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# On the Unification of Low Luminosity Active Galactic Nuclei

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**Abstract**
We have used the distributions of observed radio properties of a subset of AGNs called low luminosity AGNs (LLAGNs) to investigate the relationship between radio-selected BL Lacs (RBLs and XBLs) and Fanaroff-Riley Type I radio galaxies (FRI) in accordance with the unified scheme. Analysis of the core and extended luminosity show that RBLs are more core dominated than XBLs which are also more core dominated than FRI radio galaxies. Distributions of core dominance parameter (R) give $R_m \approx 42.41 \pm 10.77$, $5.08 \pm 1.57$, and $0.34 \pm 0.0024$ for RBLs, XBLs, and FRI, corresponding to mean viewing angles ($\phi_m$) in the range $\phi \approx 7^0 - 17^0$, $13^0 - 24^0$ and $20^0 - 28^0$ respectively. Regression analysis of the three samples shows that the core to extended luminosity plot ($L_C - L_E$) yields a strong correlation of $r \approx 0.67\pm0.52$, $0.85\pm0.46$, and $0.79\pm0.61$ for FRI, RBLs and XBLs at 95% confidence level. Similarly, regression analysis of R and $\phi$ for FRI, RBLs, and XBLs all yield a strong anti-correlation of $r \approx -0.98\pm0.02$, $-0.64\pm40.52$, and $0.75\pm4.45$. These results are consistent with BL Lac/FRI unification and suggest that relativistic beaming of the core and extended emissions is significant in low luminosity AGNs and thus a strong support for the unified scheme.

**Keywords:** Active Galaxies, Relativistic beaming, BL Lacertae

## 1.0 Introduction

Since the first example was discovered in the 1940s, Active galactic Nuclei (AGNs) have captured the interest of a growing number of astronomers. According to Longair (2010), one-fifth of all astronomers' research focuses on active galaxies, indicating their importance in astronomy and astrophysics. Fan et al. (2007); Longair (2010) showed that the ultimate source of AGN's luminosity is the gravitational potential of gas in the vicinity of a supermassive black hole with a mass of $M = 10^9 M_0$, which loses angular momentum through viscous or turbulent processes with a typical luminosity range of $10^{33}$–$10^{40}W$. AGNs are divided into two classes, conventionally called Radio-quiet (RQ) and Radio-loud (RL) Urry & Padovani (1995). The Radio-loud objects have emission contributions from both the jet(s) and the lobes such that these emission contributions dominate the luminosity of the AGNs at radio wavelengths while Radio-quite has luminosity less dominated by the radio wavelengths. Experiments have shown e.g., Sramek & Weedman (1980); Stocke et al. (1992) that the ratio of 5$GHz$ total radio to optical power in the blue band ($L_{5GHz}/L_B$) of Radio-loud AGNs ≥ 10 with radio luminosity at $L_{5GHz} \geq 10^{25}$W otherwise they are called Radio-quite.

### 1.1 Low Luminosity AGNs (LLAGNs)

LLAGNs are a sub-class of radio-quiet AGN that exhibit extreme luminosities, superluminal velocities and relativistic beaming from both the lobe and the core of the galaxies (Pei et al., 2020). LLAGNs can be separated into BL lacerate (BL Lacs) and Fanaroff-Riley Type I (FRI) radio galaxies. The BL Lacs were found as a unique type of Radio-loud low luminosity AGNs with extraordinary luminosities and variability (Urry & Padovani, 1995), with Radio-selected BL Lacs (RBLs) and X-ray-selected BL Lacs (XBLs) as two sub-categories (Fossati et al., 1998). The RBLs peaked in the infrared to the optical range of the electromagnetic spectrum (E.M), while XBLs peaked in the x-ray region of the E.M spectrum to the higher energy regime, with emission dominated by a relativistic jet aligned within the line of sight. Observations and studies have demonstrated that these two sub-classes have distinct characteristics, which can be described using a relativistic beaming model (Jackson & Wall, 1999; Seal Braun, 2010; Wang et al., 2006). On the other hand, FRI radio sources have symmetric radio jets with prolonged twin lobe structures and emission intensities that decline away from the core (Jackson & Wall, 1999).

### 1.2 Unification Scheme LLAGNs

The construction of "a unified scheme," a paradigm in which the observational qualities of distinct classes of AGNs may be explained as being inherently similar yet viewed at different orientation angles to the line of sight, has been a major focus of contemporary AGN research (Kollgaard et al., 1995; Odo et al., 2012; Pei et al., 2019; Ubachukwu & Chukwude, 2002; Urry & Padovani, 1995). This kind of unification of extragalactic radio sources is based on one of two assumptions: pure orientation (for radio-quiet AGNs) or orientation together with relativistic beaming (for radio-loud AGNs), thus connecting BL Lacs to radio galaxies with low luminosity (FRI). However, recent studies e.g. Odo et al. (2012) have shown a striking dichotomy in the spectral energy distributions (SED) of XBLs and RBLs suggesting that the two BL Lac populations may be intrinsically different. Furthermore, in a study by (Kollgaard et al., 1996; Odo et al. (2012), it was found that BL Lacs objects and FRI radio galaxies show similar extended radio luminosity and that BL Lacs objects and FRI radio galaxies are located in the same luminosity and core-dominance plot. The motivation for this research lies within these premises.

Many authors e.g. Fossati et al. (1998); Odo et al. (2017); Wang et al. (2006), agree that XBLs and RBLs are manifestations of similar physical phenomena that differ only in orientation, thus suggesting that RBLs and XBLs are the extremes of a continuous distribution of the BL Lac population. However, recent studies e.g. Odo et al. (2017) in which a remarkable dichotomy was observed in the spectral energy distributions (SED) of XBLs and RBLs suggest that the two BL Lac populations may be intrinsically different. It is thus not clear which of the BL Lac populations form the beamed counterparts of the FRI radio galaxies. In this study, we investigate the unification of these LLAGNs by considering RBLs and XBLs as intrinsically different manifestations of the beamed counterpart FRI to understand the relationship between them and the physics of their radio source components. The purpose of this research, therefore, is to test for the unification of BL Lacs (RBLs and XBLs) with FRI using statistical techniques. Furthermore, we shall:

- assemble observed data of different sub-samples of LLAGNs from the literature.
- determine the relationship between the distributions of the sub-samples in various parameters space.

- determine the average viewing angle of each sub-sample from the observed data.
- determine the nature and strength of correlation among various beaming and orientation parameters of the three sub-samples.

In light of the above, the synopsis of the present article is: In § (2), we make a brief review of the data selection process, and the constraints imposed on their parameters. In § (3), we present the theoretical relationship underlying the physics of AGNs and their applications as widely understood by applying distribution and correlation statistics for analyzing the relationship of the sub-samples. In §(4), we present the results of the application of theoretical laws and statistics to our data sets for the three sub-samples obtained from the literature and discuss its implication as applied to the unified scheme for low luminosity radio sources. Lastly, in § (5), a general discussion is presented on our expectation for the unified scheme, and conclusions are drawn respectively.

## 2.0 Data Description and Source

BL Lac objects (RBLs and XBLs) are inherently identical to other low luminosity radio counterparts under the present relativistic beaming and orientation-based unification approach for LLAGNs The relativistic effects on the apparent luminosity of emitting matter moving at relativistic speeds are known as relativistic beaming (Iyida et al., 2022; F. Odo et al., 2012; Odo et al., 2017); Onuchukwu & Ubachukwu 2013). A direct comparison of the extended luminosity ($L_E$) and core luminosity ($L_C$) of the two sub-classes of BL Lacs with that of the FRI should show a significant correlation, thereby supporting the FRI-BL Lacs unified scheme (Kollgaard et al., 1996). For a better understanding of BL Lacs-FRI unification based on the premise of the unified scheme, we will quantitatively investigate the relationship between these sub-classes based on already established relativistic models for which the beaming exponent (n) can be either 2 (jet model) or 3 (blob model) using statistical methods. These statistical analyses are aimed at comprehending the behaviour of the two-sub class of BL Lacs population (RBLs and XBLs) with that of FRI samples under relatively near physical conditions. This implies that we will determine if there is continuity in the observed beaming and orientation parameters of the three sub-samples (i.e., the core dominance parameter (R), viewing angle ($\phi$), and projected linear size (D). Notwithstanding, we will also bear in mind that FRI is the parent population of BL Lacs (RBLs and XBLs). These analyses if properly carried out will mean that any observed properties of these parameters over a range of frequencies will be compared for the support of the unified scheme.

## 2.1 Sources of Data

Kollgaard et al. (1996) compiled x-ray-selected BL Lacs objects (XBLs) detected by First High-Energy Astrophysics Observatory Large Area Sky Survey (HEAO-1 LASS ) and compared their core and extended luminosity with radio-selected BL Lacertae objects (RBLs) from Einstein Medium Sensitivity Survey (EMSS) (Rector et al., 2000) in the range of $10^{23}$ to $10^{25}$WHz$^{-1}$. Zhou et al. (2007) later used these same data to do a statistical study of XBLs, RBLs and FSRQs at 1.5GHz. In our work however, we did a careful selection of the data from Zhou et al. (2007) and Rector et al. (2000) with specific reference to the core luminosity ($L_C$), extended luminosity ($L_E$), core dominance parameter (R), and Redshift (z) for the RBL and XBL samples. Furthermore, we employed Fanaroff-Riley Type I (FRI) radio samples from Wang et al. (2006) to complete our sample selection. Information regarding the projected linear sizes (D) for RBLs and XBLs samples was updated using 1.5GHz data from Odo et al. (2011). Finally, after completing the sample parameters at 5GHz for the FRI sample, their luminosity information from Odo et al. (2011); Wang et al., (2006) at 5GHz was tuned down to 1.5GHz assuming Fan et al. (2011); Pei et al., (2019, 2020) using the equations:

$$L^{5\ \ GHz} = L^{5}{}_{GHz}{}^{5}\left(\frac{5}{1.5}\right)^{\alpha_{core}-\alpha_{ext}} . \tag{1}$$

For the core luminosity in which $\alpha_{Core} = 0$, we obtained:

$$L_C^{5GHz}{}_{GHz} = L_C^{1.5GHz} \tag{2}$$

while for the extended luminosity for which $\alpha_{ext} = 1$ we obtained:

$$L_E^{5GHz} = L_E^{1.5GHz}\left(\frac{1.5}{5}\right)^{0-1} = L_E^{1.5GHz}\left(\frac{1.5}{5}\right)^{-1} = 3.33L_E^{1.5GHz} \tag{3}$$

After tuning down the frequencies, the core dominance parameter (R) for the FRI samples was then recalculated using the expression Fan et al. (2011); Pei et al., (2019, 2020):

$$R = \frac{L_{core}}{L_{ext}}(1-z)^{\alpha_{core}-\alpha_{ext}} \tag{4}$$

In Equations 1 to 4, $\alpha$ is the spectral index ($S_v \alpha S^{\pm\alpha}$) and $z$ is the Redshift for the FRI samples. We have assumed $\alpha_{ext} = 1$ for the extended luminosity because they are lobe-dominated sources with steep spectra and $\alpha_{core} = 0$ for the core luminosity because they are core-dominated sources with flat spectra (see Fan et al. (2007); Fan et al. (2011); Odo et al. (2012); Pei et al. (2020); Pei et al. (2019); Ubachukwu (1997); Zhou et al. (2007))

The ratio of the two luminosity components $L_C/L_E$ is called the core dominance parameter (R). Some authors use the ratio of the flux densities while others e.g. Ubachukwu (1997) use the ratio of core-to-lobe luminosity to quantify this parameter. After our compilations and frequency reductions, we were left with a total of 24 RBLs, 17 XBLs, and 28 FRI radio galaxies with complete information on Redshift (z), core luminosity ($L_C$), extended luminosity ($L_E$), core dominance parameter(R), and projected linear size (D), making a total of sixty-eight (68) samples. Other parameters displayed in Table 1 (columns 8 and 9) were determined by calculation.

Table 1: RBL data table compiled from Odo et al. (2011); Zhou et al., (2007) at 1.5GHz

| Source | ID | $z$ | $LogL_C$ $(WHz^{-1})$ | $LogL_E$ $(WHz^{-1})$ | $R$ | $D$ $(KPc)$ | $\phi_n$=2 | $\phi_n$=3 |
|---|---|---|---|---|---|---|---|---|
| 0118–272 | RBL | 0.559 | 26.90 | 25.60 | 14.70 | 186.20 | 7.70 | 17.20 |
| 0235+164 | RBL | 0.94 | 27.70 | 25.70 | 61.40 | 181.00 | 5.38 | 13.57 |
| 0426–380 | RBL | 1.03 | 27.30 | 26.10 | 8.60 | 71.40 | 8.81 | 18.81 |
| 0537–441 | RBL | 0.894 | 28.00 | 26.00 | 63.20 | 152.60 | 5.35 | 13.50 |
| 0735+178 | RBL | 0.424 | 27.10 | 25.10 | 72.60 | 41.60 | 5.16 | 13.20 |
| 0814+425 | RBL | 0.258 | 26.60 | 25.20 | 20.40 | 163.10 | 7.10 | 16.29 |
| 0820+225 | RBL | 0.951 | 27.60 | 27.20 | 1.70 | 153.40 | 13.21 | 24.66 |
| 0823+033 | RBL | 0.506 | 27.10 | 24.60 | 242.60 | 212.40 | 3.82 | 10.81 |
| 0851+202 | RBL | 0.306 | 26.80 | 24.80 | 83.90 | 245.80 | 4.98 | 12.88 |
| 0954+658 | RBL | 0.367 | 26.80 | 25.00 | 52.00 | 53.90 | 5.61 | 13.95 |
| 1308+326 | RBL | 0.996 | 27.20 | 26.20 | 18.40 | 296.40 | 7.28 | 16.57 |
| 1418+546 | RBL | 0.152 | 26.00 | 24.20 | 48.90 | 171.80 | 5.70 | 14.09 |
| 1514–241 | RBL | 0.049 | 25.40 | 23.50 | 77.40 | 495.60 | 5.08 | 13.06 |
| 1538+149 | RBL | 0.605 | 27.20 | 26.30 | 6.40 | 41.40 | 9.49 | 19.76 |
| 1652+398 | RBL | 0.033 | 24.80 | 23.60 | 18.00 | 37.40 | 7.32 | 16.63 |
| 1749+701 | RBL | 0.77 | 27.10 | 25.90 | 10.20 | 47.90 | 8.44 | 18.28 |
| 1803+784 | RBL | 0.684 | 27.40 | 26.00 | 15.90 | 623.00 | 7.55 | 16.98 |
| 1807+698 | RBL | 0.051 | 25.20 | 25.00 | 1.40 | 129.80 | 14.10 | 25.79 |
| 1823+568 | RBL | 0.664 | 27.10 | 26.80 | 1.70 | 273.00 | 13.20 | 24.66 |
| 2007+777 | RBL | 0.342 | 26.50 | 25.10 | 23.00 | 304.10 | 6.89 | 15.97 |
| 2131–021 | RBL | 0.557 | 27.30 | 25.70 | 30.50 | 253.90 | 6.42 | 15.24 |
| 2200+420 | RBL | 0.069 | 25.80 | 23.90 | 79.00 | 120.40 | 5.05 | 13.01 |
| 2240-420 | RBL | 0.774 | 27.30 | 25.80 | 2.40 | 296.80 | 6.84 | 15.90 |
| 0219–164 | XBL | 0.698 | 26.70 | 26.10 | 3.00 | 96.00 | 11.47 | 22.42 |

Table 2: XBL data table compiled from Odo et al. (2011); Wang et al. (2006); Zhou et al. (2007) at 1.5GHz

| Source | ID | $z$ | $LogL_C$ $(WHz^{-1})$ | $LogL_E$ $(WHz^{-1})$ | $R$ | $D$ $(KPc)$ | $\phi_n$=2 | $\phi_n$=3 |
|---|---|---|---|---|---|---|---|---|
| 0219–164 | XBL | 0.698 | 26.70 | 26.10 | 3.00 | 96.00 | 11.47 | 22.42 |
| 0323+022 | XBL | 0.147 | 24.70 | 23.80 | 7.00 | 17.00 | 9.28 | 19.46 |
| 0414+009 | XBL | 0.287 | 25.30 | 25.10 | 1.30 | 262.00 | 14.15 | 25.79 |
| 0506–039 | XBL | 0.304 | 25.00 | 24.90 | 0.90 | 61.00 | 15.53 | 27.44 |
| 0521–365 | XBL | 0.055 | 25.70 | 26.20 | 0.30 | 51.00 | 20.48 | 33.03 |
| 0548–322 | XBL | 0.069 | 24.20 | 24.90 | 0.20 | 82.00 | 22.69 | 35.39 |
| 0829+046 | XBL | 0.18 | 25.90 | 24.90 | 8.30 | 114.00 | 8.89 | 18.92 |
| 1011+496 | XBL | 0.2 | 25.80 | 24.80 | 7.90 | 30.00 | 9.04 | 19.08 |
| 1101+384 | XBL | 0.031 | 24.00 | 24.10 | 1.60 | 90.00 | 13.43 | 24.91 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1101–232 | XBL | 0.186 | 24.80 | 25.00 | 0.50 | 204.00 | 18.00 | 30.30 |
| 1133+704 | XBL | 0.046 | 24.00 | 24.30 | 0.50 | 96.00 | 18.00 | 30.30 |
| 1652+398 | XBL | 0.034 | 24.80 | 23.60 | 18.00 | 65.00 | 7.32 | 16.63 |
| 1722+119 | XBL | 0.018 | 23.00 | 22.20 | 6.40 | 3.00 | 9.49 | 19.76 |
| 1727+502 | XBL | 0.055 | 24.20 | 23.80 | 2.60 | 15.00 | 11.89 | 22.96 |
| 1807+698 | XBL | 0.051 | 25.20 | 25.00 | 1.30 | 341.00 | 14.15 | 25.79 |
| 2007+777 | XBL | 0.342 | 26.50 | 25.10 | 23.00 | 166.00 | 6.89 | 15.97 |
| 2356–309 | XBL | 0.165 | 24.70 | 24.10 | 3.50 | 154.00 | 11.04 | 21.85 |

Table 3: FRI data table compiled from Odo et al. (2011); Wang et al. (2006) at 1.5GHz

| **Source** | **ID** | ***z*** | ***LogL<sub>C</sub>*** ***(WHz<sup>-1</sup>)*** | ***Log L<sub>E</sub>*** ***(WHz<sup>-1</sup>)*** | ***R*** | ***D (KPc)*** | $\phi_n$**=2** | $\phi_n$**=3** |
|---|---|---|---|---|---|---|---|---|
| 0034+25 | FRI | 0.0321 | 23.18 | 71.84 | 0.33 | 120.00 | 19.94 | 19.94 |
| 0055+26 | FRI | 0.0472 | 24.61 | 73.59 | 0.35 | 150.00 | 19.69 | 28.61 |
| 0104+32 | FRI | 0.0169 | 24.17 | 74.67 | 0.33 | 452.00 | 20.01 | 28.92 |
| 0116+31 | FRI | 0.0592 | 24.95 | 75.24 | 0.35 | 0.10 | 19.66 | 28.59 |
| 0149+35 | FRI | 0.0160 | 22.25 | 71.08 | 0.32 | 43.00 | 20.18 | 29.09 |
| 0755+37 | FRI | 0.0413 | 24.43 | 77.84 | 0.33 | 76.00 | 20.04 | 28.95 |
| 0915+32 | FRI | 0.0620 | 23.97 | 74.44 | 0.34 | 432.00 | 19.80 | 28.72 |
| 0924+30 | FRI | 0.0266 | 23.52 | 67.65 | 0.36 | 264.00 | 19.60 | 28.53 |
| 1113+24 | FRI | 0.1021 | 23.63 | 73.75 | 0.36 | 37.00 | 19.60 | 28.53 |
| 1256+28 | FRI | 0.0224 | 23.05 | 69.39 | 0.34 | 81.00 | 19.85 | 28.77 |
| 1321+31 | FRI | 0.0161 | 23.85 | 71.84 | 0.34 | 163.00 | 19.88 | 28.80 |
| 1322+36 | FRI | 0.0175 | 24.55 | 73.85 | 0.34 | 13.00 | 19.87 | 28.79 |
| 1339+26 | FRI | 0.0757 | 24.29 | 74.25 | 0.35 | 271.00 | 19.64 | 28.57 |
| 1346+26 | FRI | 0.0633 | 24.52 | 77.12 | 0.34 | 19.00 | 19.85 | 28.77 |
| 1357+28 | FRI | 0.0629 | 24.02 | 74.08 | 0.35 | 113.00 | 19.76 | 28.68 |
| 1422+26 | FRI | 0.0370 | 23.99 | 73.39 | 0.34 | 70.00 | 19.85 | 28.77 |
| 1430+28 | FRI | 0.0813 | 24.20 | 72.27 | 0.36 | 51.00 | 19.50 | 28.43 |
| 1502+26 | FRI | 0.0540 | 23.36 | 77.22 | 0.32 | 184.00 | 20.15 | 29.06 |
| 1511+26 | FRI | 0.1078 | 25.30 | 80.19 | 0.35 | 160.00 | 19.65 | 28.57 |
| 1521+28 | FRI | 0.0825 | 24.54 | 77.55 | 0.34 | 205.00 | 19.77 | 28.69 |
| 1525+29 | FRI | 0.0653 | 23.97 | 72.93 | 0.35 | 19.00 | 19.68 | 28.60 |
| 1527+30 | FRI | 0.1143 | 24.02 | 75.24 | 0.36 | 60.00 | 19.55 | 28.48 |
| 1610+29 | FRI | 0.0313 | 22.92 | 69.30 | 0.34 | 58.00 | 19.82 | 28.74 |
| 1613+27 | FRI | 0.0647 | 24.01 | 74.87 | 0.34 | 26.00 | 19.80 | 28.72 |
| 1626+39 | FRI | 0.0303 | 24.47 | 76.42 | 0.33 | 43.00 | 19.99 | 28.91 |
| 1827+32 | FRI | 0.0659 | 24.02 | 76.16 | 0.34 | 304.00 | 19.88 | 28.80 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2116+39 | FRI | 0.0164 | 22.70 | 72.86 | 0.32 | 106.00 | 20.20 | 29.11 |
| 2229+35 | FRI | 0.0181 | 24.02 | 72.96 | 0.34 | 312.00 | 19.91 | 28.83 |

## 3.0 Theory of Relationship

Various beaming and orientation parameters utilized for the testing of the unified scheme can be determined using these parameters (i) viewing Angle ($\Phi_m$), (ii) core dominance parameter (R), and (iii) Projected Linear Size (D). In the relativistic beaming and source orientation paradigm, these parameters are used to test for the unification of LLAGNs. Following, we shall discuss briefly these parameters and see how they are used in this work.

I. The viewing angle is defined as the angle between the jet and the line of sight of the observer(s). It is such that it gives rise to the different sub-classes of AGNs because it accounts for the orientation of the AGNs. Mathematically, the viewing angle can be computed by the expression:

$$\Phi_m = cos^{-1}\left[1-\left(\frac{2^{n-1}R_m)}{R_T}\right)^{-\frac{1}{n+\alpha}}\right] \quad (5)$$

where $R_m$ = the mean value of R for RBLs, XBLs, and FRIs, $n$ = relativistic beaming model (for continuous jet $n$ = 2 or blobs $n$ = 3), but generally $2 \leq n \leq 3$, $\alpha$ = spectral index (We have adopted $\alpha = 0$ because they are core dominated sources with flat spectra dominated by synchrotron process), $R_T$ is the minimum value of $R$, at $\phi = 90^0$. We also adopt $R_T = 0.0024$ because it appears to be consistent with the FRI–BL Lac's unification scheme for low luminosity surveys (Odo & Ubachukwu, 2013).

II. The core dominance parameter also known as the core-to-lobe ratio of an AGN is the ratio of the core luminosity to the extended or lobe luminosity of any active galactic nuclei. This parameter is a measure of the relative orientation of the source (Zhou et al., 2007). Mathematically, considering the relativistic property of the source for two opposite jets, the core dominance parameter was given by Fan (2003); Iyida et al. (2022); Odo et al. (2012); Ubachukwu & Chukwude (2002) as:

$$R = \left(\frac{L_C}{L_E}\right) = \frac{R_T}{2}\left[(1-\beta cos\phi)^{-n+\alpha} + (1+\beta cos\phi)^{-n+\alpha}\right] \quad (6)$$

where "β" denotes the source's relativistic velocity, "α" denotes its spectral index, and "ϕ" denotes its viewing angle.

III. Marchã et al. (2005); and Odo et al. (2012) demonstrated that the core dominance parameter (R) may appear to be due to beaming and projection effects, and as such should be related to the projected linear size (D) and the viewing angle (ϕ). Thus, the apparent relativistic effect of the core dominance parameter (R) and the projected linear size (D) depends on the viewing angle ($\phi$) of the source by the relationship:

$$D = D_0 \sin\phi \quad (7)$$

where $D_0$ is the intrinsic linear size and $\phi$ is the viewing angle obtained from equation (5) of the source.

## 4.0 Results and Discussion

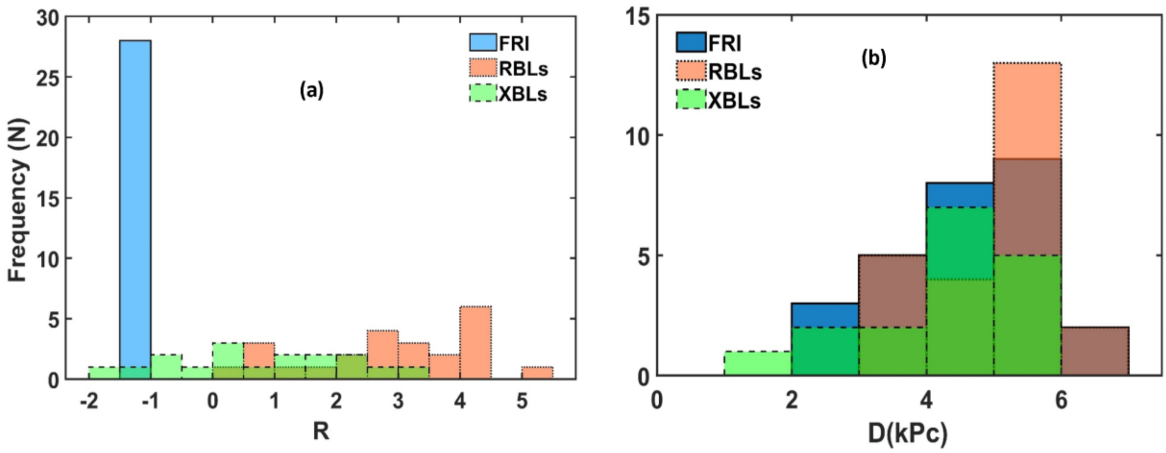
To investigate a unification scheme and comprehend the relationship between the numerous classes and subclasses of LLAGN, distributive analyses of the observed source parameters are crucial (Iyida et al., 2021). Similarly, correlations between different orientation parameters of these radio sources can be used to probe the physics of their radio source components (Ubachukwu, 1997). The core dominance parameter (R) for example is also a statistical indicator of source orientation in extragalactic radio sources (Orr & Browne, 1982). On this premise, we have statistically analysed the distribution of R, $L_C$, $L_E$, D and $\phi$ (Figures 1 to 5 under the beaming and orientation scenario (n = 2 and 3). Similarly, the correlation of R with other beaming and orientation parameters [$L_C$, $L_E$, D, and $\phi$ (n = 2,3)] as shown in Figures 6 to 10 for the three sub-classes of LLAGNs (RBLs, XBLs, and FRI) were also analysed to test for unification amongst them.

## 4.1 Distributions of observed radio source parameters for RBLs, XBLs, and FRI

The plots shown in Figures 1 to 4 give the distribution of the core luminosity ($L_C$), extended Luminosity ($L_E$), core-dominance parameter (R), Projected linear size ($D$), and viewing angle ($\phi$) for the BL Lacs (RBLs, XBLs,) and FRI all in log scale for simplicity. We adopted the blue colour for FRI, brown for RBLs, and green for XBLs for simplicity. Similarly, Table 6 shows a summary of the mean distributions we obtain alongside the standard error for the three sub-samples.

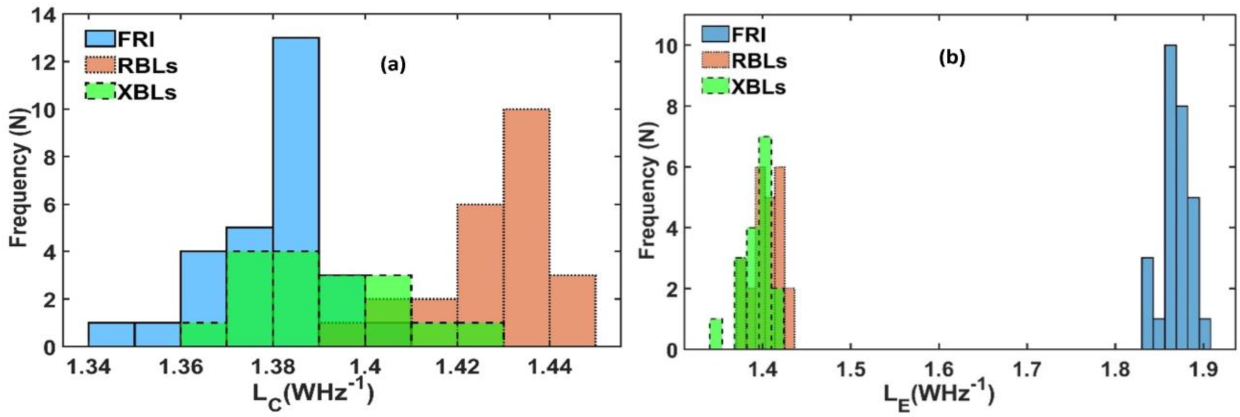
In Figure 1(a) and 1(b), $R_m$ for the three sub-samples yielded 0.34±0.0024, 42.41±10.77, and 5.08±1.57 for FRIs, RBLs, and XBLs respectively. The distribution shows that RBLs are larger than XBLs and FRI which implies that the RBLs are more core beamed than XBL and FRI as expected (Alhassan et al., 2013; Bai & Lee, 2001). Similarly, The average projected linear sizes ($D_m$) as shown in Table 6 for the three sub-samples yields 108.65 ± 22.31 for XBLs, 197.95 ± 30.23 for RBLs and 136.86 ± 23.62 FRI respectively. From Table 6, it is evident that the RBLs and FRI have larger linear sizes (D) than those of XBLs. If BL Lacs are of the same population as FRI then their linear sizes should be smaller. Our result shows that this may not be the case thus suggesting that the linear size (D) is again not a consistent parameter and thus not good for testing of unification for LLAGNs.

**Figure 1:** Distribution of (a) core dominance parameter (R) (b) projected linear size (D) for FRI, RBLs, and XBLs in a log scale



Figures 2 (a) and 2(b) show the distribution of the core and extended Luminosity. Table 6 shows that the mean value for the core luminosity ($L_C - mean$) for the FRI, RBLs, and XBLs are: 23.95 ± 0.13, 26.73 ± 0.18 and 24.97 ± 0.23 respectively. The core luminosity for the three sub-samples can be compared for unification since they all fall within the same range in Figure 2 (a) and (b). Furthermore, we noticed that the core luminosity of FRI is slightly lower for the three samples. This should mean that FRI is not as core beamed as RBLs and XBLs, thus strong support for the unified scheme. In the context of the unified scheme as shown by Fan et al. (2011), isotropic properties such as the extended luminosity are expected to be shared equally by both the RBLs and XBLs with their parent population FRI. This is not the case for our samples, as the distribution of the $L_E$ from Figure 2(b) shows otherwise. One obvious explanation for this is that the $L_E$ for FRI are more isotropic than RBLs and XBLs which is also a strong support for the unified scheme.

**Figure 1:** Distribution of (a) core dominance parameter (R) (b) projected linear size (D) for FRI, RBLs, and XBLs in a log scale




From our distributions, as shown in Figures 3(a) and 3(b) which are outlined in Table 6, using $R_T$ = 0.0024 given in Equation 5, we obtained the mean viewing angles for RBLs as $\phi_m \approx 7^0 - 17^0$, XBL as $13^0$- $24^0$ and FRI as $20^0$- $28^0$ for n = 2 and 3 respectively. In the simplest beaming and radio source orientation scenario, Odo et al. (2012); and Zhou et al. (2007) have shown that large values of R are attributed to Doppler boosting of the core emission at small viewing angles. Our result confirms this statement as the $R_m$ value for the three sub-samples showed that the least of the samples FRI with $R_m$ of 0.34 ± 0.0024 has a corresponding viewing angle $\phi \approx 20^0$ and $28^0$, followed by XBLs with $R_m$ of 5.08 ± 1.57 corresponding to $\phi_m \approx 13^0$ and $24^0$ and RBLs with $R_m$ of 42.41 ± 10.77 corresponding to $\phi_m \approx 7^0$ and $17^0$ for n = 2 and 3 respectively. Thus, there exists an inverse relationship between the core dominance parameter (R) and the viewing angle ($\phi$). This according to the unified scheme implies that the smaller the viewing angle, the more the source seems to align with the line of sight (Odo, 2012; Ubachukwu, 1997), thus supporting the unification of BL Lacs and FRI radio galaxies. On the other hand, the linear size is supposed to reduce progressively as the beaming angle reduces. This was not observed in the case of three samples, showing a strong indication that the linear size is not consistent with the BL Lac-FRI unification.

**Figure 3:** Distribution of (a) the Viewing angle (ϕ) for n=2 and (b) n=3 for FRI, RBLs, and XBLs in log scale.

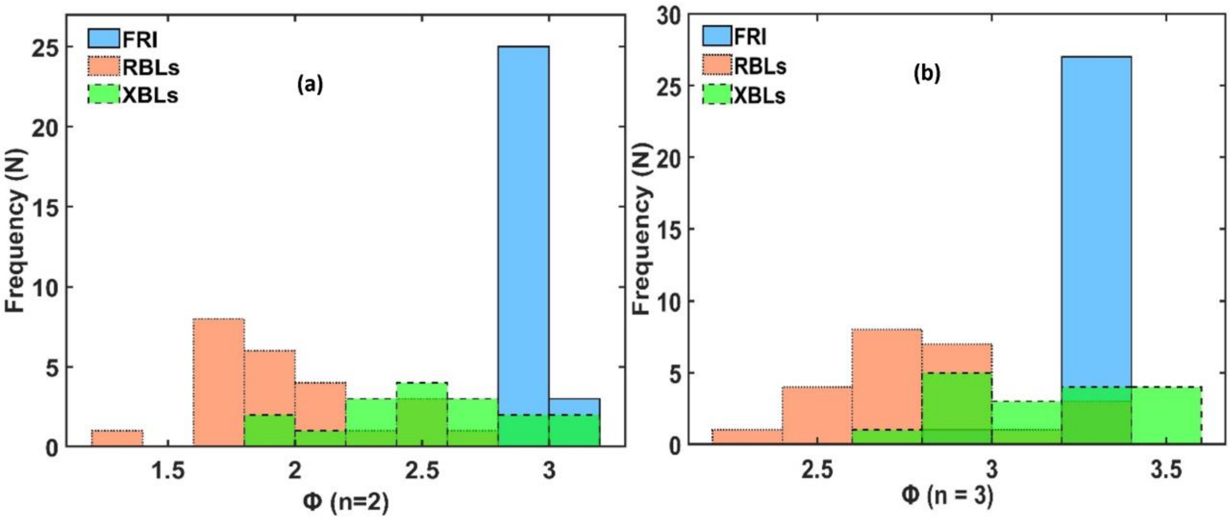



**Table 4**: Summary of Mean Distribution for RBLs, XBLs, and FRI with Standard Errors

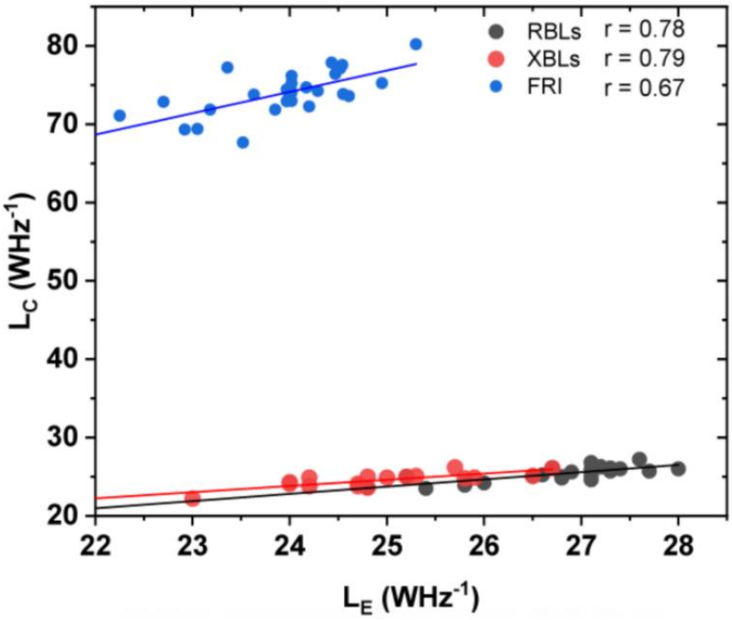
| **Parameter** | **FRI** | **RBL** | **XBL** |
|---|---|---|---|
| R | 0.34 ± 0.0024 | 42.41 ± 10.77 | 5.08 ± 1.57 |
| $L_C$ | 23.95 ± 0.13 | 26.73 ± 0.18 | 24.97 ± 0.23 |
| $L_E$ | 74 ± 0.53 | 25.3 ± 0.20 | 24.58 ± 0.23 |
| D | 136.86 ± 23.62 | 197.95 ± 30.23 | 108.65 ± 22.31 |
| Φ (n = 2) | 19.83 ± 0.04 | 7.42 ± 0.58 | 13.05 ± 1.13 |
| Φ (n = 3) | 28.43 ± 0.32 | 16.56 ± 0.83 | 24.12 ± 1.39 |
| Sample Size | 28 | 23 | 17 |
| Range of ϕ | $20^{0}$- $28^{\circ}$ | $7^{0}$- $17^{\circ}$ | $13^{0}$- $24^{\circ}$ |

### 4.2 Correlation/Regression Analysis

The strength of the relationship between two or more variables is determined by the correlation coefficient (r) and the coefficient of determination ($r^2$) gotten from the scatter plot of the variables involved. In this section of our work, we have carried out a regression analysis between core luminosity ($L_C$) and extended luminosity ($L_E$), as well as a check for the dependence of the core dominance parameter (R) with the projected linear size (D) and viewing angle (ϕ), core luminosity ($L_C$), and extended luminosity ($L_E$) for the three sub-samples. Table 5 shows a summary of the correlation analysis we obtain from our analysis. The scatter plots of the observed parameters for the three samples are shown in Figures 4 to 8. We used a blue circle to represent FRI, a red circle for XBLs, and a black circle for RBLs respectively.

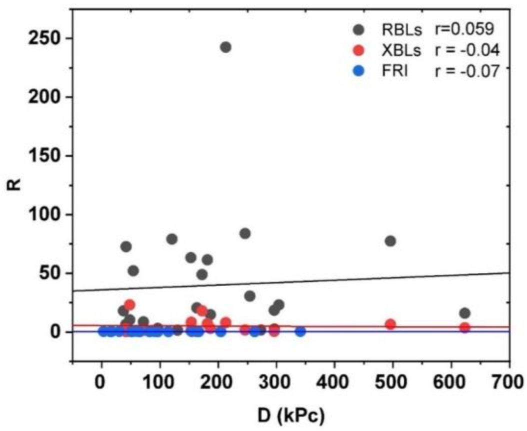
In the context of the unified scheme, as shown by Fan et al. (2011), isotropic properties such as the extended luminosity are expected to be shared by both BL Lacs and their parent population FRI. Furthermore, the relativistic beaming and radio source orientation model proposes that the core luminosity is expected to correlate with the extended luminosity for samples believed to obey BL Lacs-FRI unification. The $L_E - L_C$ scatter plot in Figure 4 shows that the three sub-sample all strongly correlate with $r \approx 0.67$, 0.78, and 0.79 for FRI, RBLs, and XBLs respectively. Regression analysis in Table 5 shows that the correlations are significant at a 95% confidence level, thus indicating strong support for the unified model. Similarly, the plot suggests that XBLs and RBLs can be classified as the same objects since they the occupy same position on the plot and also have almost the same value. FRI scatter data indicate that they are less core beamed since their $L_C - L_E$ values differ slightly from that of XBLs and RBLs. The implication is that RBLs and XBLs are more core beamed than FRI as expected, thus a strong support for the unification of LLAGNs.

**Figure 4:** Scatter plot of the core luminosity ($L_C$) against extended luminosity ($L_E$) for FRI, RBLs, and XBLs




On the other hand, our $R - D$ relations for FRI, RBLs, and XBLs, in Figure 5 showed no significant correlation with $r = -0.07$, 0.06 and −0.04 respectively. This may be a result of the projection effect when observed close to the line of sight which is still in the premise of relativistic beaming (Alhassan et al., 2013). An obvious implication of our result is that, since BL Lacs are believed to form the beamed counterparts of FRI sources, $R$–$D$ anti-correlation should be expected (Odo et al., 2012). This is also consistent with our result (Odo & Ubachukwu, 2013).

**Figure 5:** Scatter plot of the Core-dominance parameter (R) against projected linear size (D) for FRI, RBLs, and XBLs



In Figure 6, our R−ϕ log scatter plot for XBLs and RBLs produces a strong negative correlation with r = −0.75 and −0.64 respectively, signifying a close relationship between the two samples as predicted by the unified model for BL Lacs. Similarly, the $R - \phi$ relationship for FRI yields an almost perfect negative correlation with $r = -0.98$. This according to the beaming and orientation paradigm, signifies that FRI is the parent population of BL Lacs. Our result further suggests that RBLs are the more likely beamed counterpart of FRI than XBLs. Thus, according to the unified scheme, BL Lacs (RBLs and XBLs) with FRI radio galaxies will form a homogeneous class of AGNs but will appear different due to their relative orientation with respect to the line of sight (Zhou et al., 2007) and thus responsible for the beaming and orientation effect observed by these sub-classes of AGNs.

In the context of the current unified scheme, Wang et al. (2006) confirmed that the $R{-}L_C$ plot should also correlate for generally non-isotropic sources. Our result for $R{-}L_C$ data gives a correlation coefficient of $r$ = 0.78, 0.31, and -0.*59*, for RBLs, XBLs and FRI respectively, thus in agreement with this premise for non-isotropic sources. This result implies that amongst the three samples, FRI seems to be more isotropic than RBLs and XBLs due to the negative correlation we obtained. This is generally the case for FRI which is mostly isotropic, thus in support of the unification scheme for LLAGNs.

**Figure 6:** Scatter plot of the Core-dominance parameter (R) against viewing angle (ϕ) for FRI, RBLs, and XBLs (n=2)

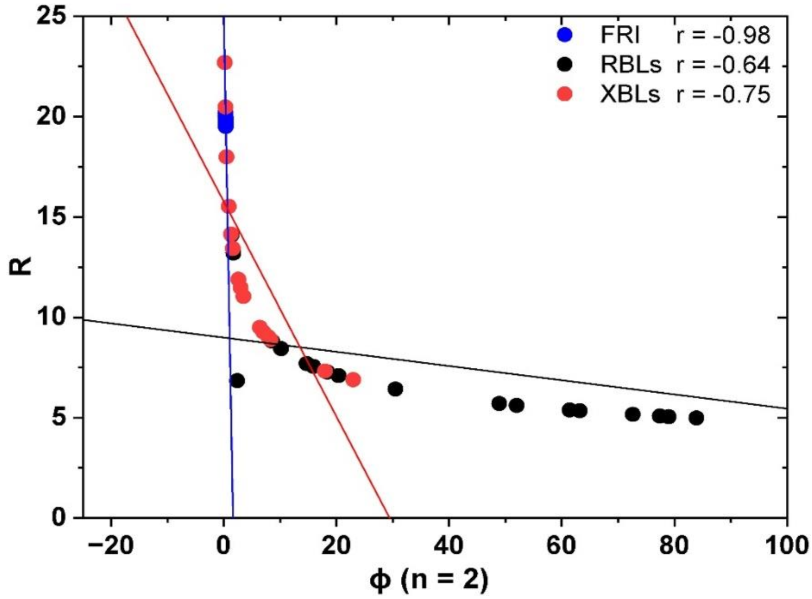
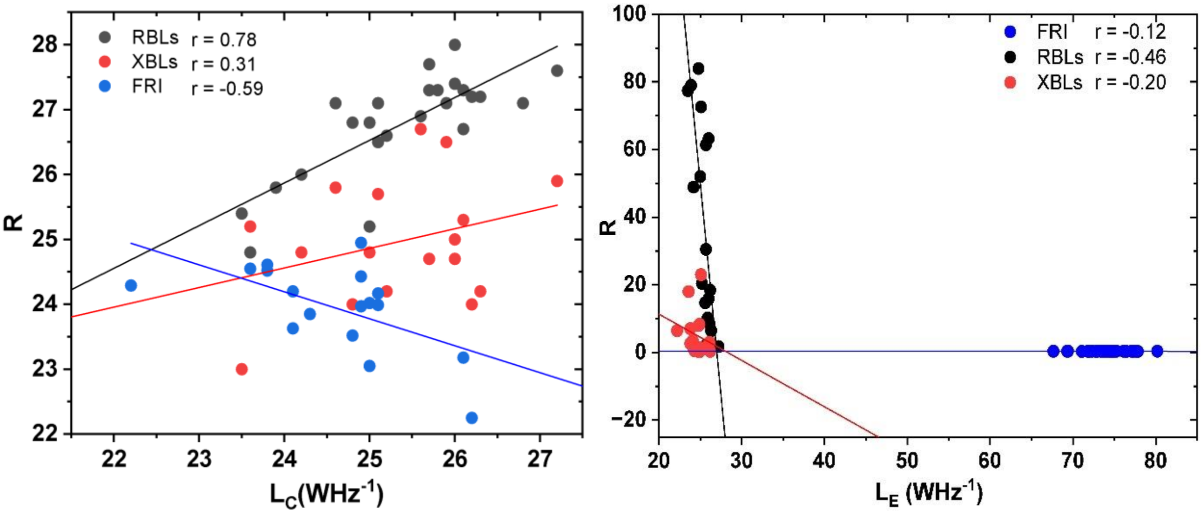


**Figure 7:** Scatter plot of (a) the core dominance parameter (R) against Core luminosity ($L_C$) and (b) the core dominance parameter (R) against extended luminosity ($L_E$) for RBLs, XBLs and FRI.




On the other hand, Fan et al. (2011) and Qin et al. (1996) have shown that the extended power or luminosity and the core dominance parameter should be anti-correlates very weakly. Our $R - L_E$ plot yields a vert weak anti-correlation of $r = -0.12$, $-0.46$, and $-0.20$ for FRI, RBLs, and XBLs respectively. Regression analysis showed that there is no significant correlation for the $R{-}L_E$ plot, but since all our results showed very weak anti-correlation, it implies that our three sub-sample are in accordance with the unification scheme for LLAGNs.

**Table 5:** Summary of correlation analysis of RBLs, XBLs, and FRI samples

| | FRI | | | RBLS | | | XBLs | | |
|---|---|---|---|---|---|---|---|---|---|
| **Regression Analysis** | ***r*** | ***r*²** | **Std Error** | ***r*** | ***r*²** | **Std Error** | ***r*** | ***r*²** | **Std Error** |
| $R - L_C$ | -0.59 | 0.34 | 0.01 | 0.78 | 0.61 | 52.81 | 0.31 | 0.09 | 40.52 |
| $R - L_E$ | -0.12 | 0.00 | 0.01 | -0.46 | 0.17 | 48.02 | -0.20 | 0.04 | 6.55 |
| $L_C - L_E$ | 0.67 | 0.45 | 0.52 | 0.78 | 0.72 | 0.46 | 0.79 | 0.62 | 0.61 |
| $R - D$ | 0.09 | 0.01 | 0.01 | 0.05 | 0.00 | 52.80 | 0.11 | 0.01 | 6.65 |
| $(R - \phi)_2$ | -0.98 | 1.00 | 0.00 | -0.64 | 0.41 | 40.52 | -0.75 | 0.56 | 4.45 |

### 4.3 Expectations of the Unified Scheme for LLAGNs

Table 7 presents a summary that compares the mean ratios of the BL Lacs (RBLs and XBLs) parameters with their parent population FRI radio galaxy in other to investigate if these sub-classes of BL Lacs with their parent population FRI agree with the BL Lacs/FRI the unification scheme.

**Table 7**: Mean ratio of BL Lacs (RBLs and XBLs) with FRI

| Parameters | RBLs/FRI | XBLs/FRI | BLs/FRI |
|---|---|---|---|
| $L_C$ | 1.12 | 1.09 | 1.03 |
| $L_E$ | 0.34 | 1.02 | 0.33 |
| $R$ | 124.73 | 8.34 | 14.94 |
| $D$ | 1.44 | 1.82 | 0.79 |
| $\phi$ | 0.37 | 0.56 | 0.65 |

**Table 8:** K-S test for the viewing angle for BL Lacs (RBLs and XBLs) with FRI

| Test | RBL/FRI | RBL/XBL | XBL/FRI |
|---|---|---|---|
| $r$ | 0.22 | −0.16 | 0.52 |
| $P-value$ | 0.395 | 0.532 | 0.031 |
| $K-S\ test$ | *Weak +ve* | *Nil* | *Strong + ve* |

In the context of the unified scheme as shown by Fan et al. (2011) and Stickel et al. (1991) isotropic properties such as the extended luminosity are expected to be shared equally by both the RBLs and XBLs with their parent population FRI. The ratio of the extended luminosity ($L_E$) as shown in Table 5 is in disagreement with this premise. The comparison showed that this statement may not entirely be correct, as the ratio of RBLs: FRI, RBLs:XBL, and XBL:FRI are > 1. The implication will be that the extended luminosity may not be entirely isotropic as previously assumed for our sample irrespective of the combination of the sub-classes they still give values > 1. On the other hand, the core luminosity ($L_C$) ratio for the three sub-samples all yield values > 1. This implies also that the core luminosity is certainly not isotropic as predicted by the unified scheme. Thus, in agreement with the unification scheme for BL Lac-FRI unification.

The ratio of the mean value of the core-dominance parameter ($R_m$) for RBLs to that of FRI and XBLs is expected to be greater if RBLs are more beamed than XBLs and FRI. Similarly, those of the XBLs to FRI should also be greater if the XBLs are more beamed than FRI. Our comparison is in agreement with this premise for sources assumed to obey the BL Lacs-FRI unification scheme. Furthermore, Alhassan et al. (20013), confirm that large values of the core-dominance parameter (R) will yield small linear sizes (D) since the linear size is directly proportional to the viewing angle as shown in Equation 7. Thus. it is expected that the ratio of the BL Lacs (RBL/XBL) should be greater than that of (RBL/FRI and XBL/FRI). This was not the case from Table 7 as the projected linear size (D) appears to be greater for the XBL/FRI ratio followed by RBL/FRI and the smallest for the XBL/FRI ratio. The possible explanation for this difference could be that the RBL/XBL unification is more valid than other sub-combinations thus in agreement with the unified scheme for low luminosity AGNs.

According to the unified scheme, large values of the core-dominance parameter (R) will always yield a smaller viewing angle ($\phi$) as shown in Figure 6's inverse relation. This according to the unified scheme implies that since RBLs have a large value of R, they should be more beamed than XBLs and FRI. Further implications of this mean that the ratio of the viewing angle ($\phi$) for RBLs: FRI and RBLs: XBLs should be < 1. Similarly, the ratio of XBLs: FRI should also be < 1 by comparison. Table 7 clearly shows that this is true as the RBL/FRI, RBL/XRL, and XBL/FRI ratios are all < 1 thus in agreement with the unification scheme for low luminosity radio AGNs. Furthermore, the K-S test for the viewing angle in Table 8 identifies the largest distance d between two given cumulative distributions (FRI/RBL and FRI/XBL), then reading out the probability p that a value as large as d could have resulted by chance if the two given distributions come from the same parent distribution (FRI). A small p means that the two samples are significantly close. Our K-S text of the viewing angle for the three samples of BL Lacs and FRI show that there is a medium, positive correlation between the variables FRI/RBL with r = 0.32 and a p-value of 0.395. Thus, a medium, positive association between FRI and RBL. On the other hand, there is a high, positive correlation between sample FRI/XBL with r = 0.52 and a p-value of 0.031. Thus, high, positive association between FRI and XBLs. The implication of this is that XBLs are significantly similar to FRI as their parent population than RBLs which is also in agreement with the unified scheme for LLAGNs.

## 5.0 Conclusion

We have statistically tested for the unification of BL Lacs and FRI radio galaxies using distributions of the core dominance parameter (R), core luminosity ($L_C$), extended luminosity ($L_E$), and projected linear size ($D$) from a total of sixty-eight (68) LLAGN samples employed from literature. The frequencies of our data sets for BL Lacs (RBLs and XBLs) were at 1.5$GHz$ while that of FRI radio galaxies was at 5$GHz$ then tuned down to 1.5$GHz$. The mean values (see Table 4) and their correlations (see Table 5) for the three sub-samples beaming and orientation parameters were determined and discussed. Furthermore, regression analysis was carried out to test the relationship between the core dominance parameter ($R$) with the core luminosity ($L_C$), extended luminosity ($L_E$), and projected linear size ($D$) respectively. The results and the variations of their correlation coefficients were discussed to test for unification on the premise of the "BL Lacs-FRI Unified scheme."

A comparison of the correlations amongst the different sub-classes and their parameters produced two remarkable results as a major consequence of the beaming and orientation scenario. Our results show that the scatter plot of the core dominance parameter ($R$) and the viewing angle ($R{-}\phi$) are strongly anti-correlated with $R \approx -0.98 \pm 0$, $-0.64 \pm 40.52$, and $-0.75 \pm 4.45$ for FRI, RBLs and XBLs respectively. In the same light, the core luminosity versus extended luminosity scatter plot ($L_C - L_E$) for the three sub-sample all show a strong correlation with $R \approx 0.67 \pm 0.52$, $0.78 \pm 0.46$ and $0.79 \pm 0.61$ for FRI, RBLs, and XBLs respectively, suggesting that RBLs, XBLs and FRI agree with the unified model at 95% confidence level. In both comparisons, the core dominance parameter versus the viewing angle plot ($R - \phi$) and the core luminosity versus the extended luminosity ($L_C - L_E$) show that our XBLs sample seems to be an intermediate population, correlating more with RBLs and less with FRI as expected. This implies that XBLs are more similar to FRI than RBLs even though FRI is the parent population of both XBLs and RBLs, thus suggesting that the XBLs/FRI unification seems to agree more with the unified scheme than RBLs/FRI for LLAGNs. $K - S$ test for the viewing angle also reveals that the unification of XBLs and FRI is more valid than that of RBLs and FRI, meaning RBLs are more strongly beamed counterparts of FRI than XBLs.

## 6.0 Acknowledgement

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## 7.0 Data Availability

No new data were generated in support of this research. The data underlying this article were derived from journals or article sources in the public domain. It was a careful compilation of different articles based on the parameters we needed for the paper. A few other parameters were calculated and added to our already compiled data after tuning down the frequency. Listed below are the articles and URLs from which our data was sourced.

1. https://iopscience.iop.org/article/10.1086/321179/meta
2. https://iopscience.iop.org/article/10.1086/301587/meta
3. https://adsabs.harvard.edu/full/record/seri/ApJ../0465/1996ApJ...465. .115K.html
4. https://iopscience.iop.org/article/10.1088/1009-9271/6/S2/68/meta
5. https://iopscience.iop.org/article/10.1088/1009-9271/7/5/03/meta
6. https://adsabs.harvard.edu/pdf/2011AfrSk..15...15O. The data for this link cannot be found on the article but was extracted from my supervisor's M.Sc. dissertation further information can be gotten from finbarr.odo@unn.edu.ng

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