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Security in Spatially Correlated MIMO OSFBC OFDM Multicast System over Frequency Selective Fading Channel

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Abstract

The system performance can be improved significantly by employing orthogonal space-frequency block coding(OSFBC) and orthogonal frequency division multiplexing (OFDM) because in this combination the receiver suffers lesser complexity. Recently, the wireless systems are deploying by the increasing number of closely spaced antennas. So the assumption of independency among the antenna brunches is not valid. Realizing the advantages of this combination of system and taking account the effect of antenna correlation, this work demonstrates the insight of secured wireless multicasting network consisting multiple antennas both in transmitter and receiver over spatially correlated frequency selective fading channels. The desired signals have been protected from eavesdropping by useful impact of antenna diversity and antenna correlation. The main contribution of this work is the developmentof mathematical models of the probability of non-zero secrecy multicast capacity (PNSMC) and the secure outage probability for multicasting (SOPM) to secure the system from eavesdropping. The analytical results are verified via Monte-Carlo simulation to ensure the validity of the developed mathematical model. Moreover, the numerical analysis gives a clear indication that security in OSFBC OFDM system with antenna diversity over spatially correlated frequency selective fading channel upgrades due to the effect of transmit antenna correlation and degradesdue to receive antenna correlation. But the loss of this security can be significantly reduced employing antenna diversity with OSFBC-OFDM systems.

Index Terms

Spatial correlation, probability of non-zero secrecy multicast capacity, frequency selective fading, secure outageprobability, Orthogonal space frequency block coding, Orthogonal frequency division

multiplexing.

I. INTRODUCTION

Recently, the researchers of wireless communication systems concentrate their attention to the use of multiple antennas at the transmitter and receiver. But the correlation between two antennas depends on the spacing between them and the wavelength of transmitted signals. Antenna correlation occurs, when $d \leq \frac{\lambda}{2}$, where d denotes the antenna spacing and λ is the wavelength of transmitted signals from the antennas. Normally, spacing between two antennas of an array remain fixed, but the wavelength of different transmitted signals are not fixed. Therefore, the assumption of independency among antenna brunches is not valid. In order to obtain the accurate result, correlation among antenna brunches should be considered properly. There are several models of antenna correlation such as exponential correlation, constant correlation, arbitrary correlation models etc. Among them, exponential model of correlation (i.e. spatial correlation) is more accurate [1]. And also multicasting is an effective radio communication technique to send a common stream of information to a group of receivers. Since, the main objective of multicasting is to send information to a particular group of receivers protecting it from other receivers. Therefore, security in multicasting is a vital issue in communication system. Moreover, the medium of wireless multicasting is open which is vulnerable to eavesdropping and fraud [2]. But the system performance can significantly be improved by employing OSFBC with OFDM, because it reduces the complexity in the receivers [3].

A. Related Works

There are many publications available in the literature analyzing the secrecy measures in frequency selective fading environment. But none of these studies demonstrates the secrecy measures in OSFBC-OFDM system over spatially correlated frequency selective fading. To give a clear vision of current status of the selected research topic some recent papers have been presented. These papers focus on the important contributions in the proposed research domain.

Recently, in the literature from [4]–[9] authours dealt with security in α - μ fading channel. In [4], L. Kong et. al. presented highly accurate and assymptotic closed form expressions for SOP over SIMO α - μ fading channel. L. Kong et. al. introduced and characterized cascaded α - μ fading which is a generalizationof the cascaded Rayleigh, Weibull and Nakagami-m fading distribution in [5]. They studied SOP, PNSC and ASC in presence of an active eavesdropper and analyzed the effects of system parameters. In [6] Sunil Yadav studied physical layer security for an underlay cognitive radio sensor network over α - μ fading channel. In this manner they developed exact and asymptotic expressions for the secrecy outage probability. They found that the system achieved a a secrecy diversity order of $\underline{\alpha m \mu m}$ when SNR of main link tend to infinity but the secrecy diversity order becomes zero when SNR of both main and eavesdropper's channel tend to infinity. A SIMO based underlay cognitive radio network was studied over α-μ fading channelin [7]. In this study Sunil Yadav showed that the higher number of eavesdroppers antenna can have amore deleterious impact on the system secrecy. J. M. Moouuleu et. al. in [8], investigated the secrecy performance of a multiple input multiple output (MIMO) system with transmit antenna selection (TAS) and maximal-ratio-combining (MRC) over α-μ fading.They developed SOP and SPSC in this manner. In [9], J. m. Moualeu et. al. investigated the secrecy performance of TAS in MIMO system over α-μ fading. They derived SOP, SPSC and ASC and demonstrated the impact of fading parameters.

In [10] a multicasting scenario was studied over α μ fading. In this paper, Hanif et. al. showed the impacts of α μ fading parameters on the security performance of multicasting scenario.

In the literature [3], [11]–[13], authors considered OSFBC -OFDM system. Employing OSFBC in MIMO-OFDM system, the system performance can be significantly improved which is shown in [3]. The MIMO-OFDM system was considered in [11] to investigate the secrecy performance. Ch. Siva et. al.in [12], investigated the spectrum efficiency for spatially correlated MIMO OSFBC-OFDM system over frequency selective fading channel. They developed capacity per unit bandwidth for different adaptation policies. They showed that the capacity improves with an increase in the correlation coefficient and withan increase in the number of antennas. In [13], Hasan et. al. studied the performance of secrecy measures for multicast channels using MIMO-OSFBC-OFDM system over α μ fading channel.

We found a number of literatures, such as from [14]–[25], which deal with the Correlation over fading channels. Recently, Mathur et. al. [14] investigated the effect of correlation on the security in α μ fading channel through asymptotic analysis. In [15], X. Sun et. al. studied the secure outage probability (SOP) and average secrecy capacity over correlated fading channel and they showed that correlation between channels degraded the secrecy capacity and the outage probability. The effect of the spatial correlation was analyzed in [16], where two relay antenna selection (RAS) model over Nakagami-m fading was used. In [17] the secure outage probability was investigated with correlated main and eavesdropper's channel. The adverse effect of correlated shadowing on secrecy measures over correlated composite Nakagami-m/Gamma fading channel were analyzed in [18]. In [19], Nuwan S. Ferdinand et. al. investigated the secrecy performance of multiple-input single-output wiretap channel when the eavesdropper channel is correlated with the main channel and transmitter employs transmit antenna selection scheme. They showed that correlations in the channel enhances security in the high SNR region but correlation degrades security at low SNR region. Jiangfeng et. al. in [20], studied the secrecy performance of single-input multiple-output systems over correlated κ-μ shadowed fading channel. They derived mathematical models for SOP and probability of non-zero secrecy capacity (PNSC). They found that, when the SNR of main channel is lower than that of eavesdropper's channel, the larger value of coefficient is helpful to improve thesystem secrecy capacity. In [21], Ibrahim et. al. analyzed the security over correlated κ-μ shadowedfading multicast channel. They developed an analytical mathematical model of PNSMC and SOPM. They found that the impact of correlation is beneficial for secrecy performance. M. K. Kundu et. al. in [22], investigated the effect of correlation on the secrecy performance over η-μ fading multicast channel. They found the detrimental effect of correlation on the secrecy capacity. A. S. M. Badrudduza et. al [23] examined enhancement of the security in wireless multicasting through correlated Nakagami=m fading channel with opportunistic relaying technique. They showed that the impact of constant correlation ismore significant than the exponential and arbitrary correlations. Salam Al Jaboori [24] studied the effectof correlation among diversity branches. They found that the correlation among diversity branches causesa detrimental effect on the performance of detection. In [25], R. Sultana et. al. investigated the impactof exponential correlation on the secrecy performance of correlated MIMO network for Nakagami-m fading multicast channel. They found that system performance increases with the decrease in the valueof correlation coefficient.

A number of works on secrecy measure performance using MIMO-OFDM system, MIMO-OFDM-

OSFBC system in correlated and spatially correlated system have been presented above. However, inthe works mentioned above, no one investigated the prformance analysis of PNSMC and SOPM overthe ”frequency selective fading channel” considereing the spatially correlated multicasting system using MIMO OSFBC OFDM system.

B. Contributions

On the basis of the literature mentioned in the previous section motivated by the fact that, security in multicasting and analysis of antenna correlation have an intense importance and realizing the advantagesof OSFBC-OFDM system in this work authors studied a secure radio multicasting network over spatially correlated frequency selective fading channels in the presence of multiple eavesdropping receivers. Authorshave developed mathematical models for PNSMC and SOPM for the proposed model. The summary ofthe vital contributions of this research are as follows;

- Firstly, on the basis of the probability density function (PDF) of MIMO OSFBC-OFDM system over spatially correlated frequency selective fading channels, authors derived the expressions for the PDFs of the minimum signal-to-noise ratio (SNR) of multicast channels and the maximum SNR of eavesdropper’s channels, and denote them by f_{dmin} (γ_M) and f_{dmax} (γ_E), respectively.
- Secondly, using the analytical expressions of f_{dmin} (γ_M) and f_{dmax} (γ_E), the closed-form analytical
  mathematical expressions for the PNSMC and SOPM were formulated.
- Finally, authors investigate the effects of the number of transmitting and receiving antennas, the number of eavesdroppers, transmitting and receiving antenna correlation on the security of spatially correlated frequency selective fading channels.

The organization of remaining part of the paper is as follows. System model and problem formulation are in the section II and III respectively. Section IV and V deal with the derivation of mathematical modelsfor PNSMC and SOPM respectively. Section VI contains the numerical results and finally the conclusions have been drawn in section VII. In appendix the calculation of some required parameters has been shown.

II. SYSTEM MODEL

Fig.1 describes a wireless multicast network. This network uses OSFBC-OFDM technique with multiple antennas in transmitter and receiver experiencing frequency selective fading while propagation of signals. In this network the transmitter is sending confidential information to the M number of legitimate receiver where N number of eavesdroppers are present to decode the information and authors are intended toprotect this eavesdropping. The transmitter is equipped with n_t antennas. Each receiver and eavesdropper are equipped with n_r and n_e antennas, respectively. The propagation paths between transmitter and legitimate receivers are multicast channels and the propagation path between transmitter and eavesdropper eavesdropper’s channel. The correlation coefficients at the transmitter, receiver and eavesdropper are denoted by ρ_t, ρ_r and ρ_e, respectively.

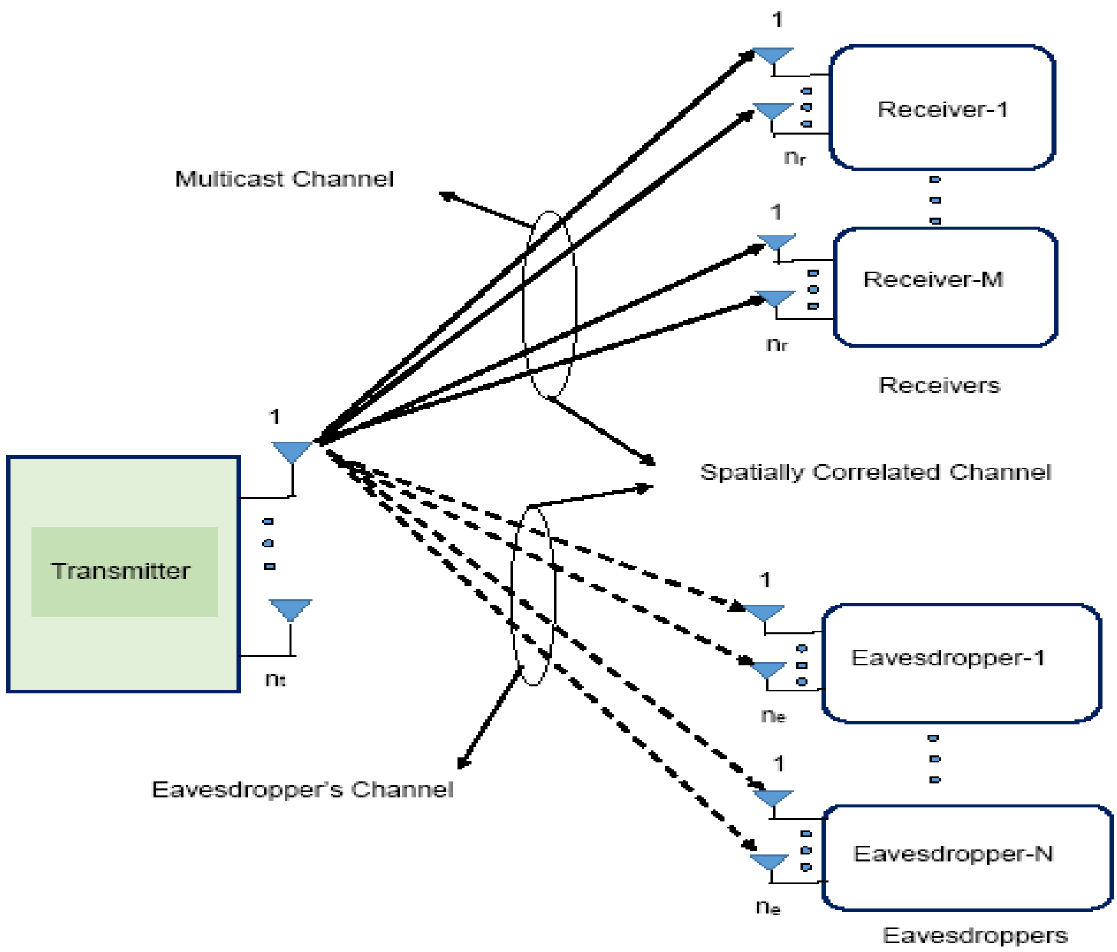

Figure 1 System Model

III. PROBLEM FORMULATION

In this section, we find the PDFs of the multicast channels and eavesdropper's channels from the PDFs of their sub-channels. The PDF of SNR of spatially correlated MIMO OSFBC-OFDM system over frequency selective fading channel is expressd by [26],

$$f_{\Upsilon}(\Upsilon) = \sum_{i=1}^{z}\sum_{j=1}^{wi}\frac{\mu_{ij}\,\gamma^{j-1}}{(j-1)!}\left\{\frac{\eta}{\bar{\gamma}}\,\frac{1}{\lambda_i}\right\}^{j} \times e^{\frac{\eta}{\bar{\gamma}}\frac{\gamma}{\lambda_i}}$$

$$= A\,\gamma^{j-1}\,e^{-B\gamma} \qquad [1]$$

Where, $A = \sum_{i=1}^{z}\sum_{j=1}^{wi}\frac{\mu_{ij}}{(j-1)!}\left\{\frac{\eta}{\bar{\gamma}}\,\frac{1}{\lambda_i}\right\}^{j}$ and $B = \sum_{i=1}^{z}\frac{\eta}{\bar{\gamma}}\,\frac{1}{\lambda_i}$;

For the value of μ_{ij} go to [26].

$\eta = R_c * n_t$, Here R_c is the code rate of OSFBC transmitter.

Z_i = Igen values of matrix R and i= 1,2,3,4 ….. z

$\sum_{i=1}^{z} w_i$= Rank of Matrix R

Where, $R = R_T^t \otimes R_R$

$R_T = n_t * n_t$ Transmit antenna correlation Matrix

$R_R = n_r * n_r$ Receive antenna correlation Matrix.

In antenna correlation exponential matrix has been used.

$(.)^t$ = Transpose matrix
$\otimes$ = Kronecker Product.

A. PDF and CDF of Each Sub-channel of Multicast Channels:

Let γMp is the signal-to-noise ratio (SNR) of pth sub-channel of multicast channel. Then, the PDF of γ_{M_p} for spatially correlated MIMO OSFBC-OFDM system over frequency selective fading is expressed by [26],

$$f\gamma_{M_p}(\gamma_{M_p}) = \sum_{i_1=1}^{z_1}\sum_{j_1=1}^{w_{i_1}} \frac{\mu_{i_1 j_1}\, {\gamma_{M_p}}^{j_1-1}}{(j_1-1)!}\left\{\frac{\eta_1}{\overline{\gamma_1}}\,\frac{1}{\lambda_{i_1}}\right\}^{j} \times e^{\frac{\eta_1}{\overline{\gamma_1}}\frac{\gamma_{M_p}}{\lambda_{i_1}}}$$

$$= A_1\, {\gamma_{M_p}}^{j_1-1}\, e^{-B_1\gamma_{M_p}} \qquad [2]$$

Where,

$A_1 = \sum_{i_1=1}^{z_1}\sum_{j_1=1}^{w_{i_1}} \frac{\mu_{i_1 j_1}}{(j_1-1)!}\left\{\frac{\eta_1}{\overline{\gamma_1}}\,\frac{1}{\lambda_{i_1}}\right\}^{j}$ and

$B_1 = \frac{\eta_1}{\overline{\gamma_1}}\,\frac{1}{\lambda_{i_1}}$

The CDF of γ_{M_p} denoted by $F\gamma_{M_p}(\gamma_{M_p})$ is defined as

$$F_{\gamma_{M_p}}\left(\gamma_{M_p}\right) = \int_0^{\gamma_{M_p}} f_{\gamma_{M_p}}\left(\gamma_{M_p}\right)\, d\gamma_{M_p} \qquad [3]$$

Substituting the value of $f_{\gamma_{M_p}}\left(\gamma_{M_p}\right)$ in equation (3) and evaluating by integration by using identity 3.381(8) of [27]

$\int_0^u x^m\, e^{-\beta x^n} dx = \frac{\gamma(v,\ \beta u^n)}{n\beta^v}$, where $v = \frac{m+1}{n}$ .and identity 8.354(1) of [27], $\gamma\,(x,\alpha) = \sum_{n=0}^{\infty}\frac{-1^n\, x^{\alpha+n}}{n!(\alpha+n)}$, it is found,

$$F_{\gamma_{M_p}}\left(\gamma_{M_p}\right) = A_1\, A_3\, {\gamma_{M_p}}^{n_1+j_1} \qquad [4]$$

Where, $A_3 = \sum_{n_1=0}^{\infty}\frac{-1^{n_1}\, {B_1}^{n_1}}{n_1!(n_1+j_1)}$.

B. PDF and CDF of Each Sub-channel of Eavesdropper's Channels:

Let γ_{E_q} is the SNR of qth sub-channel of eavesdropper's channel. Then, the PDF of γ_{E_q} for spatially correlated MIMO OSFBC-OFDM system over frequency selective fading is expressed by [26],

$$f\gamma_{E_q}(\gamma_{E_q}) = \sum_{i_2=1}^{z_2}\sum_{j_2=1}^{w_{i_2}} \frac{\mu_{i_2 j_2}\, {\gamma_{E_q}}^{j_2-1}}{(j_2-1)!}\left\{\frac{\eta_2}{\overline{\gamma_2}}\,\frac{1}{\lambda_{i_2}}\right\}^{j_2} \times e^{\frac{\eta_2}{\overline{\gamma_2}}\frac{\gamma_{E_q}}{\lambda_{i_2}}}$$

$$= A_2\, {\gamma_{E_q}}^{j_2-1}\, e^{-B_2\gamma_{E_q}} \qquad [5]$$

Where, $A_2 = \sum_{i_2=1}^{z_2}\sum_{j_2=1}^{w_{i_2}} \frac{\mu_{i_2 j_2}}{(j_2-1)!}\left\{\frac{\eta_2}{\overline{\gamma_2}}\,\frac{1}{\lambda_{i_2}}\right\}^{j_2}$ and $B_2 = \frac{\eta_2}{\overline{\gamma_2}}\,\frac{1}{\lambda_{i_2}}$

The CDF of γ_{E_q} denoted by $F\gamma_{E_q}(\gamma_{E_q})$ is defined as

$$F_{\gamma_{E_q}}\left(\gamma_{E_q}\right) = \int_0^{\gamma_{E_q}} f_{\gamma_{E_q}}\left(\gamma_{E_q}\right)\, d\gamma_{E_q} \qquad [6]$$

Substituting the value of $f_{\gamma_{E_q}}\left(\gamma_{E_q}\right)$ in equation (6) and evaluating by integration by using identity 3.381(8) of [27]

$\int_0^u x^m\, e^{-\beta x^n} dx = \frac{\gamma(v,\ \beta u^n)}{n\beta^v}$, where $v = \frac{m+1}{n}$

and identity 8.354(1) of [27], $\gamma\ (x,\alpha) = \sum_{n=0}^{\infty} \frac{-1^{n}\ x^{\alpha+n}}{n!(\alpha+n)}$, it is found,

$$F_{\gamma_{E_q}}\left(\gamma_{E_q}\right) = A_2\ \gamma_{E_q}{}^{j_2}\ \sum_{n_2=0}^{\infty} \frac{-1^{n_2} B_2{}^{n_2}\ \gamma_{E_q}{}^{n_2}}{n_2!(n_2+j_2)} \quad [7]$$

C. PDF of Minimum SNR of Multicast Channels:

We can ensure security if the minimum SNR of the multicast channel is higher than the maximum SNR of Eavesdropper channel. Let $d_{min} = \min_{1\le p\le M} \gamma_{Mp}$. Then, the PDF of d_{min} denoted by f_{dmin} (Υ_{Mp}) can be defined as,

$$f_{dmin}(\gamma_{M_p}) = f_{\gamma_{M_p}}\left(\gamma_{M_p}\right)\{1 - F_{\gamma_{M_p}}(\gamma_{M_p})\}^{M-1} \quad [8]$$

Substituting the values of $f_{\gamma_{M_p}}\left(\gamma_{M_p}\right)$ and $F_{\gamma_{M_p}}\left(\gamma_{M_p}\right)$ into equation (8) and evaluating by integration using the identity 1.110 of Table of integral series [27],

$(1+x)^q = \sum_{k=0}^{\infty}\binom{q}{k} x^k$, we have,

$$f_{dmin}(\gamma_{M_p}) = \frac{MA_1\ \gamma_M{}^{j_1-1}}{e^{B_1\gamma_M}} - \frac{MA_1{}^2\ A_3\ (M-1)}{e^{B_1\gamma_M}\ \gamma_m{}^{-(2j_1+n_1-1)}} + \frac{MA_1{}^3\ A_3{}^2\ (M-2)\ (M-1)}{2!\ e^{B_1\gamma_M}\ \gamma_M{}^{-(3j_1+2n_1+1)}} \quad [9]$$

D. PDF of Maximum SNR of Eavesdropper's Channels:

Let $d_{max} = \min_{1\le q\le E} \gamma_E$ Then, the PDF of d_{max} denoted by f_{dmax} (Υ_{Eq}) can be defined as

$$f_{dmax}\ (\Upsilon_{Eq}) = N f_{E_q}\left(\gamma_{E_q}\right)\{F_{E_q}\left(\gamma_{E_q}\right)\}^{N-1} \quad [10]$$

Substituting the values of $f_{E_q}\left(\gamma_{E_q}\right)$ and $F_{E_q}\left(\gamma_{E_q}\right)$ into equation (10) and evaluating by integration by Using 0.314 of Table of integral series [27],

$(\sum_{k=0}^{\infty} a_k\ x^k)^n = \sum_{k=0}^{\infty} C_k\ x^k$,

where, $C_m = [A_0]^n$ for m=0 and $C_m = \frac{1}{mA_0} \sum_{k=1}^{m}(kn - m + k)A_k C_{m-k}$ for m > 0

we have,

$$f_{dmax}\ (\Upsilon_{Eq}) = \frac{NA_2{}^N}{e^{B_2\gamma_e}}\ \{C_0\ \gamma_E{}^{Nj_2-1} + C_1\ \gamma_e{}^{Nj_2}\}, \quad [11]$$

where $C_0 = j_2{}^{1-N}$ and $C_1 = \frac{(1-N)\ j_2{}^{2-N}\ B_2}{1+\ j_2}$.

IV. Probability of Non-Zero Secrecy Multicast Capacity

The probability of non-zero secrecy multicast capacity denotes in form of, $Pr\ (C_{smcast} > 0)$, and defined mathematically as [28],

$$\Pr(C_{smcast>0}) = \int_0^{\infty} f_{dmin}\ (\gamma_M) \times \{\int_0^{\gamma_M} f_{dmax}(\gamma_E)\ d\gamma_E\} d\gamma_M \quad [12]$$

Substituting the values of $f_{dmin}\ (\gamma_M)$ and $f_{dmax}(\Upsilon_{Eq})$ in equation (12) and evaluating by integration by using identity 3.381(8), 3.381(10) and 8.354(1) of [27] we get ,

$$\Pr(C_{smcast} > 0) = MNA_1A_2{}^N\left\{C_0\ A_4\ \frac{\Gamma(v_{p_1})}{B_1{}^{v_{p_1}}} + C_1\ A_5\ \frac{\Gamma(v_{p_2})}{B_1{}^{v_{p_2}}}\right\} - MN(M-1)A_1{}^2A_2{}^N A_3$$

$$\left\{C_0\, A_4\, \frac{\Gamma(v_{p_3})}{{B_1}^{v_{p_3}}} + \, C_1\, A_5\, \frac{\Gamma(v_{p_4})}{{B_1}^{v_{p_4}}}\right\} - MN(M-1)(M-2)\, {A_1}^2 {A_2}^N {A_3}^2 \left\{C_0\, A_4\, \frac{\Gamma(v_{p_5})}{{B_1}^{v_{p_5}}} + \, C_1\, A_5\, \frac{\Gamma(v_{p_6})}{{B_1}^{v_{p_6}}}\right\} \quad [13]$$

Where, $A_4 = \sum_{n_3=0}^{\infty} \frac{-1^{n_3} {B_2}^{n_3}}{n_3!(Nj_2+n_3)}$; $A_5 = \sum_{n_4=0}^{\infty} \frac{-1^{n_4} {B_2}^{n_4}}{n_4!(Nj_2+1+n_4)}$; $v_{p_1} = Nj_2 + n_3 + j_1$; $v_{p_2} = Nj_2 + 1 + n_4 + j_1$; $v_{p_3} = Nj_2 + n_3 + 2j_1 + n_1$; $v_{p_4} = Nj_2 + 1 + n_4 + 2j_1 + n_1$; $v_{p_5} = Nj_2 + n_3 + 3j_1 + 2n_1$; $v_{p_6} = Nj_2 + 1 + n_4 + 3j_1 + 2n_1$.

V. Secure Outage Probability for multicasting

The secure outage probability for multicasting denoted by, Pout (Rsmcast), can be expressed as [29],

$$P_{out}(R_{smcast}) = 1 - \int_0^{\infty} f_{dmax}\,(\gamma_E) \times \left\{\int_x^{\infty} f_{dmin}(\gamma_M)\; d\gamma_M\right\} d\gamma_E, \quad [14]$$

Where $x = e^{2R_{smcast}}(1+\gamma) - 1$ and R_{smcast} denotes the target secrecy multicast rate.

Substituting the values of $f_{dmin}\,(\gamma_M)$ and $f_{dmax}(\Upsilon_{Eq})$ in equation (14) and evaluating by integration by using identity 3.381(8), 3.381(10) and 8.354(1) of [27] we get ,

$$P_{out}(R_{smcast}) = 1 - MNA_1{A_2}^N A_{12}\Big[A_6 A_9 \left\{C_0 \frac{\Gamma(U_1)}{{B_4}^{U_1}} + \, C_1 \frac{\Gamma(U_2)}{{B_4}^{U_2}}\right\}$$
$$- A_1 A_3 A_7 A_{10} \left\{C_0 \frac{\Gamma(U_3)}{{B_4}^{U_3}} + \, C_1 \frac{\Gamma(U_4)}{{B_4}^{U_4}}\right\} + \, {A_1}^2\, {A_3}^2\, A_8\, A_{11} \left\{C_0 \frac{\Gamma(U_5)}{{B_4}^{U_5}} + \, C_1 \frac{\Gamma(U_6)}{{B_4}^{U_6}}\right\}\Big] \quad [15]$$

where

$A_6 = \Gamma(j_1) \sum_{q_1=0}^{j_1-1} \frac{{B_1}^{q_1}}{q_1!}$; $A_7 = \Gamma(2j_1 + n_1) \sum_{q_2=0}^{2j_1+n_1-1} \frac{{B_1}^{q_2}}{q_2!}$; $A_8 = \Gamma(3j_1 + 2n_1) \sum_{q_3=0}^{3j_1+2n_1-1} \frac{{B_1}^{q_3}}{q_3!}$; $A_9 = \sum_{k_1=0}^{q_1} \frac{q_1!}{k_1!(q_1-k_1)!} \frac{(e^{2R_S}-1)^{k_1}}{{e^{2R_S}}^{k_1-q_1}}$; $A_{10} = \sum_{k_2=0}^{q_2} \frac{q_2!}{k_2!(q_2-k_2)!} \frac{(e^{2R_S}-1)^{k_2}}{{e^{2R_S}}^{k_2-q_2}}$; $A_{11} = \sum_{k_3=0}^{q_3} \frac{q_3!}{k_3!(q_3-k_3)!} \frac{(e^{2R_S}-1)^{k_3}}{{e^{2R_S}}^{k_3-q_3}}$; $A_{12} = e^{-B_1(e^{2R_S-1})}$; $U_1 = Nj_2 - 1 + q_1 - k_1 + 1$; $U_2 = Nj_2 + q_1 - k_1 + 1$; $U_3 = Nj_2 - 1 + q_2 - k_2 + 1$; $U_4 = Nj_2 + q_2 - k_2 + 1$; $U_5 = Nj_2 - 1 + q_3 - k_3 + 1$; $U_6 = Nj_2 + q_3 - k_3 + 1$.

VI. Numerical Results

In this section, the analytical results of PNSMC and the SOPM are verified via Monte-Carlo simulation. The simulation results of PNSMC and the SOPM are generated using MATLAB code. More than 110000 realizations are taken for the PNSMC and the SOPM and averaged to find the final simulation results of PNSMC and the SOPM.

Figure 2 describes the $Pr\ (C_{smcast} > 0)$ which is a function of the average (SNR) of the multicastchannel, $\overline{\gamma}_{Mp}$, for some specific values of the average SNR of the eavesdropper channel, γ_{Eq} while other system parameters are kept fixed. We find that $Pr\,(C_{smcast} > 0)$ decreases when $\overline{\gamma}_e$ is increased gradually from 12dB (Dash dot line) to 12.5dB (dot line), 13db (short dash line) and 14dB (solid line). Because enhencement in $\overline{\gamma}_e$ gives an increment in the capacity of eavesdropper channel and it reduces the secrecy capacity.

Figure 3 describes the $Pr\,(C_{smcast} > 0)$ which is a function of $\overline{\gamma}_{Mp}$, for some specific values of n_t while other system parameters are kept unchanged. It is observed that $Pr\ (C_{smcast} > 0)$ increases whenn_t is increased gradually from 2 (solid line) to 3 (short dash line) and 4 (dash dot line). Because, the transmit diversity provided by the transmit antennas increases the secrecy capacity.

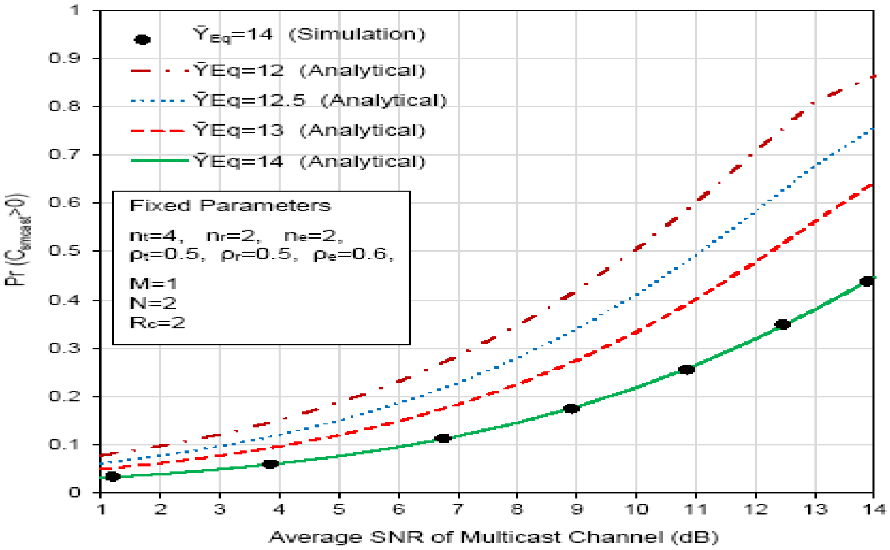

Fig. 2. The effects of the average SNR of eavesdropper's channel, γ_{Eq}, on the Pr (C_{smcast} > 0) with n_t=2, n_r=2, n_e=2, ρ_t=0.5, ρ_r=0.5, ρ_e=0.6, M=1, N=2, R_c=2.

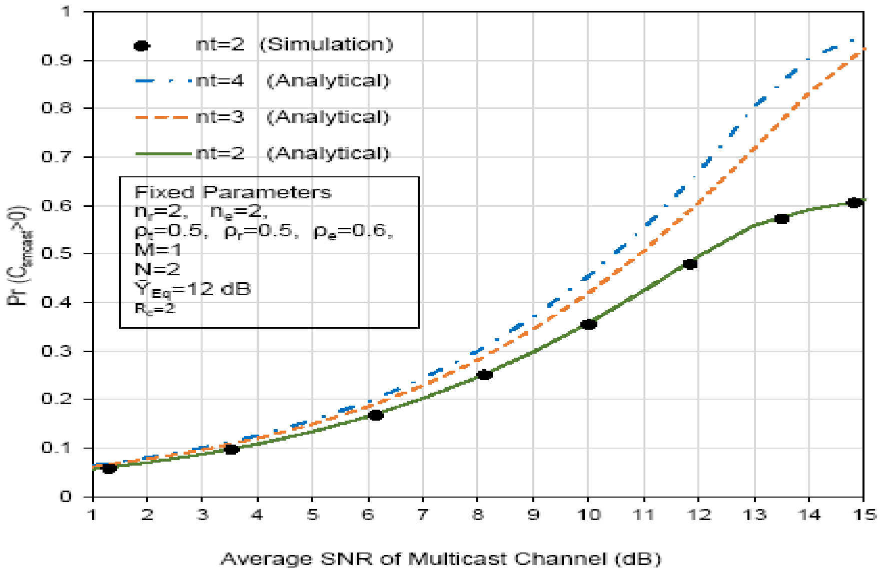

Fig. 3. The effects of the number of transmit antenna, nt, on the Pr ($Csmcast$ > 0) with nr =2, ne=2, ρt=0.5, ρr =0.5, ρe=0.6, M=1, N=2, R_c=2 and γ_{Eq} =12dB.

The Pr (C_{smcast} > 0) is shown in Fig.4, as a function of $\overline{\gamma}_{Mp}$, for some selected values of n_r while other system parameters are kept unchanged. We see that Pr (C_{smcast} > 0) increases gradually when number of receive antenna n_r is gradually increased from 2 (solid line) to 3 (short dash line), 4 (dash dot line) and6 (long dash line). Because, the diversity gain provided by the receiving antennas increases the secrecycapacity.

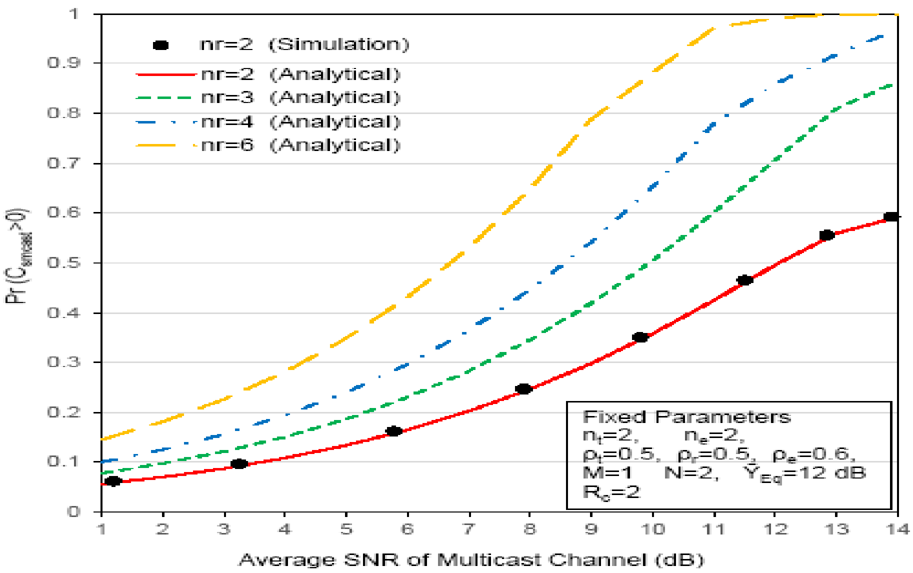

Fig. 4. The effects of the number of receive antenna, nr, on the $Pr\ (Csmcast > 0)$ with nt=2, ne=2, ρt=0.5, ρr=0.5, ρe=0.6, M=1, N=2, R_c=2 and γ_{Eq} =12dB.

Figure 5 describes $Pr\,(C_{smcast} > 0)$ as a function of $\overline{\gamma}_{Mp}$ for some specific values of ρ_t, keeping the other system parameters unchanged. We see that $Pr\ (C_{smcast} > 0)$ gradually increases when ρ_t is increased gradually from 0.5 (solid line) to 0.6 (short dash line) and 0.7 (dash dot line). Because, the correlation atthe transmit antennas upgrades the received SNR which causes an increment in the secrecy capacity.

Figure 6 describes $Pr\,(C_{smcast} > 0)$ as a function of $\overline{\gamma}_{Mp}$ for some specific values of the correlation coefficient of receive antenna ρ_r, keeping the other system parameters unchanged. We see that $Pr\ (C_{smcast} > 0)$ gradually decreases when ρ_r is increased gradually from 0.5 (solid line) to 0.6 (long dash line), 0.7 (dash dot line), 0.8 (short dash line) and 0.9 (long dash double dot line). Because, the correlation at the receivingantennas degrades the received SNR which causes a reduction in the secrecy capacity.

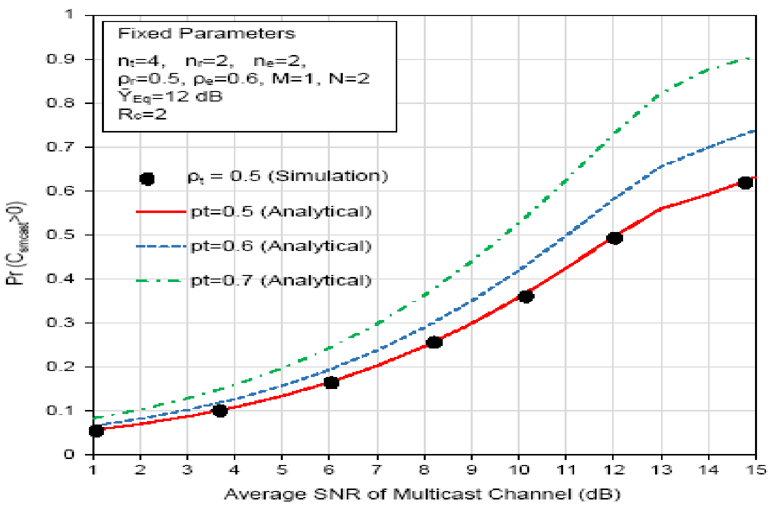

Fig. 5. The effects of the correlation coefficient of transmit antenna, ρt, on the $Pr\ (Csmcast > 0)$

with n_t=2, n_r =2, n_e=2, ρ_r =0.5, ρ_e=0.6,M=1, N=2, R_c=2 and γ_{Eq} =12dB.

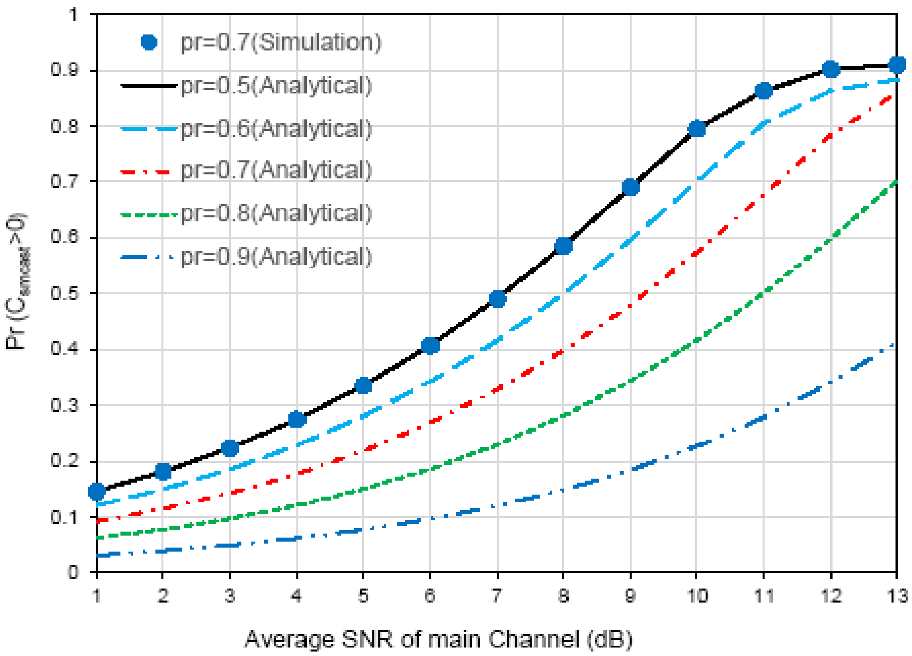

Fig. 6. The effects of the correlation coefficient of receive antenna, ρ_r, on the $Pr\ (C_{smcast} > 0)$ with n_t=2, n_r=4, n_e=2, ρ_t=0.5, ρ_e=0.6,M=1, N=2, R_c=2 and γ_{Eq} =12dB.

The figure 7 describes the $Pr\,(C_{smcast} > 0)$ which is a function of $\overline{\gamma}_{Mp}$, for some specific values of n_r and n_t. This figure differentiate the effects of n_r and n_t on the $Pr\ (C_{smcast} > 0)$. We see that $Pr\ (C_{smcast} > 0)$ improves when the value of n_r increases from 2 (solid line) to 3 (small dash line) keeping $n_t = 2$. On the other hand, $Pr\ (C_{smcast} > 0)$ also improves when the value of n_t increases from 2 (solid line) to 3 (dash dot line) keeping $n_r = 2$. But the improvement of $Pr\ (C_{smcast} > 0)$ by changing n_r is more significant than by changing n_t.

Figure 8 describes the impact of n_t and ρ_r on the $Pr\,(C_{smcast} > 0)$ as a function of $\overline{\gamma}_{Mp}$. We see that PNSMC decreases with ρ_r keeping the value of n_t=2. On other hand, PNSMC increases with n_t, keepingthe value of $\rho_r = 0.6$. This result demonstrates that the loss of secrecy capacity due to the effects of antenna correlation can be compensated by increasing the number of transmit antennas without increasingthe transmit signal power.

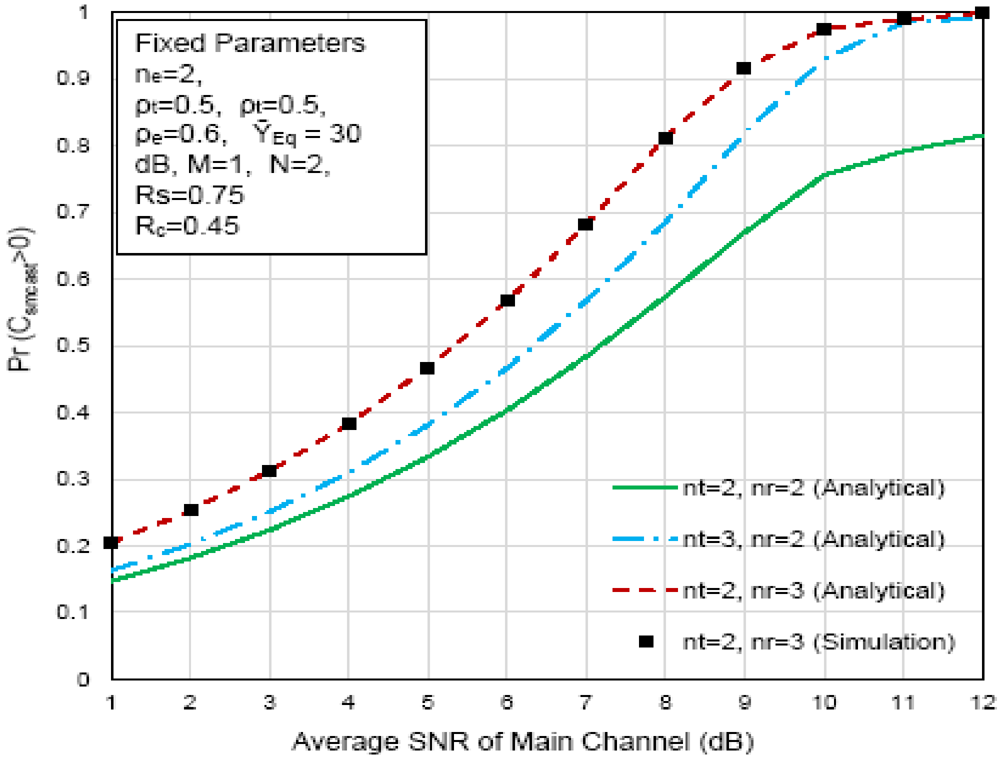

Fig. 7. The simultaneous effects of n_t and n_r on the improvement of the $Pr\ (C_{smcast} > 0)$ with n_e=2, ρ_t=0.5, ρ_r=0.5, ρ_e=0.6, M=1,N=2, R_c=2 and γ_{Eq} =12dB.

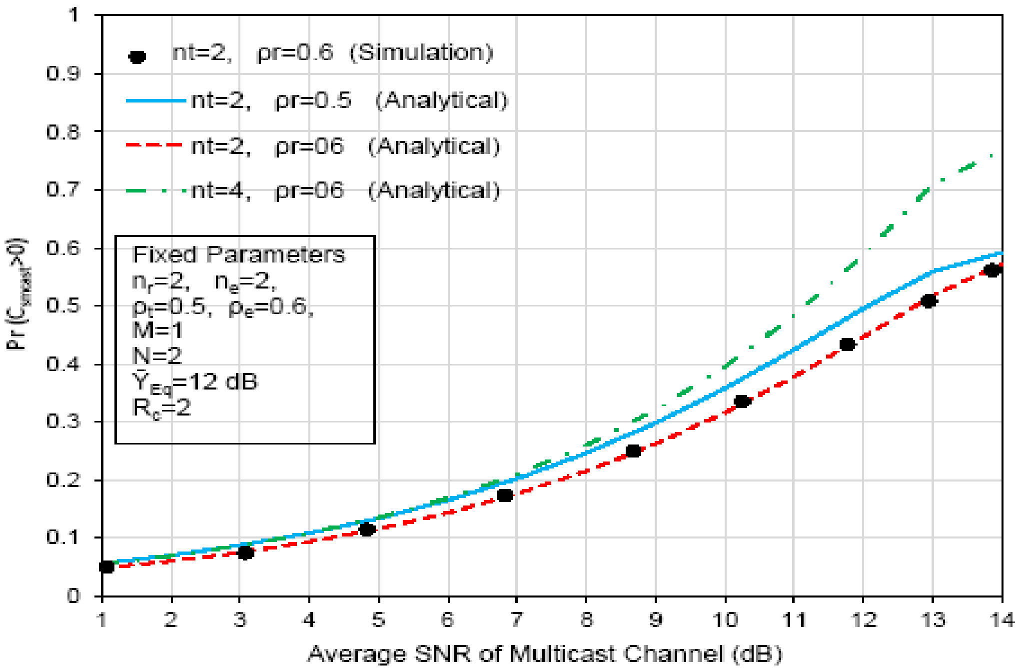

Fig. 8. The effects of n_t and ρ_r on the $Pr\ (C_{smcast} > 0)$ with n_r=2, n_e=2, ρ_t=0.5, ρ_e=0.6, M=1, N=2, R_c=2 and γ_{Eq} =12dB.

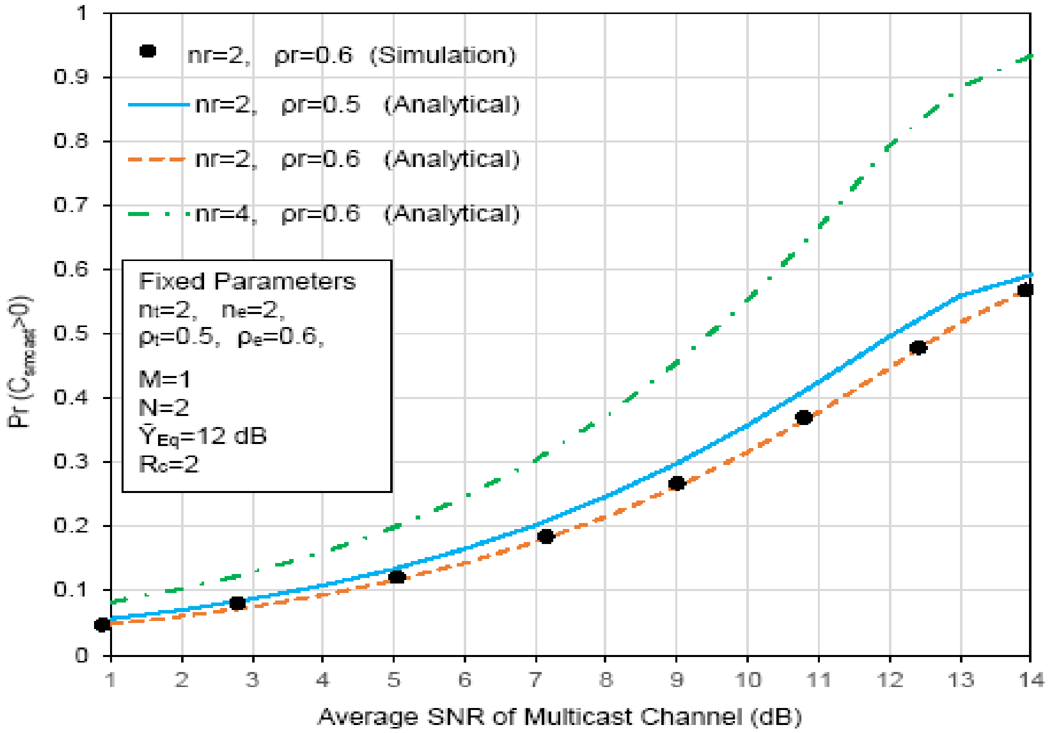

Fig. 9. The effects of n_r and ρ_r on the $Pr\,(C_{smcast} > 0)$ with n_t=2, n_e=2, ρ_t=0.5, ρ_e=0.6, M=1, N=2, R_c=2 and γ_{Eq} =12dB.

Figure 9 describes the impact of n_r and ρ_r on the $Pr\,(C_{smcast} > 0)$ as a function of $\overline{\gamma}_{Mp}$. We see that PNSMC decreases with ρ_r keeping the value of n_r=2. On other hand, PNSMC increases with n_r, keeping the value of $\rho_r = 0.6$. Comparing this improvement of security due to the effects of n_r with the improvement of security due to the effects of n_t, we can conclude that the loss the antenna diversity provided by the receiving antennas is more significant than the antenna diversity provided by the transmitting antennas in compensating the loss of secrecy capacity due to the effects of antenna correlation.

Figure 10 describes the effects of γ_{Eq} on the P_{out} (R_{smcast}) for selected values of γ_{Eq} while other system parameters are kept unchanged. We see that P_{out} (R_{smcast}) increases when γ_{Eq} increases gradually from 20dB (solid line) to 21 dB (small dash line) and 22 dB(dash dot line). which means that secrecy capacity decreases with γ_{Eq}. This is because, γ_{Eq} gives an increment in the capacity of eavesdropper's channel which reduces the secrecy capacity.

Figure 11 illustrates the effects of the number of user M and the number of eavesdropper N on the P_{out} (R_{smcast}). We see that P_{out} (R_{smcast}) increases with N which means that secrecy capacity decreases with increment of N. This is because increase in eavesdropper number increases the probability of eavesdropping. And also We observe that P_{out} (R_{smcast}) increases with M which means that secrecy capacity decreases with increment of M. This is because when number of user increases the bandwidth is shared to all.

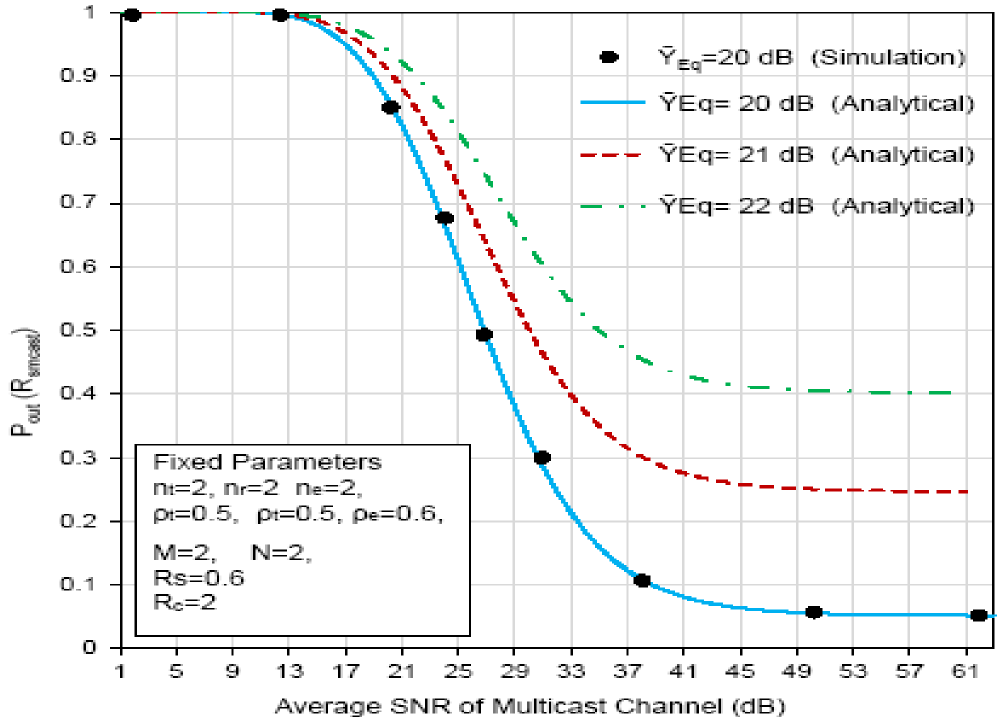

Fig. 10. The effects of the average SNR of eavesdropper channel, γ_{Eq} , on the P_{out} (R_{smcast}) with n_t=2, n_r=2, n_e=2, ρ_t=0.5, ρ_r=0.5, ρ_e=0.6, M=2, N=2, R_c=1.5, $\bar{\gamma}_0$=-5 dB, R_s=0.6.

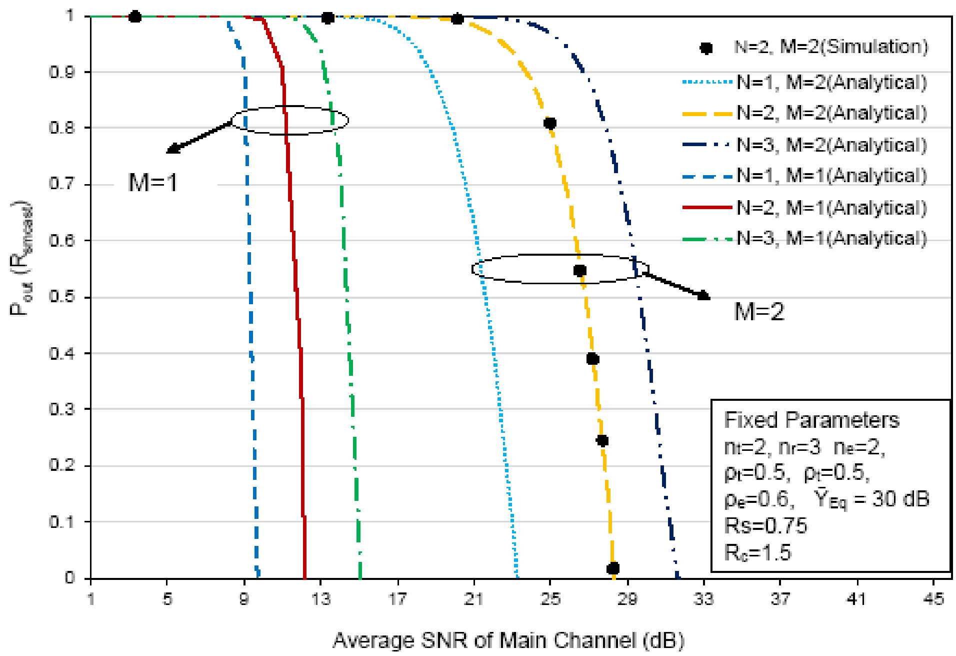

Fig. 11. The effects of the number of eavesdropper N and number of user M on the $Pout$ ($Rsmcast$) with nt=2, nr =3, ne=2, ρt=0.5, ρr =0.5, ρe=0.6, γEq =30 dB, Rc=2, $\bar{\gamma}0$= 5dB, Rs=0.75.

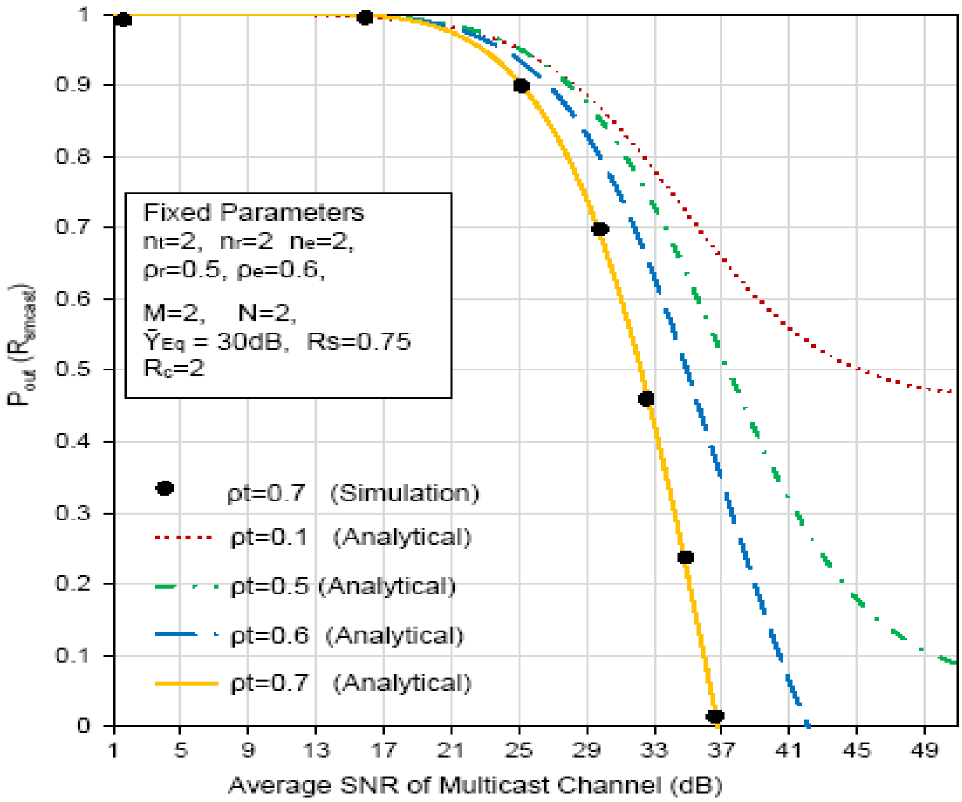

Fig. 12. The effects of transmit antenna correlation on the P_{out} (R_{smcast}) with n_t=2, n_r =2, n_e=2, ρ_r =0.5, ρ_e=0.6, M=2, N=2, γ_{Eq} =30dB, R_c=1.5, $\bar{\gamma}_0$= 0 dB, R_s=0.75.

Figure 12 describes the effects of transmit antenna correlation on the P_{out} (R_{smcast}) as a function of $\bar{\gamma}_{Mp}$. We see that P_{out} (R_{smcast}) decreases when transmit antenna correlation coefficient ρ_t is gradually increased from 0.1 (dot line) to 0.5 (dash dot line), 0.6 (long dash line) and 0.7 (solid line) at high SNR region. That means in high SNR region, security of the system increases with ρ_t. It is also observed that at low SNR region for every cases P_{out} (R_{smcast}) is almost 1. This is because, in the low SNR region, capacity of both the multicast channel and eavesdropper's channel are very low due to the effect of correlation, but in the high SNR region, the capacity of multicast channel enhances that improves the secrecy capacity.

Figure 13 describes the effects of transmit antenna correlation on the P_{out} (R_{smcast}) as a function of $\bar{\gamma}_{Mp}$. We see that P_{out} (R_{smcast}) increases when receive antenna correlation coefficient ρ_r is gradually increased from 0.1 (short dash line) to 0.5 (solid line), 0.7 (long dash dot line) and 0.9 (long dash double dot line) at high SNR region. That means in high SNR region, security of the system decreases with ρ_r.It is also observed that at low SNR region for every cases P_{out} (R_{smcast}) is almost 1. This is because, inthe low SNR region, capacity of both the multicast channel and eavesdropper's channel are very low due to the effect of correlation, but in the high SNR region, the capacity of multicast channel enhances that improves the secrecy capacity. Comparing this result with the effect of transmit antenna correlation, we see that in the low SNRregion, the effect of transmit antenna correlation on the P_{out} (R_{smcast}) is less significant than the effectof receive antenna correlation. But in the high SNR region, the effect of transmit antenna correlation onthe P_{out} (R_{smcast}) is more significant than the effect of receive antenna correlation.

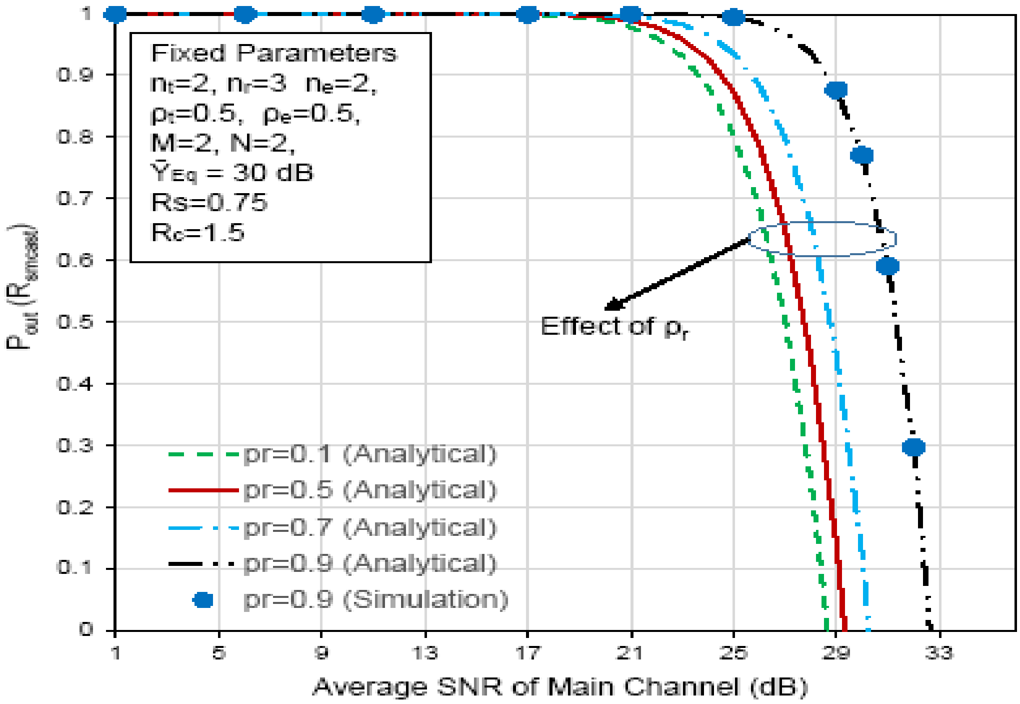

Fig. 13. The effects of receive antenna correlation on the P_{out} (R_{smcast}) with n_t=2, n_r=2, n_e=2, ρ_t=0.5, ρ_e=0.5, M=2, N=2, γ_{Eq} =30 dB, R_c=1.5, $\bar{\gamma}_0$= 5dB, R_s=0.75

On the basis of the analysis of numerical results, the summary of vital outcomes of this work is as follows:

- The mathematical model developed in this paper to ensure security in the spatially correlated MIMO OSFBC-OFDM Multicast system over frequency selective fading channels is a valid model and this model can be further extended to enhance the security level employing opportunistic relaying technique.
- Security levels in multicasting increases when number of transmit antenna n_t, receive antenna n_r and Transmit antenna correlation ρ_t increases. Both mathematical model for PNSMC and SOPM has exhibited almost the identical security performance.
- Security levels in multicasting decreases when number of eavesdropper N , the SNR of eavesdropper's channel γ_{Eq} and receive antenna correlation ρ_r increases. Both mathematical model for PNSMC and SOPM has exhibited almost the identical security performance.
- The degradation of security levels in multicasting due to the effects of receive antenna correlation, the number of eavesdroppers and the SNR of eavesdropper's channel can be compensated by increasing the number of antennas at the transmitter and receiver. But the improvement of security by changing n_r is more significant than by changing n_t.
- In the low SNR region, the effect of transmit antenna correlation on the P_{out} (R_{smcast}) is less significant than the effect of receive antenna correlation. But in the high SNR region, the effect of transmit antenna correlation on the P_{out} (R_{smcast}) is more significant than the effect of receive antenna correlation.

VII. CONCLUSION

This work is dedicated on the development of analytical mathematical models for PNSMC and SOPM which can ensure a secured radio network using MIMO OSFBC-OFDM system over frequency selective fading channel. The developed analytical model is helpful to quantify and realize the effects of system parameters on the security of proposed model. It is found that, secrecy performance of the proposed system enhances with the increase in number of transmit antenna n_t, receive antenna n_r and Transmit antenna correlation ρ_t and secrecy performance of the proposed system degrades with the increase in number of eavesdropper N, the number of user M, the SNR of eavesdropper's channel $\overline{\gamma}_e$ and receive antenna correlation ρ_r. Based on the mathematical model and the observations of numerical results, it can be concluded that in the low SNR region, security degrades with the antenna correlation significantly and the effect of transmit antenna correlation on the security is less significant than the effect of receive antenna correlation. But in the high SNR region, security enhances with the antenna correlation significantly and the effect of transmit antenna correlation on the security is more significant than the effect of receive antenna correlation. The degradation of secrecy measures due to the effects of receive antenna correlation and the number of eavesdroppers can be compensated by using antenna diversity and the improvement of security by changing n_r is more significant than by changing n_t. Moreover, this research pave the way of enhancing security of the proposed model applying opportunistic relaying technique without increasing the number of antennas and the transmit signal power.

REFERENCES

1. S. L. Loyka, "Channel capacity of MIMO architecture using the exponential correlation matrix," *IEEE Communications letters*, vol. 5, no. 9, pp. 369–371, 2001.
2. D. K. Sarker, M. Z. I. Sarkar, and M. S. Anower, "Secure wireless multicasting with linear equalization," *Physical Communication, Elsevier*, vol. 25, no. 1, pp. 201–213, 2017.
3. U. Raj and V. Bhaskar, "Performance analysis of scheduling schemes for MIMO OSFBC-OFDM system in α μ fading channel scenarios," in *proceedings, International Conference on Communication and Signal Processing*, 2013, pp. 144–148.
4. L. Kong, G. Kaddoum, and Z. Rezki, "Highly accurate and asymptotic analysis on the sop over simo α μ fading channels," *IEEE Communications Letters*, vol. 22, no. 10, pp. 2088–2091, 2018.
5. L. Kong, G. Kaddoum, and D. B. Da Costa, "Cascaded α μ fading channels: Reliability and security analysis," *IEEE Access*, vol. 6, pp. 41 978–41 992, 2018.
6. S. Yadav, "Secrecy performance of cognitive radio sensor networks over α μ fading channels," *IEEE Sensors Letters*, vol. 4, no. 9, pp. 1–4, 2020.
7. S. Yadav and D. S. Gurjar, "Secrecy performance of simo underlay cognitive radio networks over α–μ fading channels," *IEEE Access*, vol. 9, pp. 62 616–62 629, 2021.
8. J. M. Moualeu, D. B. da Costa, F. J. Lopez-Martinez, W. Hamouda, T. M. Ngatched, and U. S. Dias, "Secrecy analysis of a tas/mrc scheme in α μ fading channels," in *2019 IEEE Wireless Communications and Networking Conference (WCNC)*. IEEE, 2019, pp. 1–6.
9. J. M. Moualeu, D. B. da Costa, F. J. Lopez-Martinez, W. Hamouda, T. M. Nkouatchah, and U. S. Dias, "Transmit antenna selection in secure mimo systems over α μ fading channels," *IEEE Transactions on Communications*, vol. 67, no. 9, pp. 6483–6498, 2019.
10. A. Hanif, A. Badrudduza, M. Hossen, M. Kundu, and M. Sarkar, "Secure outage performance analysis of multicasting through α- μ fading channel," in *2019 3rd International Conference on Electrical,*

Computer & Telecommunication Engineering (ICECTE). IEEE, 2019, pp. 65–68.
11. C. Yuan, X. Tao, W. Ni, N. Li, A. Jamalipour, and R. P. Liu, “Joint power allocation and beamforming for overlaid secrecy transmissions in mimo-ofdm channels,” *IEEE Transactions on Vehicular Technology*, vol. 69, no. 9, pp. 10 019–10 032, 2020.
12. S. R. Krishna, “Spectrum efficiency for rate-adaptive MIMO OSFBC-OFDM systems with limited and full feedback over various adaptation policies,” 2011.
13. M. M. Hasan and M. Z. I. Sarkar, “Security in multicast over α-μ fading channels with orthogonal frequency division multiplexing,” *Journal of Computer and Communications*, vol. 10, no. 11, pp. 72–90, 2022.
14. A. Mathur, Y. Ai, M. Cheffena, and G. Kaddoum, “Secrecy performance of correlated α-μ fading channels,” *IEEE CommunicationsLetters*, vol. 23, no. 8, pp. 1323–1327, 2019.
15. X. Sun, J. Wang, W. Xu, and C. Zhao, “Performance of secure communications over correlated fading channels,” *IEEE Signal ProcessingLetters*, vol. 19, no. 8, pp. 479–482, 2012.
16. K. Yang, N. Yang, C. Xing, and J. Wu, “Relay antenna selection in mimo two-way relay networks over nakagami-*m* fading channels,” *IEEE Transactions on Vehicular Technology*, vol. 63, no. 5, pp. 2349–2362, 2013.
17. X. Liu, “Outage probability of secrecy capacity over correlated log-normal fading channels,” *IEEE communications letters*, vol. 17,no. 2, pp. 289–292, 2012.
18. G. C. Alexandropoulos and K. P. Peppas, “Secrecy outage analysis over correlated composite nakagami-*m*/gamma fading channels,” *IEEE Communications Letters*, vol. 22, no. 1, pp. 77–80, 2017.
19. N. S. Ferdinand, D. B. da Costa, A. L. de Almeida, and M. Latva-aho, “Physical layer secrecy performance of tas wiretap channelswith correlated main and eavesdropper channels,” *IEEE Wireless Communications Letters*, vol. 3, no. 1, pp. 86–89, 2013.
20. J. Sun, H. Bie, X. Li, J. Zhang, G. Pan, and K. M. Rabie, “Secrecy performance analysis of SIMO systems over correlated κ–μ shadowed fading channels,” *IEEE Access*, vol. 7, pp. 86 090–86 101, 2019.
21. M. Ibrahim, M. Sarkar, A. Badrudduza, M. Kundu, and S. Dev, “Impact of correlation on the security in multicasting through κ-μ shadowed fading channels,” in *2020 IEEE Region 10 Symposium (TENSYMP)*. IEEE, 2020, pp. 1396–1399.
22. M. Kundu, A. Sumona, A. Badrudduza, and S. Shabab, “Analysis of secrecy performance over correlated η-μ fading channels,” in *2019 IEEE International Conference on Signal Processing, Information, Communication & Systems (SPICSCON)*. IEEE, 2019, pp. 100–103.
23. A. Badrudduza, M. Sarkar, and M. K. Kundu, “Enhancing security in multicasting through correlated Nakagami-*m* fading channels with opportunistic relaying,” *Physical Communication*, vol. 43, p. 101177, 2020.
24. S. Al-Juboori and X. N. Fernando, “Multiantenna spectrum sensing over correlated Nakagami-*m* channels with MRC and EGC diversity receptions,” *IEEE Transactions on Vehicular Technology*, vol. 67, no. 3, pp. 2155–2164, 2017.
25. R. Sultana, M. Sarkar, and M. Hossain, “Security in wireless multicast networks with exponential correlation over Nakagami-*m* fading channels,” in *proceedings, 3rd International Conference on Electrical, Computer & Telecommunication Engineering (ICECTE)*, 2019, pp. 129–132.
26. C. S. R. Krishna and V. Bhaskar, “Spectrum efficiency for spatially correlated MIMO OSFBC-OFDM

systems over various adaptation policies," *International Journal of Soft Computing and Engineering*, vol. 2, no. 3, pp. 81–85, July 2012.
27. I. Gradshteyn and I. Ryzhik, "Asymptotic series," 2000.
28. M. Z. I. Sarkar and T. Ratnarajah, "Secure wireless multicasting through nakagami-m fading miso channel," in *2011 Conference Record of the Forty Fifth Asilomar Conference on Signals, Systems and Computers (ASILOMAR)*. IEEE, 2011, pp. 300–304.
29. M. Z. I. Sarkar and T. Ratnarajah, "Information-theoretic security in wireless multicasting," in *International Conference on Electrical & Computer Engineering (ICECE 2010)*. IEEE, 2010, pp. 53–56.