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# Enhancing Limited Authority Adaptive Control: Peudocontrol Hedging For Addressing Input Saturation

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#### Abstract:

This paper introduces an innovative approach to address the challenges of input saturation and interconnected subsystem control in limited authority adaptive control systems. Limited authority adaptive control faces issues when encountering input saturation, which can severely impact the system's performance and adaptability. Additionally, when dealing with cascaded control systems, ensuring smooth adaptation for both subsystems is essential. To overcome these challenges, we propose a method known as Pseudocontrol Hedging (PCH), which allows adaptation to continue even during input saturation and ensures effective control in interconnected subsystems. The PCH technique modifies the reference model response to accommodate expected system behavior, enabling reliable adaptation regardless of system response.

Keywords: Neural Network, Dynamic Inversion, Adaptive Control, Pitch Control, PCH

### I. INTRODUCTION

Limited authority adaptive control systems encounter difficulties when input saturation occurs, affecting the system's performance and adaptability. This paper explores a novel approach called Pseudocontrol Hedging (PCH) to address these challenges effectively. PCH involves feeding back expected system responses to the reference model, allowing adaptation to proceed seamlessly during input saturation. Additionally, PCH provides a solution for managing interconnected subsystems, ensuring that adaptive control can operate independently and effectively for each subsystem. Section 2 talks about limited authority adaptive control in interconnected systems. Pseudocontrol hedging is discussed in Section 3. The system architecture and some of the key components of using the Dynamic inversion based adaptive control design method are described in Section 4. Section 5 talks about the role of neural network to perform the adaptation of the gains. Adaptation law based on Lyapunov's direct method is described in section 6. Simulation parameters are given in section 7.

### II. LIMITED AUTHORITY ADAPTIVE CONTROL

In limited authority adaptive control systems [1-3], multiple subsystems often interact, where the state of one subsystem becomes the input for another. For instance, in flight control, the attitude subsystem controls the velocity subsystem. To enable effective adaptation for both interconnected subsystems, it is vital to account for the response of one subsystem when enabling adaptation for the other. In cases where one subsystem cannot respond fully, the design should ensure that the adaptation of the other subsystem



can proceed without hindrance. Pseudocontrol hedging offers an effective solution for addressing this challenge.

#### III. Pseudocontrol Hedging

Pseudocontrol Hedging (PCH) [4-6] is a method specifically tailored for dynamic inversion-based limited authority adaptive controllers. PCH involves providing feedback on the expected system response to the reference model, allowing adaptation to function optimally regardless of system behavior. This method ensures that adaptation continues to work even during input saturation, which is a crucial phase for maintaining reliable adaptation.

#### IV. System Architecture

In the Section above method of dynamic inversion was described for the control of nonlinear systems. The true dynamics of the aircraft are replaced by desired dynamics through feedback. The gains will be changing within the controller in response to changes in the aircraft. This approach will address the problem of of unavailability of an accurate model of aircraft dynamics. An illustration of such a controller is shown in Fig. 1. Here the architecture full state feedback with an adaptive component. This architecture includes a 'nominal' controller based on the best available information about the aircraft dynamic embedded in the design. The adaptive element then works on the error in this nominal model and provides a correction. This correction can be arbitrarily accurate correction given sufficient training information, and sufficient inputs to the correction block in the form of state and inputs. And sufficient power in the adaptive element to curve fit this model error.



Figure 1: Dynamic inversion controller with MRAC

Consider a first-order system corresponding to pitch-rate control of an aircraft with a significant unknown nonlinearity in pitch damping using the elevator. The true dynamics are given as

$$\dot{q} = M_{q}q + M_{\delta}\delta + \sin(q) \tag{1}$$

This system can represented by

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{x} + f(\mathbf{x}, \mathbf{u}) \tag{2}$$

Where Ax + Bu is the linear part and f(x, u) represents the nonlinearity in the system. Let the output be given by

$$y = Cx$$
  
C=1, Taking the derivative of the output as  
 $\dot{y} = C\dot{x}$  (3)

Now the reference model tracking error dynamics can be written as  $\dot{e} = \dot{y} - \dot{r}$  (4)



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 $\dot{r}$  the term represents the desired response, substituting eq. 2 and 3, eq. 4 becomes

 $\dot{\mathbf{e}} = C(Ax + Bx + f(x,u)) - \dot{\mathbf{r}}$ 

Now,  $\dot{e} = -Ke$  represents a stable system as long as K is positive definite. Eq becomes

 $C(Ax + Bu + f(x, u)) - \dot{r} = -Ke$ 

Substituting the value of A, B, C, a puedocontrol signal v is represented as

$$v = \dot{r} + Ke - f(x, u)$$

adaptive control element will ideally be a function of both the state and input of the original system. The puedocontrol can be further written as

 $v = \dot{r} + Ke - v_{ad}(x, u, \theta)$ 

Where ke term is a linear feedback on the tracking error, and  $v_{ad}$  the new adaptive signal, which includes internal plant parameter  $\theta$ .

PCH allows the control system designer to select which system characteristics adaptation should correct for, ensuring that the controller does not attempt to correct for absolute input saturation or the responses of subsystems with their adaptive processes. To illustrate this in the context of input saturation, we introduce variables ucmd (unsaturated input) and u (saturated input). PCH modifies the reference model's motion to accommodate the expected system response, enabling adaptation to proceed effectively.

The PCH controller is illustrated in Fig.1, where the PCH block shown computes the hedge signal

$$v_h = v - \hat{f}(x, u)$$

The reference model motion is modified by PCH in a specific way as described here. Let the original model is of the form

 $\dot{\mathbf{r}} = f_{rm}(r)$ 

The modified form with PCH becomes

$$\dot{\mathbf{r}} = f_{rm}(r) - v_h$$
$$v_{rm} = f_{rm}(r)$$

Where  $v_{rm}$  is the feedforward signal from the reference model. With PCH the reference model tracking error dynamics (e) given in eq. 5, remains the same. Note that  $u_{cmd}$  does not appear in this equation which means that the adaptation is able to function properly and attempt to correct for the desired form of model error.

The dynamics are now inverted to find the plant input u and pseudocontrol input corresponding to the derivative of pitch rate is

$$\boldsymbol{\delta} = M_{\boldsymbol{\delta}}^{-1}(\boldsymbol{v} - \mathbf{M}_{\mathbf{q}}\mathbf{q})$$

Additionally,  $v_{ad}$  can be formulated by doing a nonlinear curve fitting. Specifically, we need to achieve a curve fit of the model error as a function of state and input. This adaptive controller architecture possesses significant adaptability, as it has the capability, in principle, to modify its internal parameters to achieve desired system dynamics for an unknown nonlinear system. In practical terms, it suffices for the controller to be capable of effectively rectifying errors within the specific subset of the input and state space that the aircraft has encountered. In many scenarios, we may also find satisfaction in the controller's ability to adapt exclusively to the recently encountered portion of the input and state space. In other words, it might be permissible for the controller to discard information about aspects of the aircraft that are no longer relevant. An acceptable condition for this approach would be the controller's ability to respond swiftly when confronted with new information. This differentiation allows us to distinguish between long-term learning, where the controller enhances performance upon revisiting a portion of the state space the vehicle



has encountered before, and short-term learning, where the controller must promptly readjust when returning to previous segments of the state space.

#### IV. NEURAL NETWORK ADAPTIVE CONTROL

In this section, we introduce a distinct approach for accomplishing short-term learning, rooted in the utilization of an artificial network concept. This construct has demonstrated its effectiveness in numerous curve-fitting scenarios. Within the context of our adaptive control framework, it can be aptly referred to as 'neural network adaptive control' [7-9]. Specifically, we consider a nonparametric neural network, as depicted in Figure 2, where the designer refrains from explicitly incorporating knowledge about the functional form of the model error.

A single hidden layer (SHL) is considered which utilizes a radial basis function given by

 $\sigma_j(z) = \frac{1}{1 + e^{-a_j z}}$ 

For j = 1,...N with the value of  $a_j$  chosen to be distinct for each j, N is the number of hidden layer neurons. Now the SHL NN can be written as

 $v_{ad}(x, u, \theta) = \mathbf{W}^{\mathrm{T}} \boldsymbol{\sigma}(\mathbf{V}^{\mathrm{T}} \underline{x})$ 

Where W is the output weights, v is the input weights, and  $\underline{x}$  is the input to NN, states x, plant inputs u, and a bias. The complete set of W and V are the NN adjustment parameters.

#### V. ADAPTATION LAW

The universal approximation theorem [10-11] for the SHL neural network offers valuable insights, demonstrating that we can constrain the fitting error within a defined set of system states and input parameters. A notable advantage is the ability to achieve this while catering to any chosen error threshold by augmenting the neural network with additional middle-layer neurons (N). The proofs of boundedness [12-13] frequently entail the use of a Lyapunov function candidate and the application of the universal approximation theorem to illustrate the decrease of the Lyapunov candidate beyond a compact set. This guarantees convergence to a region encompassing zero tracking error. Importantly, the considerations involved in proving boundedness often serve as inspiration for shaping the adaptive control laws themselves. A common approach to training the neural network, directly influenced by the proof of boundedness is.

 $\dot{\mathbf{W}} = -[(\boldsymbol{\sigma} - \boldsymbol{\sigma}' \mathbf{V}^{\mathrm{T}} \underline{\boldsymbol{x}})\mathbf{e}^{\mathrm{T}} + \boldsymbol{\lambda} \|\mathbf{e}\|\mathbf{W}]\boldsymbol{\Gamma}_{\mathrm{W}}$ 

 $V_{dot} = -\Gamma V [xeTWT\sigma' + \lambda ||e||V],$ 

where  $\Gamma W$  and  $\Gamma V$  are appropriately dimensioned diagonal matrices of learning rates. The matrix  $\sigma'$  is the gradient of  $\sigma$ . The e-modification scalar

 $\lambda > 0$  is necessary for the associated boundedness theorem proof. Note the important role of tracking error (e) here. When the tracking error is zero, these parameters do not change.

#### VI. SIMULATION

The first order system given in eq.. controlled using a dynamic inversion control based on adaptive control whose parameters are learned using a neural network, is simulated in MATLAB. The values of learning rate( $\lambda$ ), feedback gain(K), plant parameters (M $_{\delta}$ , M<sub>q</sub>), and adaptation gains ( $\Gamma W$ ,  $\Gamma V$ ) are given in Table 1. The feedback gain can be calculated using any technique like LQR or pole placement.



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Parameter	Value
λ	0.01
К	-1
${ m M}_\delta$	-10
Mq	-1
$\Gamma_W$	1
$\Gamma_V$	10

**Table 1: Simulation parameters** 

#### RESULTS

The historical system behavior during the execution of an aggressive square wave as the desired response is depicted in Fig. 2. Interestingly, the controller continues to exhibit improved performance with each subsequent attempt at this maneuver. This noteworthy observation holds even in the presence of substantial modeling inaccuracies and persistent input saturation. As visually confirmed in the figure, the input remains consistently saturated for a considerable portion of the responses. Remarkably, the adaptive control approach continues to function effectively, overcoming the significant input saturation, as demonstrated in Fig. 3.



**Figure 3: Input history** 



In Fig. 4, the adaptive parameters, comprising all the elements of V and W, are displayed. Following an initial transient period, it becomes evident that the weights converge to nearly constant values, even as the vehicle executes rapid maneuv*ers*.



Figure 4: Neural network parameters

#### CONCLUSION

In conclusion, the paper has explored the dynamic inversion control technique as a powerful strategy for managing and stabilizing complex nonlinear systems. Through a model-based approach and the design of an inverse model, this control architecture allows for the effective cancellation of nonlinear dynamics, thereby transforming intricate systems into more manageable linear representations. This adaptability makes it particularly well-suited for applications like aircraft control and robotics, where system dynamics may change significantly over time.

Pseudocontrol Hedging (PCH) offers a robust solution to address input saturation and interconnected subsystem control issues in limited authority adaptive control systems. By feeding back expected system responses to the reference model, PCH ensures that adaptation continues to operate optimally, even during input saturation. This approach is vital for maintaining reliable adaptation during critical periods of system behavior, allowing for enhanced control in cascaded subsystems. PCH represents a significant step forward in the pursuit of adaptive control strategies for complex systems, providing a practical and effective solution to real-world challenges.

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