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Using Sadik Transform in Differential Equation

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Abstract
Differential equations are the equations used in engineering, science and physics. The differential equations which have linear differential equations and nonlinear differential equations. There are many ways to solve problems. Transformation is a way to find solutions to equations such as Laplace transform, Elzaki transform and Sumudu transform etc. In this research, Sadik transform is applied to solve the linear and nonlinear differential equation problems. In which nonlinear differential equation use homotopy perturbation to find solutions to equations and also present examples to illustrate the preciseness of this method.

Keywords: Differential equations, Sadik transform, Homotopy Perturbation, Linear Differetial Equation, Nonlinear Differential Equation

1. Introduction

Differential equations are widely used equations. In science Engineering and physics which the differential equation is divided into ordinary differential equations (ODE) and partial differential equations (PDE). Finding solutions to differential equations, there are many ways to find the answer to differential equations. Conversion is one of the most widely used methods for finding answers. There are many transformations such as Laplace transform, Elzaki transform, Sumudu transform etc. In 2018 Sadikali Latif Shaikh [3] introduced the Sikh transform used in control theory and the next years Sadikali Latif Shaikh [2] (2019) presented various features. later, Shivaji S. Pawar and his team [4] (2019) applied the Sadik transform to the base cell function and the volterra equation. Sudhanshu Aggarwal and Kavita Bhatnagar [5] (2019) has applied the Sadik transformation method for solving population growth and decay. These problems are very important in the field of Chemical Economics, Biology, Physics, Social Sciences and Zoology.

This research will study the solution of linear differential equations and nonlinear differential equations. By using Sadik transforms.

2. Materials and methods

2.1 Differential Equations

Definitions 1 [6] Differential equations are equations in which there is a derivative of one or more dependent variables compared to one or more variables.

Differential equations are divided into 2 types which are

1. Ordinary Differential Equations : ODE

2. Partial Differential Equations : PDE

Definitions 2 [6] Linear Differential Equation

The general from is

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \ldots + a_1(x)y' + a_0(x)y = f(x) \qquad (2.1)$$

when $a_0(x), a_1(x), a_2(x), \ldots, a_n(x)$ and $f(x)$ are function of x

If the differential equation is not linear we will call nonlinear differential equation

2.2 Sadik Transform

Definitions 3 [1] Sadik transform of $f(t)$ was defined as

$$F(v^\alpha, \beta) = S\{f(t)\} = \frac{1}{v^\beta}\int_0^\infty e^{-v^\alpha t} f(t)\,dt \qquad (2.2)$$

Sadik transform properties [2–5]

1. $S\{f'(t)\} = v^\alpha S\{f(t)\} - \dfrac{1}{v^\beta} f(0)$
2. $S\{f''(t)\} = v^{2\alpha} S\{f(t)\} - \dfrac{v^\alpha}{v^\beta} f(0) - \dfrac{1}{v^\beta} f'(0)$
3. $S\{f^{(n)}(t)\} = v^{n\alpha} S\{f(t)\} - \dfrac{v^{(n-1)\alpha}}{v^\beta} f(0) - \dfrac{v^{(n-2)\alpha}}{v^\beta} f'(0) - \ldots - \dfrac{1}{v^\beta} f^{(n-1)}(0)$

Sadik transform [2–5]

1. $S\{1\} = \dfrac{1}{v^{\alpha+\beta}}$
2. $S\{t^n\} = \dfrac{n!}{v^{(n+1)\alpha+\beta}}$
3. $S\{e^{at}\} = \dfrac{1}{v^\beta(v^\alpha - a)}$

2.3 Analysis of Modified Homotopy Perturbation Transform Method (HPTM)

Consider differential equations

$$Du(x,t) + Ru(x,t) + Nu(x,t) - f(x,t) = 0 \qquad (2.3)$$

initial conditions $u(x,0) = c_1$

taking Sadik transform $S\{Du(x,t)\} + S\{Ru(x,t)\} + S\{Nu(x,t)\} = S\{f(x,t)\}$

then
$$u(x,t) = S^{-1}\left\{\frac{c_1}{v^{\alpha+\beta}}\right\} + S^{-1}\left\{\frac{1}{v^\alpha} S\{f(x,t)\}\right\} - S^{-1}\left\{\frac{1}{v^\alpha} S\{Ru(x,t) + Nu(x,t)\}\right\}$$

$$= F(x,t) - S^{-1}\left\{\frac{1}{v^\alpha} S\{Ru(x,t) + Nu(x,t)\}\right\}$$

when $$F(x,t) = S^{-1}\left\{\frac{c_1}{v^{\alpha+\beta}}\right\} + S^{-1}\left\{\frac{1}{v^{\alpha}} S\{f(x,t)\}\right\}$$

Therefor $$u(x,t) = F(x,t) - pS^{-1}\left\{\frac{1}{v^{\alpha}} S\{Ru(x,t) + Nu(x,t)\}\right\} \tag{2.4}$$

apply the homotopy perturbation method and the nonlinear term can be decomposed as

$$u(x,t) = \sum_{n=0}^{\infty} p^n u_n(x,t), \quad Nu(x,t) = \sum_{n=0}^{\infty} p^n H_n(x,t)$$

so $$\sum_{n=0}^{\infty} p^n u_n(x,t) = F(x,t) - pS^{-1}\left\{\frac{1}{v^{\alpha}} S\left\{R\left(\sum_{n=0}^{\infty} p^n u_n(x,t)\right) + \sum_{n=0}^{\infty} p^n H_n(x,t)\right\}\right\}$$

Compare the equivalent power coefficients of p

$$p^0 : \quad u_0(x,t) = F(x,t)$$

$$p^{n+1} : \quad u_{n+1}(x,t) = -S^{-1}\left\{\frac{1}{v^{\alpha}} S\left\{R\left(\sum_{n=0}^{\infty} p^n u_n(x,t)\right) + \sum_{n=0}^{\infty} p^n H_n(x,t)\right\}\right\}$$

And the solution to the homotopy perturbation method in the power series p as

$$\begin{aligned} u(x,t) &= \sum_{n=0}^{\infty} p^n u_n(x,t) \\ &= u_0(x,t) + p^1 u_1(x,t) + p^2 u_2(x,t) + p^3 u_3(x,t) + \ldots \end{aligned} \tag{2.5}$$

From (5) when $p \to 1$ the solution to the equation considered is

$$\begin{aligned} u(x,t) &= \lim_{p\to 1}\left(u_0(x,t) + p^1 u_1(x,t) + p^2 u_2(x,t) + p^3 u_3(x,t) + \ldots\right) \\ &= u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \ldots \end{aligned} \tag{2.6}$$

3. Results

Solving differential equations

Example 3.1 Consider the first order ordinary differential equation $y'' - 2y' - 3y = 5$ (3.1)

subject to the initial condition $y(0) = 0$, $y'(0) = 1$. The exact solution is $y(t) = \frac{1}{3}\left(-5 + 5e^t\cosh(2t) - e^t\sinh(2t)\right)$

Solution taking the Sadik transform to both sides in Eq. (3.1) then

$$S\{y''\} - S\{2y'\} - S\{3y\} = S\{5\}$$

$$v^{2\alpha} S\{y\} - \frac{v^{\alpha}}{v^{\beta}} y(0) - \frac{y'(0)}{v^{\beta}} - 2\left[v^{\alpha} S\{y\} - \frac{y(0)}{v^{\beta}}\right] - 3S\{y\} = \frac{5}{v^{\alpha+\beta}}$$

$$\left(v^{2\alpha} - 2v^{\alpha} - 3\right) S\{y\} = \frac{5}{v^{\alpha+\beta}} + \frac{1}{v^{\beta}}$$

$$S\{y\} = \frac{5v^{-\beta}}{v^{\alpha}\left(v^{2\alpha}-2v^{\alpha}-3\right)}+\frac{v^{-\beta}}{v^{\alpha}\left(v^{2\alpha}-2v^{\alpha}-3\right)}$$

$$y(t) = -\frac{5}{3}+\frac{2}{3}e^{3t}+e^{-t}$$

$$= \frac{1}{3}\left(-5+5e^{t}\cosh(2t)-e^{t}\sinh(2t)\right)$$

Therefore, the solution which is the same exact solution.

Example 3.2 Consider the partial differential equation $\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}-u = 0$ (3.2)

subject to the initial condition $u(x,0) = 1+\sin x$, $u_t(x,0) = 0$

Solution taking the Sadik transform to both sides in Eq. (3.2) then

$$S\left\{\frac{\partial^2 u}{\partial t^2}\right\}-S\left\{\frac{\partial^2 u}{\partial x^2}+u\right\} = 0$$

$$v^{2\alpha}S\{u(x,t)\}-\frac{v^{\alpha}}{v^{\beta}}u(x,0)-\frac{1}{v^{\beta}}u_t(x,0) = S\left\{\frac{\partial^2 u}{\partial x^2}+u\right\}$$

$$S\{u(x,t)\} = \frac{1}{v^{\alpha+\beta}}(1+\sin x)+\frac{1}{v^{2\alpha}}S\left\{\frac{\partial^2 u}{\partial x^2}+u\right\}$$

$$u(x,t) = 1+\sin x+S^{-1}\left\{\frac{1}{v^{2\alpha}}S\left\{\frac{\partial^2 u}{\partial x^2}+u\right\}\right\} \tag{3.3}$$

applying Eq. (3.3), we obtain the component of the solution as follows:

$$u_0(x,t) = 1+\sin x$$

$$u_1(x,t) = S^{-1}\left\{\frac{1}{v^{2\alpha}}S\left\{\frac{\partial^2 u_0}{\partial x^2}+u_0\right\}\right\} = \frac{t^2}{2!}$$

$$u_2(x,t) = S^{-1}\left\{\frac{1}{v^{2\alpha}}S\left\{\frac{\partial^2 u_1}{\partial x^2}+u_1\right\}\right\} = \frac{t^4}{4!}$$

$$u_3(x,t) = S^{-1}\left\{\frac{1}{v^{2\alpha}}S\left\{\frac{\partial^2 u_2}{\partial x^2}+u_2\right\}\right\} = \frac{t^6}{6!}$$

$$\vdots$$

therefor

$$u(x,t) = u_0(x,t)+u_1(x,t)+u_2(x,t)+\ldots$$

$$= \sin x+1+\frac{t^2}{2!}+\frac{t^4}{4!}+\frac{t^6}{6!}+\ldots$$

$$= \quad \sin x + \cosh(t)$$

Example 3.3 Consider the porous medium equation $\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[u(x,t)^m u_x(x,t)\right]$ (3.4)

subject to the initial condition $u(x,0) = x$

Solution From Eq. (3.4) if $m = 1$ then $\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[u(x,t)u_x(x,t)\right]$ (3.5)

taking the Sadik transform to both sides in Eq. (3.5) then

$$S\left\{\frac{\partial u}{\partial t}\right\} = S\left\{\frac{\partial}{\partial x}[u(x,t)u_x(x,t)]\right\}$$

$$v^\alpha S\{u(x,t)\} - \frac{1}{v^\beta}u(x,0) = S\left\{u(x,t)u_{xx}(x,t) + [u_x(x,t)]^2\right\}$$

$$S\{u(x,t)\} = \frac{x}{v^{\alpha+\beta}} + \frac{1}{v^\beta}S\left\{u(x,t)u_{xx}(x,t) + [u_x(x,t)]^2\right\}$$

$$u(x,t) = x + S^{-1}\left\{\frac{1}{v^\beta}S\left\{u(x,t)u_{xx}(x,t) + [u_x(x,t)]^2\right\}\right\}$$

applying Eq. (3.5), we obtain the component of the solution as follows:

$$u_0(x,t) = x$$

$$u_1(x,t) = S^{-1}\left\{\frac{1}{v^\beta}S\left\{u_0(x,t)u_{0xx}(x,t) + [u_{0x}(x,t)]^2\right\}\right\}$$

$$= S^{-1}\left\{\frac{1}{v^\beta}S\{1\}\right\}$$

$$= t$$

$$u_2(x,t) = S^{-1}\left\{\frac{1}{v^\beta}S\left\{u_1(x,t)u_{1xx}(x,t) + [u_{1x}(x,t)]^2\right\}\right\}$$

$$= 0$$

$$u_3(x,t) = S^{-1}\left\{\frac{1}{v^\beta}S\left\{u_2(x,t)u_{2xx}(x,t) + [u_{2x}(x,t)]^2\right\}\right\}$$

$$= 0$$

$$\vdots$$

therefor

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \ldots$$

$$= x + t$$

4. Conclusions

Solving problems of differential equations By using homotopy perturbation method and Sadik transformation is a simple and effective solution.

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