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Split Perfect Total Domination of An Interval Graph

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Abstract:
A Set S of vertices in a graph G(V,E) is called a dominating set,if every vertex $v \in V$ id either an element of S or is adjacent to an element of S. A Set S of vertice in a graph G(V,E) is called a Total dominating set if every vertex $v \in V$ is adjacent to an element of S, or a total dominating set of a graph G with no isolated vertices is a subset S of the vertex set such that every vertex of G is adjacent to a vertex in S. In this paper we will find the perfect total dominating set of an Interval graph. We also find out the vertex induced subgraph regarding perfect total dominating set which were splitted.

Keywords: Interval graph, Dominating set, Total dominating set,Split-dominating set and perfect total dominating set of an interval graph.

Introduction:

Let $G=(V,E)$ be a graph. A set $D \subseteq V(G)$ is a dominating set of G if every vertex in V/D is adjacent to some vertex in D. A dominating set D of G is called a split dominating set if the vertex induced subgraph $<V-D>$ is disconnected.The split domination number $\gamma_s(G)$ of the graph G is the minimum cardinality of the split dominating set.

A set $S \subseteq V$ is a perfect dominating set (PDS) if every vertex not in S is adjacent to exactly one vertex in S. The perfect domination number of G, denoted by $\gamma_p(G)$ is the minimum cardinality of the PDS of G.

A total dominating set of a graph G with no isolated vertices is a subset S of the vertex set such that every vertex of G is adjacent to a vertex in S. The concept of total domination in graph theory was first introduced by Cockayne, Dawes and Hedetniemi in [1] and it has been studied extensively by many researchers in the last years, see for example [2],[3].

A perfect total dominating set PTDS of a graph *G(V,E)* is the dominating set S every vertex not in S is adjacent to exactly one vertex in S, with no isolated vertices is a subset S of the vertex set such that every vertex of G is adjacent to a vertex in S.

A perfect total dominating set PTDS of a graph *G(V,E)* is disconnected perfect total dominating set if the vertex induced subgraph $<V-PTDS>$ is disconnected. i.e., a perfect total dominating set PTDS of a graph $G(V,E)$ is a split perfect total dominating set if the vertex induced subgraph $<V-PTDS>$ is disconnected, otherwise it is a non-split perfect total dominating set.

Split domination in graphs was introduced by V.R. Kulli[4]in 1997. They have studied these parameters for various standard graphs and obtained the bounds for them. Q.M.Mahyoub and N.D.Sonerinitiate the split dominating set and split domination number in fuzzy graphs. The concept of perfect domination was introduced by J.A. Telle and A. Proskurowski albeit [7] indirectly, as a vertex partitioning problem. The concept of split and non split domination in graphs and also in Maheswari, B et all [5,6] .

MAIN THEOREMS

1. Theorem:

Let G be an interval graph corresponding to an n interval family $I = \{i_1, i_2, ..., i_n\}$. If i, j, k, l are any four intervals in I , l$\neq$ i such that l $\in PTDS$, and if there are m vertices to the left or right of the vertex which belongs to PTDS and also d(l) = m+1, then split perfect total domination occurs in G and the split perfect total dominating set <V-PTDS> is disconnected as $|PTDS| = 2$.

Proof:

Let $I = \{i_1, i_2, ..., i_n\}$ be the given n interval family and G is an interval graph corresponding to I. If i, j, k, l are any four intervals in I , l$\neq$ i such that l $\in PTDS$, and if there are m vertices to the left or right of the vertex which belongs to PTDS and also d(l) = m+1, then split perfect total domination occurs in G . If otherwise , we take the contradiction that d(l) = m+2, then the vertex which intersect the vertex l also intersect another vertex in the PTDS, which leads to the contradiction of the perfect domination that every vertex intersects exactly one vertex in the perfect domination. So d(l) = m+1, and the induced subgraph , <V-PTDS> will be splitted, it gives split perfect total domination.

Illustration:

We will find the perfect total dominating set as follows from an interval family as follows,

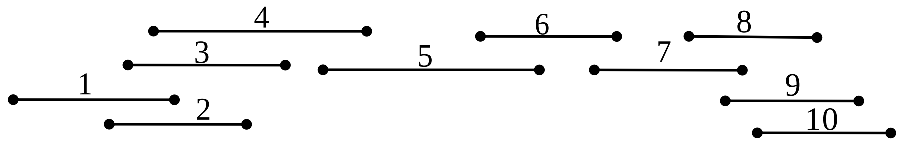

Fig.1: Interval family *I*

The corresponding interval gra mily *I* is as follows,

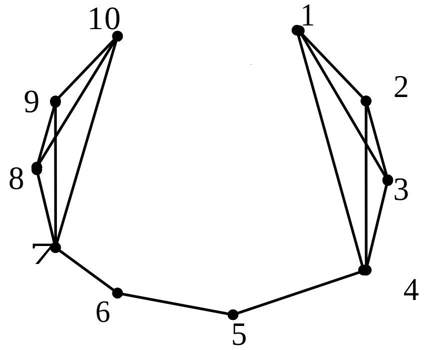

Fig. 2: Interval graph *G*

From the above interval graph G, the perfect total dominating set was {4,7}

Thus we get the vertex induced subgraph $<V - PTDS>$ as follows,

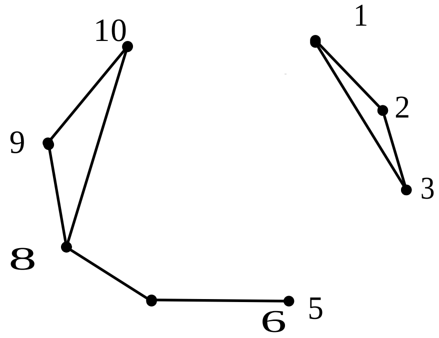

Fig.3: Vertex induced subgraph $<V - PTDS>$ - Disconnected graph from G

Hence the theorem is proved.

Theorem:

Let $I = \{i_1, i_2, ..., i_n\}$ be an n interval family and G is an interval graph corresponding to I. If i, j, k are any three intervals in I such that k intersects both i, j to the left of k, k $\in$ PTDS, where PTDS is the perfect total dominating set and there is atleast one interval to the right of k, other than the vertex which was adjacent to k, which does not intersects both i and j. Then split perfect total domination occurs in G and the split perfect total dominating set $< V - PTDS >$ is disconnected as $|PTDS|$=2.

Proof:

Let $I = \{i_1, i_2, ..., i_n\}$ be an n interval family and G is an interval graph corresponding to I. If i, j, k are any three intervals in I such that k intersects both i, j to the left of k, k $\in$ PTDS, where PTDS is the perfect total dominating set and there is at least one interval to the right of k, other than the vertex which was adjacent to k, which does not intersects both i and j. If that interval exists, then intersect not only one interval in the PTDS, but also the other vertex, which leads to the contradiction of the perfect domination.

Illustration:

Consider the interval family I as follows,

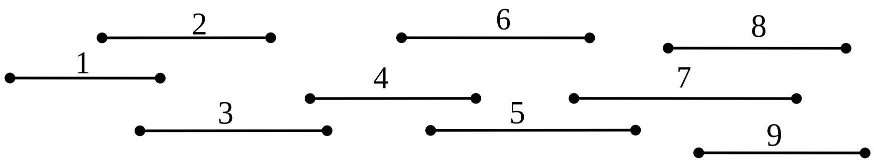

Fig.4: Interval family I

The corresponding interval graph *G* is as follows,

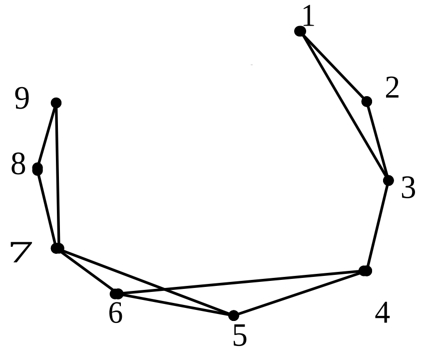

Fig.5: Interval graph *G*

The Perfect Total Dominating Set PTDS = {3,7}

Induced Subgraph G[PDS] is as follows,

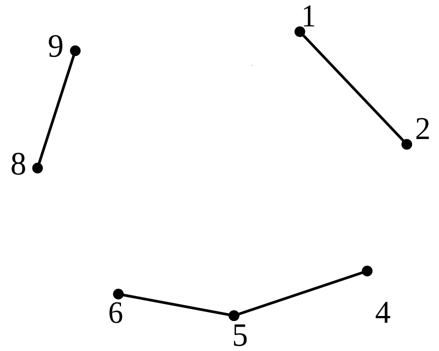

Fig.6: Vertex induced subgraph <*V - PTDS*> - Disconnected graph from *G*

Hence the theorem is proved.

Theorem:

Let us consider an n interval family $I = \{i_1, i_2, ..., i_n\}$ and G be an interval graph of *I*. If *i, j, k* are any three intervals such that $j \in$ PTDS and j intersects both i and k. The vertex j of the interval family I intersect exactly one vertex to the left of j and exactly one vertex to the right of j i.e., d(j)=2, for atleast two vertices in the PTDS, then split – perfect total domination occurs in G and the split perfect total dominating set <V-PTDS> is disconnected as $|PTDS| = 2$

Proof:

Let $I = \{i_1, i_2, ..., i_n\}$ be the given n interval family and G is an interval graph of *I*. Let i, j, k be three intervals satisfies the hypothesis, such that $j \in$ PTDS and j intersects both i and k. The vertex j of the interval family I intersect exactly one vertex to the left of j and exactly one vertex to the right of j i.e., d(j)=2, for atleast two vertices in the PTDS.Otherwise the vertex j intersect one more vertex to the right of itself then there is no non-split total domination occurs and does not contradict our statement but it contradict the perfect domination statement. Hence d(j) = 2,for atleast two vertices in the PTDS, then the split perfect total domination occurs in G and the split perfect total dominating set <V-PTDS> is disconnected as $|PTDS| = 2$

Illustration:

We will find the perfect total dominating set using the interval family as follows, for this consider the following interval family *I*,

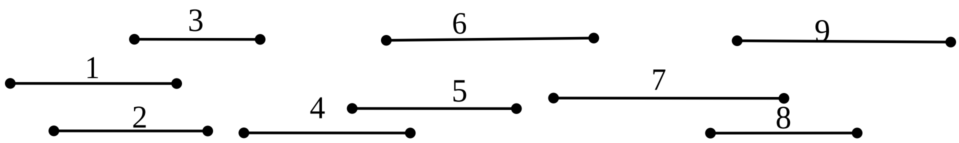

Fig.7: Interval family *I*

The corresponding interval graph *G* ... ly *I* is as follows,

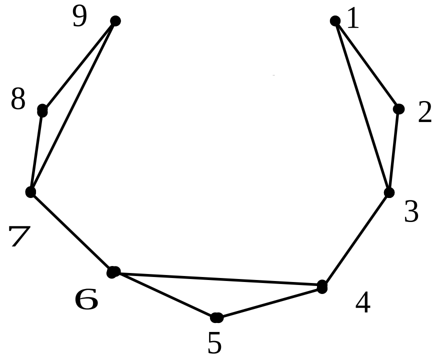

Fig.8: Interval graph *G*

From the above interval graph *G*, the perfect total dominating set was {1,5,7}
Thus we get the vertex induced subgraph <*V* - *PTDS*> as follows,

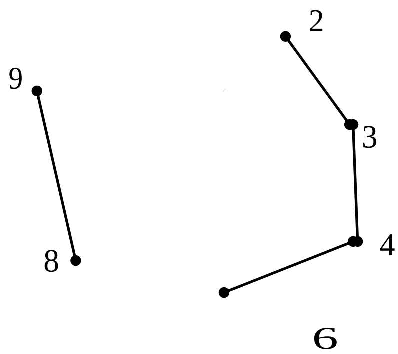

Fig.9: Vertex induced subgraph <*V* - *PTDS*> - Disconnected graph from *G*

The theorem is proved.

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