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Mathematical Formulation of Memory Model for Magnetization of Magnetic Fluid and its Application Using Numerical Methods

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Abstract:

In this paper we propose memory model for magnetization of magnetic fluid - a colloidal suspension of magnetic nano particles in a liquid carrier.The idea originates from the mathematical formulation of constitutive equations of solids with memory given by Boltzmann.

Keywords: Memory Model, Magnetization and Magnetic fluid

1. Introduction

In continuum mechanics we find a class of materials which possess memory [1]. For this class of materials, the present state of deformation cannot be determined completely unless the entire history of loading is known. In other words, these materials remember how they have been loaded to the present state and respond accordingly. Mathematical formulation of constitutive equations of solids with memory was given by Boltzmann [1]. To incorporate memory in the constitutive equations of elastic solids, Boltzmann assumed that the linear relationship between load and deflection depends on a third parameter, time. Thus, for solids with memory the deflection u(t) is proportional to the force F(t) but the constant of proportionality c is a function of time t. For solids which do not possess memory the constant of proportionality c is a simple constant, independent of time.

In sec.2, we give Boltzmann’s formulation of solids with memory as it is necessary to understand the memory model of magnetization of magnetic fluid.

In sec.3, we summarize Langevin’s approach to find magnetization of magnetic fluid. In his work magnetic fluid is subject to constant magnetic field H at a given absolute temperature T and magnetization of the magnetic fluid is obtained in terms of energy ratio mH/kT(where m is magnetic moment of magnetic particle and k is Boltzmann’s constant).

In sec.4, we propose a memory model for magnetization of magnetic fluid. It is shown that if magnetic fluid is subject to a time varying magnetic field it will acquire memory. That is, in this case magnetization of magnetic fluid at time t will not depend on the energy ratio at time t but it will depend on the time history of energy ratio up to the time t.

In sec.5, we take a numerical example and calculate magnetization of magnetic fluid using equations of the proposed memory model.

In the last section we calculate magnetization of magnetic fluid by Langevin's formula for the example of sec 5. It is found that the magnetization obtained by equations of memory model is more than that calculated by using Langevin's formula. The increase in magnetization is the effect of memory acquired by the magnetic fluid.

2. Mathematical Formulation of Elastic Solids with Memory given by Boltzmann.

A class of materials for which the load-deflection relationship is linear but this linear relationship depends on a third parameter, the time, possess memory. For these materials, the present state of deformation cannot be determined completely unless the entire history of loading is known. Mathematically, the load-deflection relationship for these materials can be written as [1].

$$u(t) = c(t)\, F(t), \qquad (2.1)$$

Where u(t) is the elongation produced by the time varying force F(t) and c is the constant of proportionality and is supposed to be a function of time.

If a simple bar fixed at one end is subject to a time varying force F(t) in the direction of the axis at the other end, then in a small time interval dτ at time t = τ, the increment in loading is $(\frac{dF}{dt})d\tau$ at $t = \tau$.

. This increment remains active on the bar and contributes an element du(t) to the elongation at time t, with a proportionality constant c depending on the time interval (t-τ.) Hence

$$du(t) = c(t - \tau)F'(\tau)\, d\tau, \qquad (2.2)$$

where F′(τ) denotes derivative of F(τ).

Integrating this, we get

$u(t) = \int_0^t c(t-\tau)\, F'(\tau)\, d\tau$, where t=0 is the time at which force F(t) starts acting. (2.3)

If F(t)=I(t), the unistep function then eq (2.3) gives

u(t)=c(t)

Thus, physically the constant of proportionality c(t) in eq (2.1) is the elongation produced by application of the force F(t)=I(t) at time t=τ.

3. Langevin's Approach to find Magnetization of Magnetic Fluid

Magnetic fluid is a colloidal suspension of magnetic nano particles in a liquid carrier. The important property of this fluid is that it can be made to flow by applying magnetic field and its flow can be controlled by controlling the magnetic field. Because of this peculiar property, this fluid finds many applications in technology including Nanotechnology and Biomedical applications. Hence in-depth study of its properties is necessary. The first author of this paper has published papers on Magnetic fluid properties and other areas[3-5].

In this section we summarize Langevin's approach[1] to find magnetization of magnetic fluid. In the absence of an applied magnetic field the magnetic particles of magnetic fluid are randomly oriented and the fluid has no net magnetization. When a magnetic field is applied, the magnetic particles try to align with the field. This tendency of alignment is overcome by thermal agitation. The Probability p(Θ) that magnetic particle has orientation Θ is proportional to Boltzmann factor [2] and is given by

$$p(\Theta) = c e^{-\alpha(1-\cos\theta)} \qquad (3.1)$$

Where c is constant of proportionality and α is energy ratio given by

$$\alpha = \frac{mH}{kT} \qquad (3.2)$$

Where m is the magnetic moment of the magnetic particles, H is the applied magnetic field, k is Boltzmann's constant and T is absolute temperature.

The number of particles lying in the configuration space between Θ and Θ + dΘ is given by [2]

$$n(\Theta)d\Theta = \frac{N}{2} sin\theta \, . ce^{-\alpha(1-cos\theta)} \, d\theta \qquad (3.3)$$

The constant of proportionality c must satisfy the condition

$$\int_0^{\pi} n(\theta) \, d\theta = N \qquad (3.4)$$

Where N is the total number of magnetic particles in the magnetic fluid.
Using the condition (3.4), we get

$$c = \frac{2\alpha}{1 - e^{-2\alpha}} \qquad (3.5)$$

In Langevin's work, magnetic fluid is subject to a constant magnetic field H at a given absolute temperature T. Hence the energy ratio α (given by equation (3.2)) is constant. This implies that the constant of proportionality c given by equation (3.5) is also constant. Hence, according to Boltzmann, Langevin's approach to find magnetization of magnetic fluid does not involve the memory concept.

The effective dipole moment of a particle is its component along the field direction and is equal to mcosΘ. Langevin calculated average value $\overline{m}$ of mcosΘ by using the formula

$$\overline{m} = \int_0^{\pi} m \, cos\theta \, n(\theta) \, d\theta / \int_0^{\pi} n(\theta) \, d\theta$$

and obtained

$$\overline{m} = mL(\alpha) \qquad ....(3.6)$$

Where L(α) denotes Langevin's function and is given by

$$L(\alpha) = coth\alpha - \frac{1}{\alpha} \qquad (3.7)$$

The magnitude of magnetization M of the magnetic fluid is the total of the effective dipole moments of the magnetic particles in its unit volume and is given by

$$\mu_0 M = n\overline{m} \qquad (3.8)$$

Where $\overline{m}$ is given by equation (3.6) and n is the number of magnetic particles per unit volume of the magnetic fluid. The direction of magnetization is in the direction of the applied magnetic field.

4. Memory Model for Magnetization of Magnetic Fluid

As explained in sec. 3, when magnetic fluid is subject to constant (independent of time) magnetic field H(t) at a given absolute temperature T, its magnetization does not involve memory. But if the applied magnetic field H and/or the absolute temperature T are functions of time t, the energy ratio α is a function of time. Hence the constant of proportionality c (given by eq. 3.5)and appearing in the (eq. 3.1) for the probability of orientation Θ of magnetic particles also becomes function of time t. According to Boltzmann [1] this implies that probability of orientation Θ is a memory function of time t. That is, probability of orientation of the particle at time t does not depend on the value of energy ratio α at time t but it depends on the time history of energy ratio α. In the proposed memory model for magnetization, magnetic fluid is subject to magnetic field Ht at a given absolute temperature T_0. Hence the energy ratio α becomes a function of time t and we denote it by α_1. Thus,

$$\alpha_1 = \mathrm{m}H(t)/kT_0 \qquad (4.1)$$

The eq. (3.1) for probability of orientation Θ of magnetic particle in magnetic fluid modifies to

$$p_1(\Theta) = c_1(t).e^{-\alpha_1(1-\cos\theta)} \qquad \ldots(4.2)$$

Where, c_1(t) is the constant of proportionality and is obtained by using the condition given by eq.3.4. Using this condition, we get

$$c_1(t) = \frac{2\alpha_1}{1-e^{-2\alpha_1}} \qquad \ldots(4.3)$$

The change in probability $p_1(\Theta)$ of orientation Θ of the magnetic particle at time t=τ in a small time interval dτ is

$$dp_1(\Theta) = c_1(t-\tau)\left[\frac{d}{dt}e^{-\alpha_1(1-\cos\theta)}\right]_{t=\tau}d\tau$$
$$= c_1(t-\tau)\left[e^{-\alpha_1(1-\cos\theta)}.(\cos\theta - 1)\frac{d}{dt}\alpha_1\right]_{t=\tau}d\tau \qquad \ldots(4.4)$$

On integrating this, we get probability of orientation Θ at time t as

$$p_1(\Theta) = p_0(\Theta) + (\cos\Theta - 1)\int_0^t c_1(t-\tau)\left[\left[e^{\alpha_1(\cos\theta-1)}\left(\frac{d}{dt}\alpha_1\right)\right]\right]_{t=\tau}d\tau \qquad \ldots(4.5)$$

Where $p_0(\Theta)$ is the probability of orientation Θ at time t=0. If at time t=0, the applied magnetic field is 0 then $p_0(\Theta) = 1$.

The number of particles lying at time t in the configuration space between Θ and $\theta + d\theta$ is given by [2] pp.(56-57).

$$n_1(\Theta)\,d\Theta = \frac{N}{2}(\sin\Theta).p_1(\Theta)\,d\Theta \qquad \ldots(4.6)$$

Where, N is the total number of magnetic particles in magnetic fluid and $p_1(\Theta)$ is given by eq. (4.5).

The average value of effective dipole moment mcosΘ of a magnetic particle along the field is

$$\overline{m}_1 = \frac{\int_0^\pi m\cos\theta\,.n_1(\theta)\,d\theta}{\int_0^\pi n_1(\theta)\,d\theta} = \frac{\int_0^\pi m\cos\theta\,.n_1(\theta)\,d\theta}{N} \qquad \ldots(4.7)$$

Where $n_1(\Theta)$ dΘ is given by eq. (4.6) and $p_1(\Theta)$ is given by eq. (4.5).

The eq. (3.8) for magnetization of magnetic fluid modifies to

$$\mu_0 M = n\overline{m} \qquad \ldots(4.8)$$

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Where, n is the number of magnetic particles per unit volume of the magnetic fluid and $\overline{m}_1$ is given by eq. (4.7).

5. Illustrative Numerical Example

In this section we calculate magnetization of magnetic fluid at time t=4 sec. The magnetic fluid is subject to time-varying magnetic field and equations of the memory model derived in section 4 are used to calculate magnetization of the magnetic fluid. The integrals involved in the equations are calculated by using Simpson's one third rule.

We write the energy ratio α_1 given by eq. (4.6) as

$$\alpha_1 = \frac{m}{kT_0} \mathrm{H(t)} = \lambda \mathrm{H(t)} , \quad \ldots(5.1)$$

Where,

$$\lambda = \frac{m}{kT_0} \quad \ldots(5.2)$$

If we take

$$\mathrm{H(t)} = \frac{1}{2\lambda} t^2 , \quad \ldots(5.3)$$

Then

$$\alpha_1 = \frac{1}{2} t^2 \quad \ldots(5.4)$$

Substituting for α_1 from eq. (5.4) in eq. (4.3) we get

$$c_1(\mathrm{t}) = \frac{t^2}{1-e^{-t^2}} \quad \ldots(5.5)$$

From eq. (5.3), the applied magnetic field H(t) at time t=0 is zero. Hence

$$p_0(\Theta) = 1 \quad \ldots(5.6)$$

Using eq. (5.6), eq. (5.4) and eq. (5.5) in eq. (4.5), we get

$$p_1(\Theta) = 1 + (\cos\Theta - 1) \int_0^{t=4} F(\tau) d\tau \quad \ldots(5.7)$$

Where,

$$\mathrm{F}(\tau) = \frac{\tau(t-\tau)^2}{1- e^{-(t-\tau)^2}} e^{\frac{1}{2}(\cos\theta - 1)\tau^2} \quad \ldots(5.8)$$

By Simpson's one third rule

$$\int_0^4 F(\tau) d\tau = \frac{1}{3}[\{\mathrm{F(0) + F(4)}\} + 4\{\mathrm{F(1) + F(3)}\} + 2\mathrm{F(2)}] \quad \ldots(5.9)$$

Substituting the values of F(0), F(4),…. In eq. (5.9), we get

$$\int_0^4 F(\tau) d\tau = \frac{4}{3} [a_1 e^{\frac{1}{2}(\cos\theta - 1)} + a_2 e^{\frac{9}{2}(\cos\theta - 1)} + a_3 e^{2(\cos\theta - 1)}] \quad \ldots(5.10)$$

Putting this in eq. (5.7), we get

$p_1\Theta = 1 + \frac{4}{3}(\cos\ \theta\ -1)[\quad a_1\, e^{\frac{1}{2}(coscos\,\theta-1\,)} + a_2\, e^{\frac{9}{2}(\cos\,\theta-1\,)} + a_3\, e^{2(\cos\,\theta-1\,)}]$ … (5.11)

Where,

$$a_1 = \frac{9}{1-e^{-9}} = 9.001$$
$$a_2 = \frac{3}{1-e^{-1}} = 4.745$$
$$a_3 = \frac{4}{1-e^{-4}} = 4.0746$$

The number of particles in the configuration space between Θ and Θ+dΘ at time t=4 is

$$n_1(\Theta)d\Theta = \frac{N}{2}(\sin\Theta).p_1(\Theta)\, d\Theta \qquad \text{… (5.12)}$$

Where $p_1(\Theta)$ is given by eq. (5.11).

The average value of the effective dipole moment of a magnetic particle along the field is

$$\overline{m}_1 = \frac{\int_0^{\pi} m\cos\theta\, .n1(\theta)d\theta}{\int_0^{\pi} n1(\theta)d\theta} = \frac{\int_0^{\pi} m\cos\theta\, .n1(\theta)d\theta}{N} \qquad \text{…(5.13)}$$

Substituting for $n_1(\Theta)d\Theta$ from eq. (5.12), writing $\int_0^{\pi} n_1(\theta)d\theta$ = N, the total number of particles in the magnetic fluid, eq. (5.13) can be written as

$$\overline{m}_1 = \frac{m}{2}\int_0^{\pi} F_1(\theta)d\theta\ , \qquad \text{…(5.14)}$$

Where,

$$F_1(\Theta) = \sin\Theta\cos\Theta\,[\,1 + \frac{4}{3}(\cos\theta - 1)\{9.001\, e^{\frac{1}{2}(\cos\theta-1)} + 4.745\, e^{\frac{9}{2}(\cos\theta-1)} + 4.0746\, e^{2(\cos\theta-1)}\}] \qquad \text{…(5.15)}$$

Using Simpson's one-third rule

$$\int_0^{\pi} F_1(\theta)d\theta = \frac{\pi}{12}[(F_1(0) + F_1(\pi)) + 4(F_1(\tfrac{\pi}{4}) + F_1(\tfrac{3\pi}{4})) + 2\, F_1(\tfrac{\pi}{2})]$$
$$= 3.505 \qquad \text{…(5.16)}$$

Putting this in eq. (5.14), we get

$$\overline{m}_1 = 1.752\text{m} \qquad \text{…(5.17)}$$

Substituting for $\overline{m}_1$ from eq. (5.17) in eq. (4.8), we get magnetization of magnetic fluid as

$$\mu_0 M = n\overline{m}_1 = (1.752\text{m})\text{n} \qquad \text{…(5.18)}$$

Where n is the number of magnetic particles per unit volume of the magnetic fluid.

6. Comparison of the value of magnetization obtained by the proposed memory model and the value obtained by Langevin's Formula

In the previous section, we calculated magnetization of magnetic fluid using the memory model. The magnetization of magnetic fluid at time t=4 sec. was found to be

$\mu_0 M = (1.752m)n$ …(6.1)

If we calculate the magnetization of magnetic fluid using Langevin's formula for the applied magnetic field given by eq. (5.3), the energy ratio α_1 given by eq. (5.1) at time t=4sec is $\alpha_1 = 8$. Hence

the average value $\overline{m}$ of the effective dipole moment given by Langevin's formula is $\overline{m}$ = mL(α)= 0.875m Hence the magnetization of the magnetic fluid is

$\mu_0 M = (0.875\ n)m$ …(6.2)

Where, n is the number of magnetic particles per unit volume of the magnetic fluid.

Thus, the magnetization of magnetic fluid obtained by using a memory model is more than that calculated by Langevin's formula. This is the effect of memory acquired by the magnetic fluid (when a time varying magnetic field is applied to magnetize the fluid). The magnetic fluid remembers how the magnetizing field is changing and its response (magnetization of the magnetic fluid) is based on the time history of the magnetizing field.

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