Prime Labeling of Nauru Graph

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Abstract
A graph G = (V(G), E(G)) is observed to admit prime labeling, if a graph that receives prime labeling is called prime graph. In this research article we investigate that the Nauru graph admits prime labeling. We construct the mirror graph and shadow graph of the Nauru graph. We also establish prime labeling using some graph operations such as duplication. switching and fusion with few ideas.

Keywords: Nauru Graph, Prime Labeling, Duplication, Fusion

1. INTRODUCTION

In this paper, we define a connected and undirected graph name Nauru graph and we denote the vertex set by V(G) and edge set by E(G) of graph G and their corresponding cardinality by |V(G)| and |E(G)|.Here we establish that Nauru graph admits prime labeling.

2. PRELIMINARIES

Definition 2.1[1].

The Nauru graph is a symmetric bipartite 3-regular undirected graph with 24 vertices and 36 edges. It is a graph with girth 6. It is named by David Epstein after the twelve pointed star in the flag of Nauru [1].

Definition 2.2[4].

Let G be a bipartite graph with partite sets V_1 and V_2 and G' be the copy of G with corresponding partite sets V'_1 and V'_2. The mirror graph M(G) of G is obtained from G and G' by joining each vertex ofV_2 to its corresponding vertex in V_2 by an edge. The concept of Mirror graphs was introduced by Bresar et al. in 2004 as an intriguing class of graphs.

Definition 2.3[2].

Duplication of a vertex v_i of a graph G constructs a new graph G_1 by adding a vertex v'_i with $N(v_i) = N(v'_i)$ In other words, a vertex v'_i is said to be a duplication of the vertex v_i if all the vertices which are adjacent to v_i in G are now adjacent to v'_i in G_1.

Definition 2.4[2].

A vertex switching G_s, of a graph G is obtained by taking a vertex v of G and by removing the entire edges incident with u and v adding edges joining v to every vertex which are not adjacent to v in G.

Definition 2.5[2].

Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by fusing (identifying) two vertices u and v by a single vertex x in G_1 such that every edge which was incident with either u or v in G now incident with x in G_1.

3. PRIME LABELING OF NAURU GRAPH

Theorem 3.1.

The Nauru graph is a prime graph.

Proof.

Let G be the Nauru graph with 24 vertices and 36 edges. The vertex set (G) = {v_1, v_2,…v_{12},u_1. , u_{12}}. In general V(G) = {v_i , u_i/1 ≤ i ≤12} and |V(G)| = 24.

The edge set $E(G) = \{v_i v_{i+1}, 1 \le i \le 11\} U\{v_{12}v_1\} U\{u_{2i}u_{11+2i}, i = 1,2\} U\{u_{2i}u_{2i-5}, 3 \le i \le 6\} U\{u_{2i+7}u_{2i+2}, i = 1,2\} U\{u_{2i-5}u_{2i+2}, 3 \le i \le 5\} U\{u_7u_2\} U\{v_{2i}u_{2i}, 1 \le i \le 6\} U\{v_{2i+7}u_{2i+7}, i = 1,2\} U\{v_{2i-5}u_{2i-5}, 3 \le i \le 6\}$ and | EG)| = 36.

Let us define a labeling f: V(G)→{1, 2, 3, , 24} by

$f(v_i) = i, 1 \le i \le 12$, $f(u_{2i}) = 2i + 11, i = 1,2$,

$f(u_{2i+7}) = 2i + 12, i = 1,2$, $f(u_{2i-5}) = 2i + 12, 3 \le i \le 6$

Then

$$\gcd(f(v_i), f(v_{i+1})) = 1$$
$$\gcd(f(v_{12)}, f(v_1)) = 1$$
$$\gcd(f(u_{2i}) f(u_{11+2i})) = 1$$
$$\gcd(f(u_{2i}) f(u_{2i-5})) = 1,$$
$$\gcd(f(u_{2i+7}) f(u_{2i+2})) = 1,$$
$$\gcd(f(u_{2i-5}) f(u_{2i+2})) = 1$$
$$\gcd(f(u_7) f(u_2)) = 1,$$
$$\gcd(f(v_{2i}) f(u_{2i})) = 1,$$
$$\gcd(f(v_{2i+7}) f(u_{2i+7})) = 1,$$
$$\gcd(f(v_{2i-5}) f(u_{2i-5}) = 1$$

Therefore G is a prime graph.

4. CONSTRUCTION OF MIRROR GRAPH M(G) OF NAURU GRAPH.

STEP :1

Consider the Nauru graph G with 24 vertices and 36 edges.The vertex set V(G) = {v_i , u_i/1 ≤ i ≤12} . And |V(G)|=24.

The edge set $E(G) = \{v_i v_{i+1}, 1 \le i \le 11\} U\{v_{12}v_1\} U\{u_{2i}u_{11+2i}, i = 1,2\} U\{u_{2i}u_{2i-5}, 3 \le i \le 6\} U\{u_{2i+7}u_{2i+2}, i = 1,2\} U\{u_{2i-5}u_{2i+2}, 3 \le i \le 5\} U\{u_7u_2\} U\{v_{2i}u_{2i}, 1 \le i \le 6\} U\{v_{2i+7}u_{2i+7}, i = 1,2\} U\{v_{2i-5}u_{2i-5}, 3 \le i \le 6\}$.

And |E(G)|=36 . G is a bipartite graph with partite sets .

V_1(G)={v_i, u_i/i = 1,3,5,7………,11} and

V_2(G)={ v_i, u_i /i = 2,4,………,12}

STEP : 2

Let G$'$ be the copy of the Nauru graph G with 24 vertices 36 edges. The vertex set V(G$'$) ={x_i, w_i/ 1 ≤ i≤24} and |V()| = 24.

The edge set $E(G') = \{x_i x_{i+1}, 1 \le i \le 11\} U \{x_{12}x_1\} U \{w_{2i}w_{11+2i}, i = 1,2\} U \{w_{2i}w_{2i-5}, 3 \le i \le 6\} U \{w_{2i+7}w_{2i+2}, i = 1,2\} U \{w_{2i-5}w_{2i+2}, 3 \le i \le 5\} U \{w_7 w_2\} U \{x_{2i}w_{2i}, 1 \le i \le 6\} U \{x_{2i+7}w_{2i+7}, i = 1,2\} U \{x_{2i-5}w_{2i-5}, 3 \le i \le 6\}$

. And |E(G')|=36 . G' is a bipartite graph with partite sets .

$V_1'(G')$={ x_i, w_i u_i / i = 1,3,5,7,……….,11} and

$V_2'(G')$={ x_i, w_i / i = 2,4,……….,12}.

Where $V_1'$and V_2' are copies of $V_1$and V_2 respectively.

STEP :3

Let M(G) be the mirror graph of G. The mirror graph M(G) of G is obtained from G and G' by joining each vertex in V_2' by additional edges { v_i, x_i / i=1,3,5,….23}

V[M(G)] = { v_i, x_i/ $1 \le i \le 24$} is the vertex set of M(G).

$E(M(G)) = \{v_i v_{i+1}, 1 \le i \le 11\} U \{v_{12}v_1\} U \{u_{2i}u_{11+2i}, i = 1,2\} U \{u_{2i}u_{2i-5}, 3 \le i \le 6\} U \{u_{2i+7}u_{2i+2}, i = 1,2\} U \{u_{2i-5}u_{2i+2}, 3 \le i \le 5\} U \{u_7 u_2\} U \{v_{2i}u_{2i}, 1 \le i \le 6\} U \{v_{2i+7}u_{2i+7}, i = 1,2\} U \{v_{2i-5}u_{2i-5}, 3 \le i \le 6\} U \{$ v_i, x_i / i=1,3,5,….23}

In the edge set of M(G) |V(M(G))| = 48.

5. Duplication of a Vertex of Nauru Graph

Theorem 5.1

The Graph obtained by duplication of any Arbitrary Vertex of Nauru Graph is Prime Graph.

Proof:

Let G be a Nauru Graph with 24 vertex and 36 edge. The vertex set

V(G) = {v_i , u_i/1 ≤ i ≤12} and |V(G)| = 24.

The edge set $E(G) = \{v_i v_{i+1}, 1 \le i \le 11\} U \{v_{12}v_1\} U \{u_{2i}u_{11+2i}, i = 1,2\} U \{u_{2i}u_{2i-5}, 3 \le i \le 6\} U \{u_{2i+7}u_{2i+2}, i = 1,2\} U \{u_{2i-5}u_{2i+2}, 3 \le i \le 5\} U \{u_7 u_2\} U \{v_{2i}u_{2i}, 1 \le i \le 6\} U \{v_{2i+7}u_{2i+7}, i = 1,2\} U \{v_{2i-5}u_{2i-5}, 3 \le i \le 6\}$

and |E(G)|=36.

Let G_d represent duplication graph arbitrary vertex of G.

The Vertex Set V(G_d)= { v_i , u_i /1 ≤ i ≤12}U{ v_i ,or u_i /1 or 2 or…or 12} and |V(G_d)|=25

The edge set

$E(G) = \{v_i v_{i+1}, 1 \le i \le 11\} U \{v_{12}v_1\} U \{u_{2i}u_{11+2i}, i = 1,2\} U \{u_{2i}u_{2i-5}, 3 \le i \le 6\} U \{u_{2i+7}u_{2i+2}, i = 1,2\} U \{u_{2i-5}u_{2i+2}, 3 \le i \le 5\} U \{u_7 u_2\} U \{v_{2i}u_{2i}, 1 \le i \le 6\} U \{v_{2i+7}u_{2i+7}, i = 1,2\} U \{v_{2i-5}u_{2i-5}, 3 \le i \le 6\}$U{the 3 edges of v_1' or u_1' adjacent to all those vertices which are adjacent to v_1} and |E(G_d)| = 39.

Let us define a labeling f: V(G)→{1, 2, 3, , 24} by

$f(v_i) = i, 1 \le i \le 12,\ f(u_{2i}) = 2i + 11, i = 1,2,$

$f(u_{2i+7}) = 2i + 12, i = 1,2\ ,\ \ f(u_{2i-5}) = 2i + 12, 3 \le i \le 6$

$f(v_i') = 25$

this pattern of labeling admits prime .

Therefore G_d is prime graph .

6. Switching of Nauru graph

Theorem

The graph G attained by Switching of vertex v_1 of a Nauru Graph is Prime.

Proof

Let $G_s$represents the Switching vertex graph $v_1$of Nauru Graph.

$V(G_S)=\{ v_i, u_i / 1 \le i \le 12\}$ $|v(G_S)|=24$.

$E(G) = \{v_i v_{i+1}, 1 \le i \le 11\} U \{v_{12} v_1\} U \{u_{2i} u_{11+2i}, i = 1,2\} U \{ u_{2i} u_{2i-5}, 3 \le i \le 6\} U \{u_{2i+7} u_{2i+2}, i = 1,2\} U \{u_{2i-5} u_{2i+2}, 3 \le i \le 5\} U \{u_7 u_2\} U \{v_{2i} u_{2i}, 1 \le i \le 6\} U \{v_{2i+7} u_{2i+7}, i = 1,2\} U \{v_{2i-5} u_{2i-5}, 3 \le i \le 6\} U \{ v_i, x_i / 3 \le i \le 23, i \ne 8\}$

and $|E(G)|=56$.

Let us define a labelling f: V(G)→{1, 2, 3, , 24} by

$f(v_i) = i, 1 \le i \le 12, \ f(u_{2i}) = 2i + 11, i = 1,2,$

$f(u_{2i+7}) = 2i + 12, i = 1,2, \quad f(u_{2i-5}) = 2i + 12, 3 \le i \le 6$

the above pattern of labelling G_S admits prime labelling.

Therefore It is Prime Graph.

7. Fusing of two vertices.

Theorem:

The graph attained by fusing v_1 and u_{12} of a Nauru graph is prime graph.

Proof:

Let G_f be a graph attained by fusing vertices v_1,u_{12} as one of vertex u in Nauru graph.

$V(G_f) = \{ v_i, u_i / 2 \le i \le 12\} U \{u\}$ and

$|V(G_f)| = 23$.

$$E(G_f) = \{v_i v_{i+1}, 1 \le i \le 11\} U \{v_{12} v_1\} U \{u_{2i} u_{11+2i}, i = 1,2\} U \{ u_{2i} u_{2i-5}, 3 \le i \le 6\} U \{u_{2i+7} u_{2i+2}, i = 1,2\} U \{u_{2i-5} u_{2i+2}, 3 \le i \le 5\} U \{u_7 u_2\} U \{v_{2i} u_{2i}, 1 \le i \le 6\} U \{v_{2i+7} u_{2i+7}, i = 1,2\} U \{v_{2i-5} u_{2i-5}, 3 \le i \le 6\}$$

Define f: V(G)→{1, 2, 3, , 23} by

$f(v_i) = i, 1 \le i \le 12, \ f(u_{2i}) = 2i + 11, i = 1,2,$

$f(u_{2i+7}) = 2i + 12, i = 1,2, \quad f(u_{2i-5}) = 2i + 12, 3 \le i \le 6$

The above pattern admits prime labeling.

Conclusion:

In this we proceed Nauru graph admits Prime labeling and also constructed mirror graph , and also established prime labeling using operations such as duplication , switching and fusion.

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