

# Practical Understanding Thought Models as Teaching Aids for Undergraduate Students

**Dr. Harvinder Kaur**

Associate Professor Department of Mathematics, Govt. P.G.College, Sec 14 Panchkula, Haryana

## Abstract

This paper is the study of practical understanding thought Models as teaching aids to de-contextualize selected abstract algebra concepts. This will involve the use of analogy in everyday life to de-abstractise the concepts of abstract algebra. The concepts here selected for study include functions and equivalence relations. These concepts are taken for consideration because these have consequent and wide application to other areas of pure mathematics. This study would enhance the understanding of the difficult abstract concepts of function and relation. It would help in learning the basic concept with ease and ultimately better achievement in higher education pure mathematics concepts at introduction and definition level.

## Introduction

Few efforts have been made at improving teaching and learning of mathematics at the level of higher education. At this stage, Mathematics courses are often dreaded, tedious and difficult to understand and learn due to its abstract nature. It is observed that much difficulty lies in understanding the definition and introduction level. Once the introductory part is understood by the learner, its application part would be much easier. Experiences have shown that undergraduate pure mathematics presentations are difficult to follow, the absorption often slow, tedious and painstaking. To remove this difficulty and for better learning, thought models can be used to decontextualize the basic abstract algebra concepts. Once this is achieved consequent terms, theorems and proofs will be relatively easier to comprehend.

This is an initiation to de-contextualize some basic concepts of abstract algebra like group theory, mappings and functions, ideals, quotient sets, equivalence relations etc. Obviously not all concepts can be de-abstractized and even in cases where this is possible the de-abstractization may not be possible beyond the introductory stage. Caution must however be exercised in this enterprise. The idea is not to change the nature of pure mathematics but a better approach to its teaching. This 'decontextualization theory' should be limited to very basic concepts and the introduction of such concepts.

## Definition of Concepts of Equivalence relation

Equivalence relation is an old fundamental concept in pure mathematics. An equivalence relation on a set  $S$  has been defined as a relation that is

- (i) Reflexive:  $aRa$  for all  $a \in S$
- (ii) Symmetric: if  $aRb$  then  $bRa$  for all  $a, b \in S$
- (iii) Transitive: if  $aRb$  and  $bRc$  then  $aRc$  for every  $a, b, c \in S$

If a and b are related in this way we say that they are equivalent under R

**De-contextualization of Concept Using Thought Model**

To make real the otherwise abstract concept of equivalence relations, we use day to day example to represent this concept. This we call thought models because it is not a concrete model or threedimensional model rather it is compared and illustrated with real life situations and relationship. Specifically, we use the illustration to explain equivalence relations. Consider a case study of resemblance in a family.

The following illustrates are analogous to equivalence relations.

**Reflexive:** Every man resembles himself therefore  $aRa$

**Symmetric:** If 'a' resembles 'b' then b resembles a.

e.g. if a boy resembles his brother then definitely his brother resembles him. therefore If  $aRb$  then  $bRa$ .

**Transitive:** If 'a' resembles 'b' and 'b' resembles 'c' definitely 'a' resembles 'c' e.g. If a girl resembles her mother and her mother resembles her grand mother then definetly the girl resembles her grand mother therefore If  $aRb$  and  $bRc$  then  $aRc$ .

**Definition of Concept of Function**

Let X and Y be two non-empty sets. A function f defined from X to Y, is a rule of collection of rules which associate to each element x in X a unique element y in Y. The function is also known as mapping or transformation or correspondence.

e.g. Let  $X = (1,2,3,4)$  and  $Y = (2,4,6,8,10)$

Here, the rule i.e. f which associates to each element x in X, the element  $2x$  in Y is a function from X to Y. The rule  $f(x) = 2x$  is shown by the following diagram:

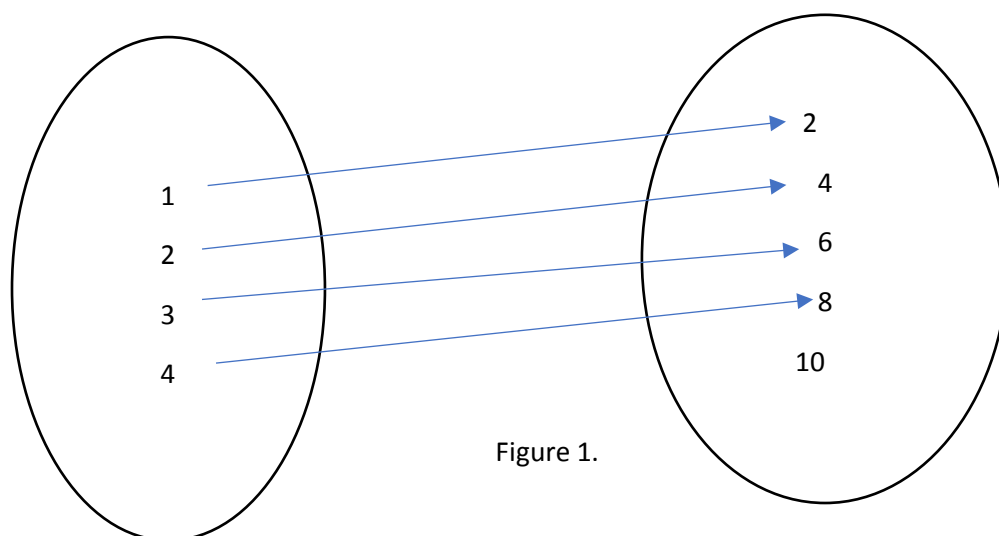


Figure 1.

1. To each element x in X there exists a unique element y in Y such that  $y=f(x)$

2. Different elements of X may be associated with the same element of Y.
3. There may exist some elements of Y, which are not associated with any element of X.

### De-contextualization of Concept Using Thought Model

X. is child of Y

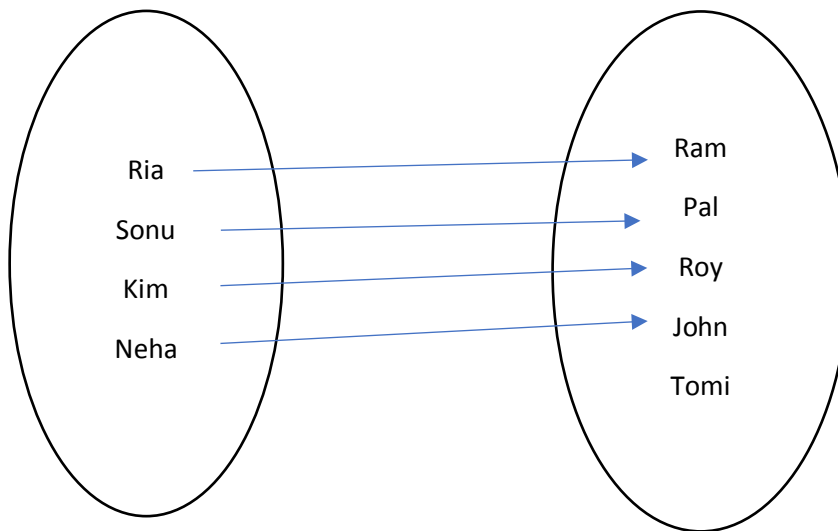


Figure 2.

$f: X \rightarrow Y$  is a function from X to Y. Here function is child of i.e. every member of X is child of some member of Y.

Figure 2 depicts that

- (i) Ria is child of Ram or Ram is father of Ria
- (ii) Sonu is child of Pal or Pal is father of Sonu
- (iii) Kim is child of Roy or Roy is father of Kim
- (iv) Neha is child of John or John is father of Neha
- (v) Tomi has no child.

Hence this illustrates a function.

1. To each element  $x$  in X there exists a unique element  $y$  in Y such that  $y=f(x)$  i.e. each member of X is child of some unique member of Y such that  $y=father\ of\ (x)$ .
2. Different elements of X may be associated with the same element of Y. i.e. different member of X (means two or more children) may be of the same father.
3. There may exist some elements of Y, which are not associated with any element of X. i.e. there may be some member of Y who does not have any child.

**Types of function**

**into Function:** A function  $f$  from  $X$  to  $Y$  is said to be into function iff there is at least one element of  $Y$  which is not the image of any element of  $X$ . Figure 2 is the example of the into function.

**Onto Function:** A function  $f$  from  $X$  to  $Y$  is said to be onto (or surjective) function iff every element in  $Y$  is an image of at least one element in  $X$ .

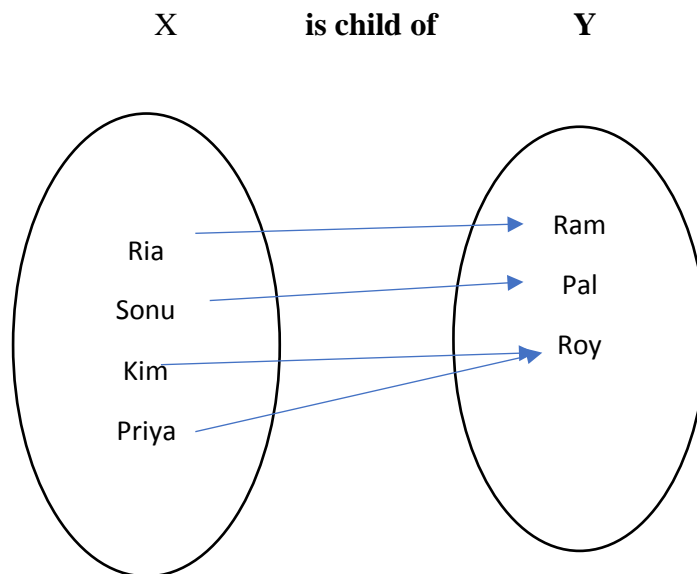


Figure 3

Figure 3 depicts that

- (i) Ria is child of Ram or Ram is father of Ria.
- (ii) Sonu is child of Pal or Pal is father of Sonu.
- (iii) Kim and Priya are children of Roy or Roy is father of two children Kim and Priya.
- (iv) There is no childless man.

**One-one Function:**

A function  $f$  from  $X$  to  $Y$  is said to be one-one (or injective) function iff different elements of  $X$  have different images in  $Y$ . Figure 2 is the example of the 1-1 function.

**one-one correspondence**

A function  $f$  from  $X$  to  $Y$  is said to be one-one correspondence (or bijective) function iff  $f$  is both oneone and onto.

Figure 4 is the example of the one-one onto function. Every member in group  $X$  is child of some unique member in  $Y$  or every member in group  $Y$  has a unique (only one) child. No father has more than one child and no father is childless.

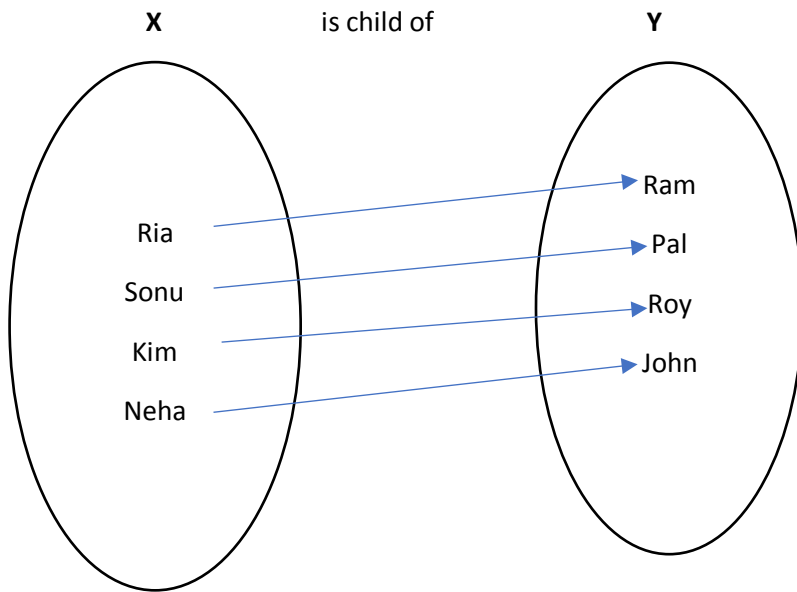
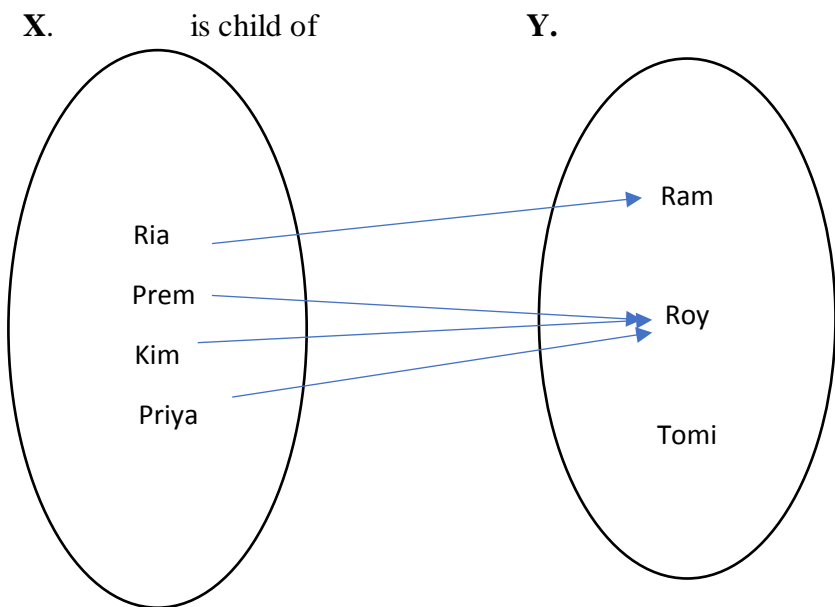


Figure 4

**Many-one Function:**

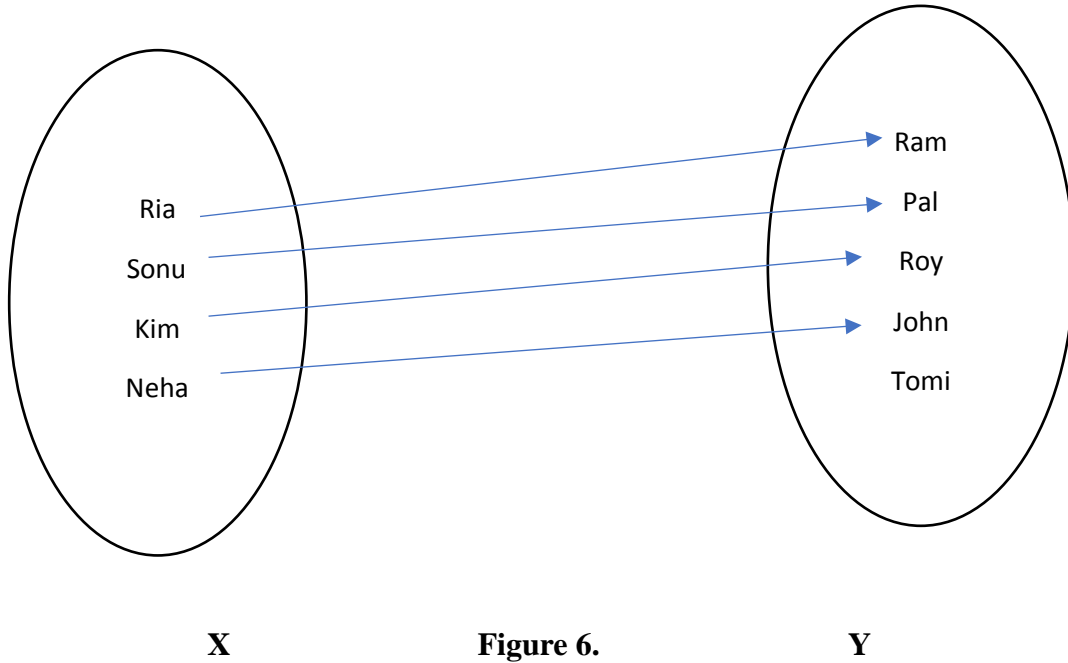
A function  $f$  from  $X$  to  $Y$  is said to be Many-one function if different elements of  $X$  have same images in  $Y$ . Figure 5 is the example of the Many-one function.



Kim, Prem and Priya are children of Roy or Roy is father of three children. There may be a person who has more than one children eg. Roy has 3 children and there may be any childless person.

**One-one but not onto Function:**

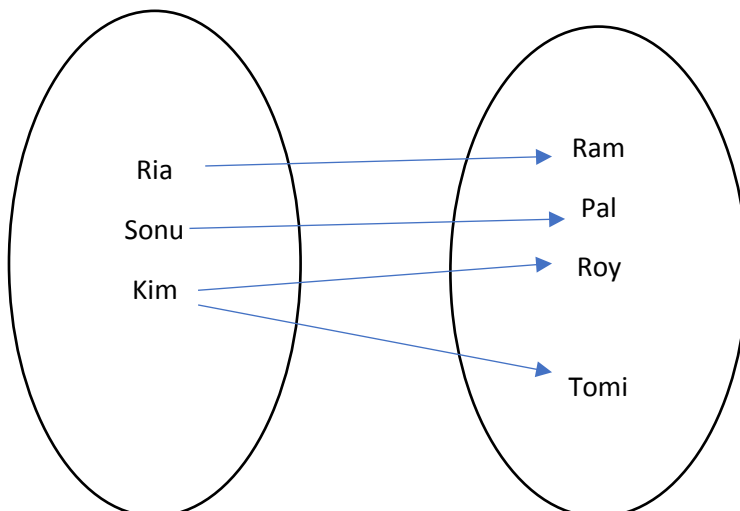
Figure 6 is example of 1-1 not onto ie, no person has more than one child but there is at least one person who does not have any child.



**Not a Function:**

If an element in X has more than one images in Y then it is not a function. i.e. No child has two fathers.

**X.** is child of **Y.** **Figure 7**



This is just the beginning and an invitation to de-contextualize some basic abstract algebra topics. It would help in learning the basic concept with ease and ultimately better achievement in higher education pure mathematics concepts at introduction and definition level.

## Conclusion

It has been illustrated in the study how thought models can be used to de-contextualize otherwise abstract concepts of equivalence relations, quotient sets and a fundamental theorem about them. This is just the beginning and an invitation to de-contextualize some other basic abstract algebra topics. This is with a view to enhancing the understanding and ultimately better achievements in higher education pure mathematics concepts. Caution must however be exercised in this endeavour. The idea is not to change the nature of pure mathematics but a better approach to its teaching. This 'de-contextualization theory should be limited to very basic concepts and the introduction of such concepts. Examples include but not limited to group theory, mappings and functions, ideals. Obviously not all concepts can be deabstractized and even in cases where this is possible the deabstraction may not be possible beyond the introductory stage. But this would have gone far in meeting the learners' needs because majority of learners problems lies at the definition and internalization of basic terms. Once this is achieved consequent terms, theorems and proofs will be relatively easier to comprehend. The call then is to higher mathematics professional educators to help deabstractised these beginning concepts. This should be done as a team in form of projects so that a collection of these de-abstractions can be placed in the hands of mathematics lecturers so as to serve thought models as teaching aids. This is much more so because actual teachers of these concepts are largely lecturers with no educational training. Routine seminars for mathematics lecturers can now ensue so as to know how to incorporate these thought models into their instructions.

## References

1. T. Brown. Towards a hermeneutical understanding of Mathematics and Mathematical learning. P.Ernest eds. The Falmer Press, London, 1994, pp141- 150.
2. A.O Ekong. Improvisation of materials from local resources for demonstrative Instruction deliveries in agricultural education. C.M. Ekpo eds., Belpot, Abak Nigeria, 2003.
3. D. Ekwo. Correlating data with model lessons. Rice University School Mathematics Project, Spring Networking Conference. Houston, 2005.
4. A. Gerber The influence of Second Language Teaching on Undergraduate Mathematics Performance. M.Sc thesis, University of Pretoria,2005.
5. Proceedings of The Third International Conference on Mathematical Sciences- ICM 2008
6. Dr.M.AliDombayci Models of thinking education and Quadruple thinking vol 5,issue4,2014.