{aligned}
 &= \left| \begin{array}{cccc} x_4^1 & & & \\ x_4 & x_6^1 & & \\ x_4 & x_6 & x_8^1 & \\ x_4 & x_6 & x_8 & x_{11}^1 \end{array} \right| \wedge \left| \begin{array}{cccc} x_4^1 & x_5 & x_6^1 & x_7 & x_8^1 & x_9 & x_{11} \end{array} \right| \\
 &= \left| \begin{array}{cccc} x_4^1 & x_5 & x_6^1 & x_7 & x_8^1 & x_9 & x_{11} \end{array} \right| \quad (50)
 \end{aligned}$$

$$K_1^1 K_2^1 K_3^1 K_4^1 K_5^1 K_6^1 K_7^1 K_8^1 K_9^1 K_{10}^1 K_{11}^1 K_{12}^1 K_{13}^1 K_{14}^1 K_{15}^1 K_{16}^1$$

$$\begin{aligned}
 &= \left| \begin{array}{cccc} x_4^1 & & & \\ x_4 & x_6^1 & & \\ x_4 & x_6 & x_8^1 & \\ x_4 & x_6 & x_8 & x_{11}^1 \end{array} \right| \wedge \left| \begin{array}{cccccc} x_4^1 & x_5 & x_6^1 & x_7 & x_8^1 & x_9 & x_{11}^1 & x_{12} \end{array} \right|
 \end{aligned}$$

$$= \left| x_4^1 \quad x_5 \quad x_6^1 \quad x_7 \quad x_8^1 \quad x_9 \quad x_{11}^1 \quad x_{12} \right| \quad (51)$$

Making use of equations (37) to (51) in (20), one can get equation (52) and from (2) one can get (53)

$$f(x_4, x_5, x_6, x_8, x_9, x_{10}, x_{11}, x_{12}) = \begin{array}{|c|} \hline x_4 \quad x_6 \quad x_8 \quad x_{11} \\ x_4 \quad x_6 \quad x_8 \quad x_{11}^1 \quad x_{12} \\ x_4^1 \quad x_5 \quad x_6 \quad x_8 \quad x_{11} \\ x_4^1 \quad x_5 \quad x_6 \quad x_8 \quad x_{11}^1 \quad x_{12} \\ x_4 \quad x_5^1 \quad x_6^1 \quad x_7 \quad x_8 \quad x_{11} \\ x_4 \quad x_5 \quad x_6^1 \quad x_7 \quad x_8 \quad x_{11} \\ x_4 \quad x_5^1 \quad x_6^1 \quad x_7 \quad x_8 \quad x_{11}^1 \quad x_{12} \\ x_4 \quad x_5 \quad x_6^1 \quad x_7 \quad x_8 \quad x_{11}^1 \quad x_{12} \\ x_4^1 \quad x_5 \quad x_6^1 \quad x_7 \quad x_8 \quad x_{11} \\ x_4^1 \quad x_5 \quad x_6^1 \quad x_7 \quad x_8 \quad x_{11}^1 \quad x_{12} \\ x_4 \quad x_5^1 \quad x_6 \quad x_7^1 \quad x_8^1 \quad x_9 \quad x_{11} \\ x_4 \quad x_5^1 \quad x_6 \quad x_7 \quad x_8^1 \quad x_9 \quad x_{11} \\ x_4 \quad x_5 \quad x_6 \quad x_7^1 \quad x_8^1 \quad x_9 \quad x_{11} \\ x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8^1 \quad x_9 \quad x_{11} \\ x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8^1 \quad x_9 \quad x_{11} \\ x_4^1 \quad x_5 \quad x_6 \quad x_7^1 \quad x_8^1 \quad x_9 \quad x_{11} \\ x_4^1 \quad x_5 \quad x_6 \quad x_7 \quad x_8^1 \quad x_9 \quad x_{11} \\ x_4^1 \quad x_5 \quad x_6 \quad x_7^1 \quad x_8^1 \quad x_9 \quad x_{11}^1 \quad x_{12} \\ x_4^1 \quad x_5 \quad x_6 \quad x_7 \quad x_8^1 \quad x_9 \quad x_{11}^1 \quad x_{12} \\ x_4 \quad x_5^1 \quad x_6^1 \quad x_7 \quad x_8^1 \quad x_9 \quad x_{11} \\ x_4 \quad x_5 \quad x_6^1 \quad x_7 \quad x_8^1 \quad x_9 \quad x_{11} \\ x_4 \quad x_5^1 \quad x_6^1 \quad x_7 \quad x_8^1 \quad x_9 \quad x_{11}^1 \quad x_{12} \\ x_4 \quad x_5 \quad x_6^1 \quad x_7 \quad x_8^1 \quad x_9 \quad x_{11}^1 \quad x_{12} \\ x_4^1 \quad x_5 \quad x_6^1 \quad x_7 \quad x_8^1 \quad x_9 \quad x_{11} \\ x_4^1 \quad x_5 \quad x_6^1 \quad x_7 \quad x_8^1 \quad x_9 \quad x_{11}^1 \quad x_{12} \\ \hline \end{array} \quad (52)$$

$$F(x_1, x_2, x_3, x_4, \dots, x_{13}) = \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_6 & x_8 & x_{10} & x_{11} & x_{13} & & & & & \\ x_1 & x_2 & x_3 & x_4 & x_6 & x_8 & x_{10} & x_{11}^1 & x_{12} & x_{13} & & & & & \\ x_1 & x_2 & x_3 & x_4^1 & x_5 & x_6 & x_8 & x_{10} & x_{11} & x_{13} & & & & & \\ x_1 & x_2 & x_3 & x_4^1 & x_5 & x_6 & x_8 & x_{10} & x_{11}^1 & x_{12} & x_{13} & & & & \\ x_1 & x_2 & x_3 & x_4 & x_5^1 & x_6^1 & x_7 & x_8 & x_{10} & x_{11} & x_{13} & & & & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6^1 & x_7 & x_8 & x_{10} & x_{11} & x_{13} & & & & \\ x_1 & x_2 & x_3 & x_4 & x_5^1 & x_6^1 & x_7 & x_8 & x_{10} & x_{11}^1 & x_{12} & x_{13} & & & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6^1 & x_7 & x_8 & x_{10} & x_{11}^1 & x_{12} & x_{13} & & & \\ x_1 & x_2 & x_3 & x_4^1 & x_5 & x_6^1 & x_7 & x_8 & x_{10} & x_{11} & x_{13} & & & & \\ x_1 & x_2 & x_3 & x_4^1 & x_5 & x_6^1 & x_7 & x_8 & x_{10} & x_{11}^1 & x_{12} & x_{13} & & & \\ x_1 & x_2 & x_3 & x_4 & x_5^1 & x_6 & x_7^1 & x_8^1 & x_9 & x_{10} & x_{11} & x_{13} & & & \\ x_1 & x_2 & x_3 & x_4 & x_5^1 & x_6 & x_7 & x_8^1 & x_9 & x_{10} & x_{11} & x_{13} & & & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7^1 & x_8^1 & x_9 & x_{10} & x_{11} & x_{13} & & & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8^1 & x_9 & x_{10} & x_{11}^1 & x_{12} & x_{13} & & \\ x_1 & x_2 & x_3 & x_4 & x_5^1 & x_6 & x_7 & x_8^1 & x_9 & x_{10} & x_{11}^1 & x_{12} & x_{13} & & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7^1 & x_8^1 & x_9 & x_{10} & x_{11}^1 & x_{12} & x_{13} & & \\ x_1 & x_2 & x_3 & x_4^1 & x_5 & x_6 & x_7 & x_8^1 & x_9 & x_{10} & x_{11} & x_{13} & & & \\ x_1 & x_2 & x_3 & x_4^1 & x_5 & x_6 & x_7^1 & x_8^1 & x_9 & x_{10} & x_{11}^1 & x_{12} & x_{13} & & \\ x_1 & x_2 & x_3 & x_4^1 & x_5 & x_6 & x_7 & x_8^1 & x_9 & x_{10} & x_{11}^1 & x_{12} & x_{13} & & \\ x_1 & x_2 & x_3 & x_4 & x_5^1 & x_6^1 & x_7 & x_8^1 & x_9 & x_{10} & x_{11} & x_{13} & & & \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6^1 & x_7 & x_8^1 & x_9 & x_{10} & x_{11}^1 & x_{12} & x_{13} & & \\ x_1 & x_2 & x_3 & x_4^1 & x_5 & x_6^1 & x_7 & x_8^1 & x_9 & x_{10} & x_{11} & x_{13} & & & \\ x_1 & x_2 & x_3 & x_4^1 & x_5 & x_6^1 & x_7 & x_8^1 & x_9 & x_{10} & x_{11}^1 & x_{12} & x_{13} & & \end{matrix} \quad (53)$$

Finally, the probability of the successful operation i.e., reliability of the complex system is given by

$$\begin{aligned}
 R_S &= P_r \{ F(x_1, x_2, x_3, x_4, \dots, x_{13}) \} \\
 &= R_1 R_2 R_3 R_4 R_6 R_8 R_{10} R_{11} R_{13} + R_1 R_2 R_3 R_4 R_6 R_8 R_{10} R_{11} R_{12} R_{13} + R_1 R_2 R_3 Q_4 R_5 R_6 R_8 R_{10} R_{11} R_{13} \\
 &+ R_1 R_2 R_3 Q_4 R_5 R_6 R_{10} R_{11} R_{12} R_{13} + R_1 R_2 R_3 Q_4 Q_5 R_6 R_7 R_8 R_{10} R_{11} R_{13}
 \end{aligned}$$

$$\begin{aligned}
 &+R_1R_2R_3R_4R_5Q_6R_7R_8R_9R_{10}R_{11}R_{13} + R_1R_2R_3R_4Q_5Q_6R_7R_8R_9Q_{10}R_{11}R_{13} \\
 &+R_1R_2R_3R_4R_5Q_6R_7R_8R_9Q_{10}R_{11}R_{13} + R_1R_2R_3Q_4R_5Q_6R_7R_8R_9R_{10}R_{11}R_{13} \\
 &+R_1R_2R_3Q_4R_5Q_6R_7R_8R_9Q_{10}R_{11}R_{13} + R_1R_2R_3R_4Q_5Q_6Q_7R_8R_9R_{10}R_{11}R_{13} \\
 &+R_1R_2R_3R_4Q_5R_6R_7Q_8R_9R_{10}R_{11}R_{13} + R_1R_2R_3R_4R_5R_6Q_7Q_8R_9R_{10}R_{11}R_{13} \\
 &+R_1R_2R_3R_4R_5R_6R_7Q_8R_9R_{10}R_{11}R_{13} + R_1R_2R_3R_4Q_5Q_6Q_7Q_8R_9R_{10}R_{11}R_{13} \\
 &+R_1R_2R_3R_4Q_5R_6R_7Q_8R_9R_{10}Q_{11}R_{12}R_{13} + R_1R_2R_3R_4R_5R_6Q_7Q_8R_9R_{10}Q_{11}R_{12}R_{13} \\
 &+R_1R_2R_3R_4R_5R_6R_7Q_8R_9R_{10}Q_{11}R_{12}R_{13} + R_1R_2R_3Q_4R_5Q_6Q_7Q_8R_9R_{10}R_{11}R_{13} \\
 &+R_1R_2R_3Q_4R_5R_6R_7Q_8R_9R_{10}R_{11}R_{13} + R_1R_2R_3Q_4R_5R_6Q_7Q_8R_9R_{10}Q_{11}R_{12}R_{13} \\
 &+R_1R_2R_3Q_4R_5R_6R_7Q_8R_9R_{10}Q_{11}R_{12}R_{13} + R_1R_2R_3R_4Q_5Q_6Q_7Q_8R_9R_{10}R_{11}R_{13} \\
 &+R_1R_2R_3R_4R_5Q_6R_7Q_8R_9R_{10}R_{11}R_{13} + R_1R_2R_3R_4Q_5Q_6R_7Q_8R_9R_{10}Q_{11}R_{12}R_{13} \\
 &+R_1R_2R_3R_4R_5Q_6R_7Q_8R_9R_{10}Q_{11}R_{12}R_{13} + R_1R_2R_3Q_4R_5Q_6R_7Q_8R_9R_{10}R_{11}R_{13} \\
 &+R_1R_2R_3Q_4R_5Q_6R_7Q_8R_9R_{10}Q_{11}R_{12}R_{13}
 \end{aligned} \tag{54}$$

$$\begin{aligned}
 R_S = &R_1R_2R_3R_4R_6R_8R_{10}R_{11}R_{13} + R_1R_2R_3R_5R_7R_8R_{10}R_{11}R_{13} - R_1R_2R_3R_4R_5R_6R_8R_{10}R_{11}R_{13} \\
 &+R_1R_2R_3R_4R_7R_8R_{10}R_{11}R_{13} + R_1R_2R_3R_5R_7R_8R_{10}R_{11}R_{13} - R_1R_2R_3R_4R_5R_7R_8R_{10}R_{11}R_{13} \\
 &-R_1R_2R_3R_4R_6R_7R_8R_{10}R_{11}R_{13} - R_1R_2R_3R_5R_6R_7R_8R_{10}R_{11}R_{13} + R_1R_2R_3R_4R_5R_6R_7R_8R_{10}R_{11}R_{13} \\
 &+R_1R_2R_3R_4R_6R_9R_{10}R_{11}R_{13} + R_1R_2R_3R_5R_6R_9R_{10}R_{11}R_{13} - R_1R_2R_3R_4R_5R_6R_9R_{10}R_{11}R_{13} \\
 &+R_1R_2R_3R_4R_7R_9R_{10}R_{11}R_{13} + R_1R_2R_3R_5R_7R_9R_{10}R_{11}R_{13} - R_1R_2R_3R_4R_5R_7R_9R_{10}R_{11}R_{13} \\
 &-R_1R_2R_3R_4R_6R_7R_9R_{10}R_{11}R_{13} - R_1R_2R_3R_5R_6R_7R_9R_{10}R_{11}R_{13} + R_1R_2R_3R_4R_5R_6R_7R_9R_{10}R_{11}R_{13} \\
 &-R_1R_2R_3R_4R_6R_8R_9R_{10}R_{11}R_{13} - R_1R_2R_3R_5R_6R_8R_9R_{10}R_{11}R_{13} + R_1R_2R_3R_4R_5R_6R_8R_9R_{10}R_{11}R_{13} \\
 &-R_1R_2R_3R_4R_7R_8R_9R_{10}R_{11}R_{13} - R_1R_2R_3R_5R_7R_8R_9R_{10}R_{11}R_{13} + R_1R_2R_3R_4R_5R_7R_8R_9R_{10}R_{11}R_{13} \\
 &+R_1R_2R_3R_4R_6R_7R_8R_9R_{10}R_{11}R_{13} + R_1R_2R_3R_5R_6R_7R_8R_9R_{10}R_{11}R_{13} \\
 &-R_1R_2R_3R_4R_5R_6R_7R_8R_9R_{10}R_{11}R_{13} + R_1R_2R_3R_4R_6R_8R_{10}R_{12}R_{13} + R_1R_2R_3R_5R_6R_8R_{10}R_{12}R_{13} \\
 &-R_1R_2R_3R_4R_5R_6R_8R_{10}R_{12}R_{13} + R_1R_2R_3R_4R_7R_8R_{10}R_{12}R_{13} + R_1R_2R_3R_5R_7R_8R_{10}R_{12}R_{13} \\
 &-R_1R_2R_3R_4R_5R_7R_8R_{10}R_{12}R_{13} - R_1R_2R_3R_4R_6R_7R_8R_{10}R_{12}R_{13} - R_1R_2R_3R_5R_6R_7R_8R_{10}R_{12}R_{13} \\
 &+R_1R_2R_3R_4R_5R_6R_7R_8R_{10}R_{12}R_{13} + R_1R_2R_3R_4R_6R_9R_{10}R_{12}R_{13} + R_1R_2R_3R_5R_6R_9R_{10}R_{12}R_{13} \\
 &-R_1R_2R_3R_4R_5R_6R_9R_{10}R_{12}R_{13} + R_1R_2R_3R_4R_7R_9R_{10}R_{12}R_{13} + R_1R_2R_3R_5R_7R_9R_{10}R_{12}R_{13} \\
 &-R_1R_2R_3R_4R_5R_7R_9R_{10}R_{12}R_{13} - R_1R_2R_3R_4R_6R_7R_9R_{10}R_{12}R_{13} - R_1R_2R_3R_5R_6R_7R_9R_{10}R_{12}R_{13} \\
 &+R_1R_2R_3R_4R_5R_6R_7R_9R_{10}R_{12}R_{13} - R_1R_2R_3R_4R_6R_8R_9R_{10}R_{12}R_{13} - R_1R_2R_3R_5R_6R_8R_9R_{10}R_{12}R_{13} \\
 &+R_1R_2R_3R_4R_5R_6R_8R_9R_{10}R_{12}R_{13} - R_1R_2R_3R_4R_7R_8R_9R_{10}R_{12}R_{13} - R_1R_2R_3R_5R_7R_8R_9R_{10}R_{12}R_{13} \\
 &+R_1R_2R_3R_4R_5R_7R_8R_9R_{10}R_{12}R_{13} + R_1R_2R_3R_4R_6R_7R_8R_9R_{10}R_{12}R_{13} - R_1R_2R_3R_4R_5R_6R_8R_{10}R_{11}R_{12}R_{13} \\
 &+R_1R_2R_3R_5R_6R_7R_8R_9R_{10}R_{12}R_{13} - R_1R_2R_3R_4R_5R_6R_7R_8R_9R_{10}R_{12}R_{13} \\
 &-R_1R_2R_3R_5R_6R_8R_{10}R_{11}R_{12}R_{13} + R_1R_2R_3R_4R_5R_6R_8R_{10}R_{11}R_{12}R_{13}
 \end{aligned}$$

$$\begin{aligned}
 & -R_1R_2R_3R_4R_7R_8R_{10}R_{11}R_{12}R_{13} -R_1R_2R_3R_5R_7R_8R_{10}R_{11}R_{12}R_{13} +R_1R_2R_3R_4R_5R_7R_8R_{10}R_{11}R_{12}R_{13} \\
 & +R_1R_2R_3R_4R_6R_7R_8R_{10}R_{11}R_{12}R_{13} +R_1R_2R_3R_5R_6R_7R_8R_{10}R_{11}R_{12}R_{13} \\
 & -R_1R_2R_3R_4R_5R_6R_7R_8R_{10}R_{11}R_{12}R_{13} -R_1R_2R_3R_4R_6R_9R_{10}R_{11}R_{12}R_{13} \\
 & -R_1R_2R_3R_5R_6R_9R_{10}R_{11}R_{12}R_{13} +R_1R_2R_3R_4R_5R_6R_9R_{10}R_{11}R_{12}R_{13} \\
 & -R_1R_2R_3R_4R_7R_9R_{10}R_{11}R_{12}R_{13} -R_1R_2R_3R_5R_7R_9R_{10}R_{11}R_{12}R_{13} +R_1R_2R_3R_4R_5R_7R_9R_{10}R_{11}R_{12}R_{13} \\
 & +R_1R_2R_3R_4R_6R_7R_9R_{10}R_{11}R_{12}R_{13} +R_1R_2R_3R_5R_6R_7R_9R_{10}R_{11}R_{12}R_{13} \\
 & -R_1R_2R_3R_4R_5R_6R_7R_9R_{10}R_{11}R_{12}R_{13} +R_1R_2R_3R_4R_6R_8R_9R_{10}R_{11}R_{12}R_{13} \\
 & +R_1R_2R_3R_5R_6R_8R_9R_{10}R_{11}R_{12}R_{13} -R_1R_2R_3R_4R_5R_6R_8R_9R_{10}R_{11}R_{12}R_{13} \\
 & +R_1R_2R_3R_4R_7R_8R_9R_{10}R_{11}R_{12}R_{13} +R_1R_2R_3R_5R_7R_8R_9R_{10}R_{11}R_{12}R_{13} \\
 & -R_1R_2R_3R_4R_5R_7R_8R_9R_{10}R_{11}R_{12}R_{13} -R_1R_2R_3R_4R_6R_7R_8R_9R_{10}R_{11}R_{12}R_{13} \\
 & -R_1R_2R_3R_5R_6R_7R_8R_9R_{10}R_{11}R_{12}R_{13}
 \end{aligned} \tag{55}$$

Where, $R_i (i = 1, 2, \dots, 13)$ is the reliability of the section state $x_i (i = 1, 2, \dots, 13)$ respectively.

6. SOME PARTICULAR CASES:

Case I:

When reliability of each component R then equation (55) yields

$$R_S = R^{13} - 8R^{12} + 24R^{11} - 32R^{10} + 16R^9 \tag{56}$$

Case II: When failure rates follow Weibull time distribution:

Let λ_i be the failure rate corresponding to system state $x_i, \forall i = 1, 2, 3, \dots, 13$, then reliability of considered system at an instant 't', is given by

$$R_{SW}(t) = \sum_{i=1}^{41} \exp\{-\alpha_i t^p\} - \sum_{j=1}^{40} \exp\{-\beta_j t^p\} \tag{57}$$

Where, p is appositve parameter and

$$\alpha_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{13}$$

$$\alpha_2 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{13}$$

$$\alpha_3 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{13}$$

$$\alpha_4 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{13}$$

$$\alpha_5 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{13}$$

$$\alpha_6 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{13}$$

$$\alpha_7 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{13}$$

$$\alpha_8 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{13}$$

$$\alpha_9 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{13}$$

$$\alpha_{10} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{13}$$

$$\alpha_{11} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{13}$$

$$\alpha_{12} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{13}$$

$$\begin{aligned}\beta_8 &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{13} \\ \beta_9 &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{13} \\ \beta_{10} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{13} \\ \beta_{11} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{13} \\ \beta_{12} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{13} \\ \beta_{13} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{13} \\ \beta_{14} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_{10} + \lambda_{12} + \lambda_{13} \\ \beta_{15} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{12} + \lambda_{13} \\ \beta_{16} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{12} + \lambda_{13} \\ \beta_{17} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{12} + \lambda_{13} \\ \beta_{18} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_9 + \lambda_{10} + \lambda_{12} + \lambda_{13} \\ \beta_{19} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{12} + \lambda_{13} \\ \beta_{20} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{12} + \lambda_{13} \\ \beta_{21} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{12} + \lambda_{13} \\ \beta_{22} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12} + \lambda_{13} \\ \beta_{23} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12} + \lambda_{13} \\ \beta_{24} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12} + \lambda_{13} \\ \beta_{25} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12} + \lambda_{13} \\ \beta_{26} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12} + \lambda_{13} \\ \beta_{27} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \\ \beta_{28} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \\ \beta_{29} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \\ \beta_{30} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \\ \beta_{31} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \\ \beta_{32} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \\ \beta_{33} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \\ \beta_{34} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \\ \beta_{35} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \\ \beta_{36} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \\ \beta_{37} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \\ \beta_{38} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} \\ \beta_{40} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13}\end{aligned}$$

Case III: When failure rates follow exponential time distribution

Exponential distribution is nothing but a particular case of Weibull distribution for $p = 1$ and is very useful for practical problems purpose. Therefore, the reliability of considered system as a whole at an instant 's' is expressed as:

$$R_{SE}(t) = \sum_{i=1}^{41} \exp\{-\alpha_i t\} - \sum_{j=1}^{40} \exp\{-\beta_j t\} \tag{58}$$

Where, α_i and β_j 's has mentioned earlier.

Also, an important reliability parameter, viz; Mean time to failure (M.T.T.F) in this case is given by

$$\begin{aligned} \text{M.T.T.F} &= \int_0^{\infty} R_{SE}(t) dt \\ &= \sum_{i=1}^{41} \left(\frac{1}{\alpha_i} \right) - \sum_{j=1}^{40} \left(\frac{1}{\beta_j} \right) \end{aligned} \tag{59}$$

7. Numerical Example:

Setting $\lambda_i = 0.01$ for $i = 1-13$ and $p = 2$ in equations (57) and(58) then

Table 1

S.No.	Time (t)	$R_S(t)$	
		Exponential Distribution	Weibull Distribution
1	0	1	1
2	1	0.95085	0.95085
3	2	0.81371	0.90342
4	3	0.61894	0.85770
5	4	0.41131	0.81371
6	5	0.23441	0.77142
7	6	0.11266	0.73082
8	7	0.04503	0.69189
9	8	0.01482	0.65461
10	9	0.00399	0.61894
11	10	0.00087	0.58486

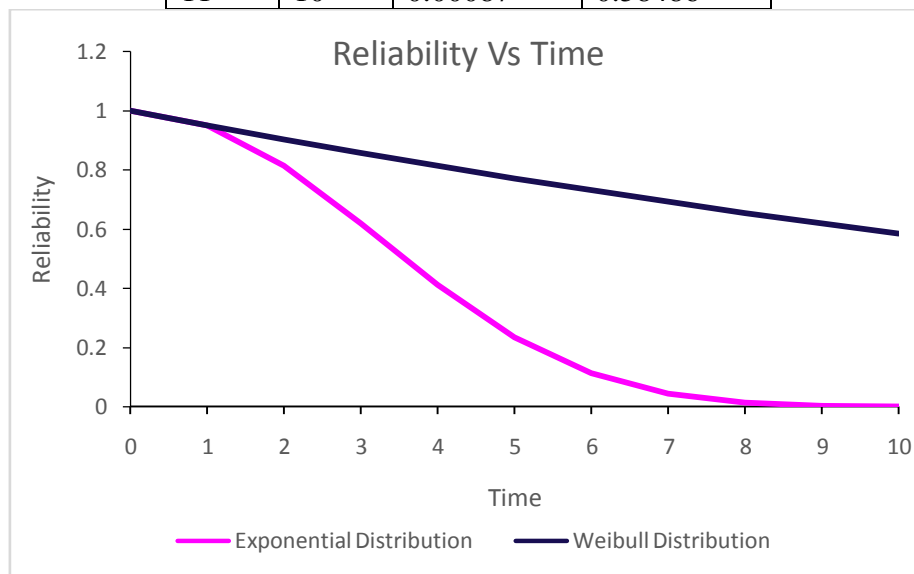


Fig.2

7. Numerical computation for MTTF:

Setting $\lambda_i = \lambda$ one can compute table 2 from equation (59)

Table 2

S.No.	λ	MTTF
1	0.0	∞
2	0.1	1.69852
3	0.2	0.84926
4	0.3	0.56617
5	0.4	0.42463
6	0.5	0.33970
7	0.6	0.28309
8	0.7	0.24265
9	0.8	0.21232
10	0.9	0.18872
11	1.0	0.16985

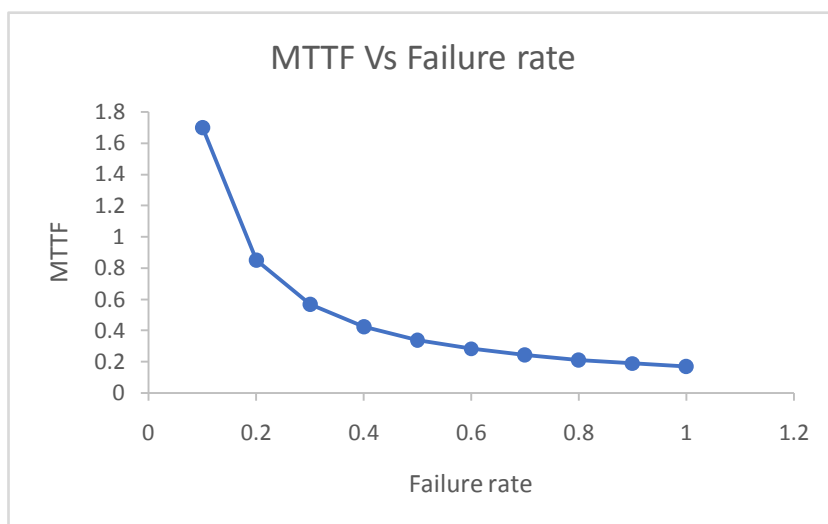


Fig.3

9. Conclusion:

In this paper, we have considered a sugar plant for analysis of various reliability parameters by employing the Boolean function technique and algebra of logics. Table 1 computes the reliability of the system with respect to time when failure rates follow exponential and Weibull time distributions. From the graph “reliability Vs Time”(Fig2) reveals that the reliability of the complex system decreases approximately at a uniformly rate in case of exponential time distribution, but decreases very rapidly when failure rates follow Weibull distribution. Table 2 and graph “M.T.T.F Vs Failure rate”(Fig.3) yields that MTTF of the system decreases catastrophically in the beginning but later it decreases approximately at a uniform rate.

10. References:

1. Gupta P.P. Kumar, Evaluation of Reliability and MTTF of a Power Plant with the aid of Boolean Function Expansion Algorithm, Microelectronic Reliability,U.K.,26,1986. PP.821-824.

2. Deepankar Sharma and Neelam Sharma, Evaluation of Some Reliability parameters for Tele communication system by Boolean Function Technique, International Journal of Research and Reviews in Applied Sciences, Volume 1, Issue 3, 2009, PP 228-235
3. Deepankar Sharma, Jitendra Kumar Rana and Sumit Gupta, Measurement of Reliability Parameters for Solar (PV) System by Employing Boolean Function Technique, Research Journal of Mathematics and Statistics 2(2): 81-85, 2010
4. Shikha Bansal, S.C. Agarwal & Kuldeep Sharma, Evaluation of Reliability factors using Boolean function technique in Milk powder manufacturing plant, IJRRAS 4(4), September 2010, PP 416-424
5. S.C. Agarwal, Deepika & Neha Sharma, Reliability analysis of sugar manufacturing plant using Boolean function technique, IJRRAS 6(2), February 2011, PP.165-171
6. Surbhi Agarwal and Deepankar Sharma, Reliability Assessment of a Four Cylinder Diesel Engine with Standby Fuel Backup through Perfect Switching Over Device, Research Journal of Mathematics and Statistics 3(1):34-38,2011.
7. Mohit Kumar, Ram Avtar Jaswal, Evaluation of Reliability Parameter of Rice mill, International Journal of Advanced Research in Electrical, Electronic and Instrumentation Engineering, vol.2, Issue 6, June 2013, PP 2360-2366
8. Jasbir Singh, Mr. Ram Avtar Jaswal, Reliability Analysis of Washing unit of Paper mill using B.F. Technique, International Journal of Advanced Engineering Research and studies/II/IV/July-Sept., 2013/84-86.