International Journal for Multidisciplinary Research (IJFMR)

E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

Mechanical Vibration Analysis of An Uav Wing Spar

Merina Amon Mwasandube

Assistant lecturer, Department of Aeronautical Engineering, National Institute of Transport.

Abstract

This paper presents the mechanical vibration analysis of a UAV (Unmanned aerial vehicle) wing spar. Theoretical and numerical calculations are performed by considering the spar as a cantilever beam. The model is created and modal analyses are performed by using the MATLAB software. The natural frequencies and the related mode shapes are obtained. The results of theoretical calculations linear, nonlinear and random vibration of both single and multiple degree of freedom (DOF) system are graphically presented. The study aims to illustrate vibration tendencies of the wing during flight.

.

Keywords: Aircraft Wing, Vibration, Cantilever Beam, Modal Analysis

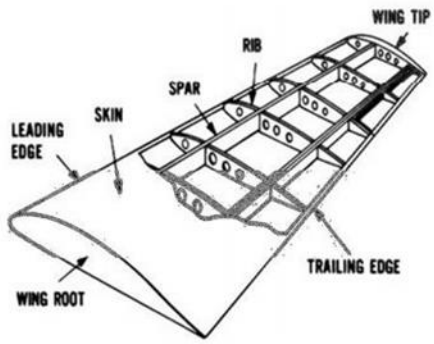
1. Introduction

1.1 Wing structure

A wing structure of an aircraft is a crucial component of flight that together with the help of airfoil profile that generates lift by the vehicle's forward airspeed and the shape of the wings. The internal structures of most wings are made up of spars and stringers running spanwise and ribs and formers or bulkheads running chordwise (leading edge to trailing edge).

The spars are the principle structural members of a wing[1]. They support all distributed loads, as well as concentrated weights such as the fuselage, landing gear, and engines. The skin, which is attached to the wing structure, carries part of the loads imposed during flight. It also transfers the stresses to the wing ribs. The ribs, in turn, transfer the loads to the wing spars.

Figure1: Wing internal structure

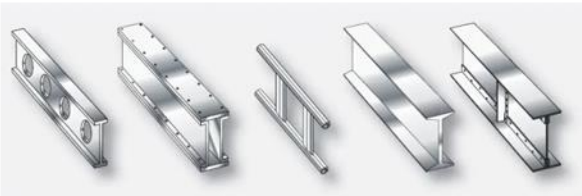


•

Spars are strong beams which run span wise in the wing and carry the force and moments due to the span wise lift distribution. They correspond to the longerons of the fuselage. They run parallel to the lateral axis of the aircraft, from the fuselage toward the tip of the wing, and are usually attached to the fuselage by wing fittings, plain beams, or a truss.

A wing has two spars. One spar is usually located near the front of the wing, and the other about two-thirds of the distance toward the wing's trailing edge Spar run parallel to the lateral axis of the aircraft, from the fuselage toward the tip of the wing, and are usually attached to the fuselage by wing fittings, plain beams, or a truss. Therefore the spar beams can be considered as a cantilever beam for the design purpose.
Spars may be made of metal, wood, or composite materials depending on the design criteria of a specific aircraft. They can be generally classified into four different types by their cross sectional configuration as shown in Figure 2. They may be solid, Box shaped, partly hollow and I–beam spar. The top and bottom of the I–beam are called the caps and the vertical section is called the web. The entire spar can be extruded from one piece of metal but often it is built up from multiple extrusions or formed angles. The web forms the principal depth portion of the spar and the cap strips are attached to it. Together, these members carry the loads caused by wing bending, with the caps providing a foundation for attaching the skin.

Figure 2: Types of Spar Configurations



In this paper, the aircraft spar is considered as a cantilever with a decreased load distribution from the root to tip. This decrease is all the same whether the spar is a rectangular, I-section, tapered Made of Aluminum alloy 2024-T4:2024-T351 characteristics.

In the present paper, linear, nonlinear and random vibration[2] was performed to both single and multiple degree of freedom model on an fixed wing UAV (Figure 3). Linear vibration on continuous model was also obtained. The simulation and calculations were performed by MATLAB and later compared to obtain a final analysis of the study. The material considered in the study is Aluminum alloys, valuable because they have a high strength-to-weight ratio, lightweight, corrosion resistant and comparatively easy to fabricate.

Figure 3: Fixed wing UAV

2. LINEAR VIBRATION OF A SINGLE DEGREE OF FREEDOM

In linear vibration of a single degree of freedom analysis we will analyse free, forced and random vibration of the models considering both damped and undamped system and obtain a natural frequency and response.

2.1Free vibration of a viscously damped single degrees of freedom system.

When a system, after an initial disturbance, is left to vibrate on its own, the ensuing vibration is known as free vibration[3]. No external force acts on the system. The oscillation of a simple pendulum is an example of free vibration. In this case the system is viscously damped so that was taken that into consideration.

A spar considered as a cantilever beam, mathematical models of the system was developed to investigate vibration in the horizontal direction. Consider the elasticity of the spar itself and introducing a damping and then mass (m) of the whole system is considered to be lumped at the end of the beam. Figure 1 shows a below is the mathematical model of a spar;

Figure 4: Viscously Damped Single Degrees of Freedom System

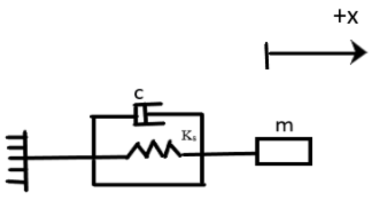


Figure 5: free body diagram

The viscous damping force *F* is proportional to the velocity or *v* and can be expressed as

F = - cx (1)

Where *c* is the damping constant or coefficient of viscous damping and the negative sign indicates that the damping force is opposite to the direction of velocity. If *x* is measured from the equilibrium position of the mass *m*, the application of Newton's law yields the equation of motion:

$m\ddot{x} + \mathrm{c}x + kx = 0$ (2)

To solve the above equation we assume a solution in the form

$\mathrm{x(t)} = \mathrm{C}e^{st}$ (3)

Where C and *s* are undetermined constants Introduce critical damping constant and damping ratio.

For any damped system, the damping ratio z is defined as the ratio of the damping constant c to the critical damping constant c_c

$\mathrm{z} = \frac{\mathrm{C}}{C_C} = \frac{\mathrm{C}}{\mathbf{2}\sqrt{mk}}$ (4)

Substituting equation (ii) and (iii) into (ii) we obtain two roots

$$s^2 + z\omega_n s + \omega_n = 0 \quad (5)$$

$$s_{1,2} = -z\omega_n \pm \omega_n\sqrt{z^2 - 1} \quad (6)$$

Thus the solution, can be written as

$$\boldsymbol{x(t) = C_1 e^{(-z+\sqrt{z^2-1})\omega_n t} + C_2 e^{(-z-\sqrt{z^2-1})\omega_n t}} \quad (7)$$

The nature of the roots will determine behavior of the solution, depending upon the magnitude of damping. . C_1 and C_2 can be obtained through initial conditions.

$$C_1 = \frac{x_0\omega_n(z+\sqrt{z^2-1})+\dot{x}_0}{2\omega_n\sqrt{z^2-1}} \quad \text{And} \quad C_2 = \frac{-x_0\omega_n(z-\sqrt{z^2-1})+\dot{x}_0}{2\omega_n\sqrt{z^2-1}} \quad (8)$$

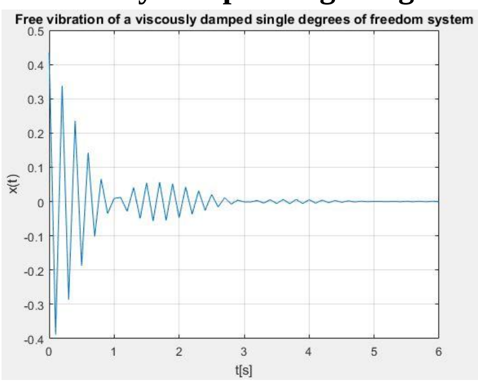
To find the free-vibration response of a viscously damped wing spar system, MATLAB program was used to find the response of a system with the Following data:

Lumped Mass (m) of the system = 62.096kg
Stiffness (K) = 3.046 x 10^6 N/m
Damping constant (c) = 1.5x10^3N.s/m
Initial displacement x_0= 0.4321 m
Initial velocity $\dot{x}_0$ =1.05m/s
The natural frequency ω_n = 221.479 and z = 0.005

2.2 Results

To find the free-vibration response of a viscously damped wing spar system, MATLAB program was used to find the response of a system with the
Following data:
Lumped Mass (m) of the system = 62.096kg
Stiffness (K) = 3.046 x 10^6 N/m
Damping constant (c) = 1.5x10^3N.s/m
Initial displacement x_0= 0.4321 m
Initial velocity $\dot{x}_0$ =1.05m/s
The natural frequency ω_n = 221.479 and z = 0.005
The results showed that the system is **underdamped as showm in the figure below.**

Figure 6: Results of viscously damped single degrees of freedom system



2.3 Forced vibration of an undamped single degree of freedom system.

In this case, the spar is an undamped system and is subjected to a harmonic force, so the damping equation;

$c\dot{x} = 0$ (9)

If a force acts on the mass *m* of an undamped system, the equation of motion reduces to;

$m\ddot{x} + kx = f_0 cos\omega t$ (10)

The homogeneous solution of this equation is given by;

$X_h = a_1 cos\omega_n t + a_2 sin\omega_n t$ (11)

Where ω_n is the natural frequency of the system. Because the exciting force (t) is harmonic, the particular solution X_p is also harmonic and has the same frequency ω Thus we assume a solution in the form;

$X_p = C_1 sin\omega t + C_2 cos\omega t$ (12)

For this case;

$\omega \neq \omega_n$ (13)

So we can obtain C_1 and C_2 as;

$C_1 = \frac{-f_0}{2m\omega_n}$, $C_2 = 0$ (14)

So the general solution for the system is;

$x(t) = x_h(t) + x_p(t)$ (15)

$a_1 cos\omega_n t + a_2 sin\omega_n t + \frac{-f_0}{m(\omega_n{}^2 - \omega^2)} sin\omega t$ (16)

The initial conditions $x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$ results to;

$a_1 = x_0$ and $a_2 = \frac{\dot{x}_0}{\omega_n} - \frac{f_0 \omega}{m(\omega_n{}^2 - \omega^2)}$ (17)

the response of the system under harmonic vibration will be:

$\mathbf{x(t)} = \boldsymbol{x_0 cos\omega_n t + \frac{\dot{x}_0}{\omega_n} - \frac{f_0 \omega}{m(\omega_n{}^2 - \omega^2)} sin\omega_n t + \frac{-f_0}{m(\omega_n{}^2 - \omega^2)} sin\omega t}$ (18)

MATLAB program was used to plot the response of a system based on the Following data:

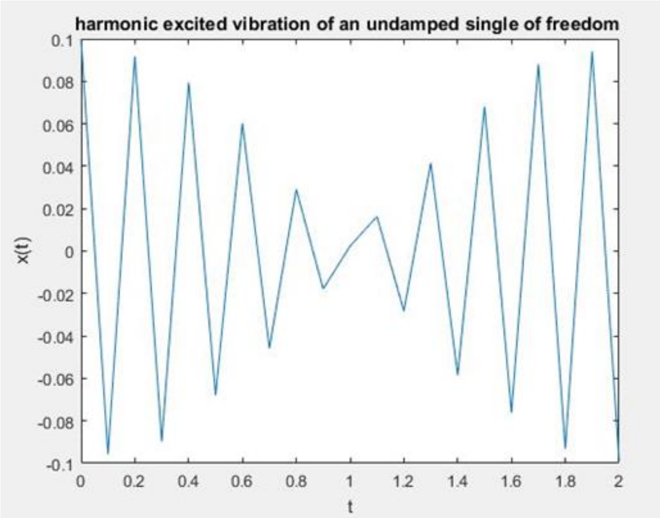
$\boldsymbol{\omega = 200\ rad/s, \omega_n = 222.479 rad/s}$,the F_0= 1277N

$\boldsymbol{f_0 = \frac{F_0}{m} = \frac{1277}{62.096} = 20.565}$

Initial conditions:

$$x_0 = 0.1m\,, \dot{x}_0 = \frac{0.2m}{s}$$

Figure 7: Results of harmonically excited single degrees of freedom system



3. Linear Vibration of a Multi Degree Freedom System

Systems that require two independent coordinates to describe their motion are called *two degree of freedom systems*. Considered a spar as a cantilever beam with the mass of a rib creating to masses with two independent coordinates creating a two degree of freedom system (DOF) and in this case we considered the free and forced vibration under damped and undamped condition.

3.1 Free vibration of a viscously damped two degrees of freedom system

Consider a viscously damped two-degree-of-freedom spring-mass system, shown in the Figure below;

Figure 8 : A Viscously Damped Two-Degree-Of-Freedom Spring-Mass System

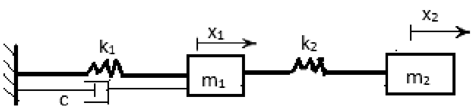


Figure 9: Free body Diagram

The motion of the system is completely described by the coordinates x_1 and x_2 which define the positions of the masses m_1 and m_2 and at any time *t* from the respective equilibrium positions. Since its free vibration no external forces and act on the masses.

The free-body diagrams of the masses and are shown in Figure 7.The application of Newton's second law of motion to each of the masses gives the equations of motion:

In matrix form;

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{matrix} \ddot{x}_1 \\ \ddot{x}_2 \end{matrix} + \begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix} \begin{matrix} \dot{x}_1 \\ \dot{x}_2 \end{matrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix} \quad (19)$$

$\boldsymbol{m_1}$ being the mass of a wing rib attached to the spar and $\boldsymbol{m_2}$ being the mass of a lumped system, $\boldsymbol{m_1}$=53 k(g , $\boldsymbol{m_2}$= 62.096 kg , $\boldsymbol{c = 150\ N.s/m}$, $\boldsymbol{k_2 = 2k_1 = k}$=3.046 x 10^6 N/m

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{matrix} \ddot{x}_1 \\ \ddot{x}_2 \end{matrix} + \begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix} \begin{matrix} \dot{x}_1 \\ \dot{x}_2 \end{matrix} + \begin{bmatrix} 2k & -k \\ -k & -k \end{bmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix} \quad (20)$$

Or $\boldsymbol{M\dot{y} + Ky = 0}$ (21)

Where $\boldsymbol{M} = \begin{bmatrix} c & M \\ M & 0 \end{bmatrix}, \boldsymbol{K} = \begin{bmatrix} -M & K \\ 0 & K \end{bmatrix}$ $\boldsymbol{and\ Y} = \begin{matrix} \dot{x} \\ x \end{matrix}$

The solution is assumed as

$\boldsymbol{y = \varphi e^{-\gamma t}}$ (22)

Where γ are the eigenvalues of $M^{-1}K$ and φ are the eigenvectors. The general solution is a linear combination over all solutions

$\boldsymbol{y = \sum_{j=1}^{4} c_j \varphi_j e^{-\gamma_j t}}$ (23)

And after introduction of initial conditions;

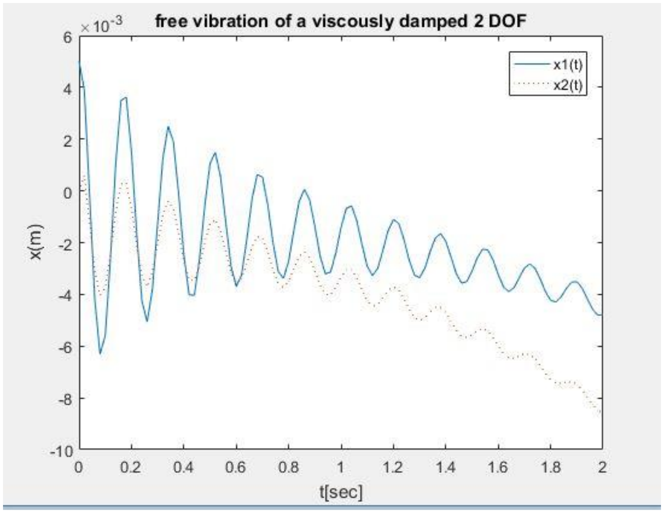
$\boldsymbol{x_1(0) = 0, x_2(0) = 0.005\ \mathrm{m}, \dot{x}_1\ (0 = 0, \dot{x}_2\ (0) = 0}$ (24)

$\boldsymbol{y = \sum_{j=1}^{4} c_j \varphi_j}$= VC (25)

$\boldsymbol{C = V^{-1} y_0}$ (26)

The MATLAB code to find the free-vibration response of the system and modal nodes and response of the system is as below;

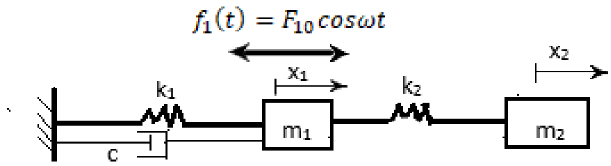
Figure 10: Results of a 2 DOF viscously damped system



3.2 Forced Vibration of Undamped Two Degrees of Freedom System

In this case harmonically excited two degree of freedom system of a wing spar was analysed. Since damping was disregarded; c =0, the below equations were utilized to obtain a steady state response and frequency-response curve was plotted.

Figure 11: harmonically excited two degree of freedom system



When masses m_1 and m_2 are excited by force

$$f_1(t) = F_{10}cos\omega t \quad (27)$$

The equation of motion can be expressed as;

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{matrix} \ddot{x}_1 \\ \ddot{x}_2 \end{matrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{matrix} F_{10}cos\omega t \\ 0 \end{matrix} \quad (28)$$

$\boldsymbol{m_1}$ being the mass of a wing rib attached to the spar and $\boldsymbol{m_2}$ being the mass of a lumped system, $\boldsymbol{m_1}$=53 kg , $\boldsymbol{m_2}$= 62.096 kg , $\boldsymbol{c = 0}$, $\boldsymbol{k_2 = 2k_1 = k}$=3.046 x 10^6N/m

$$\boldsymbol{f_1(t) = F_{10}cos\omega t\,, f_2 = 0} \quad (29)$$

We can write the steady-state solutions as

$$\mathbf{x_{j(t)} = Xje^{-i\omega t}},\ \boldsymbol{j = 1, 2} \quad (30)$$

Where X_1 and X are, in general, complex quantities that depend on ω and the system parameters. Substitution of Equation. (29) And (30) into Equation of motion leads to;

$$\begin{bmatrix} -\omega^2 m_1 + 3k & -2k \\ -2k & -\omega^2 m_2 + 2k \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_{10} \\ 0 \end{Bmatrix} \quad (31)$$

We define impedance matrix $\boldsymbol{Z_{rs}(i\omega)}$ as

$$\boldsymbol{Z_{rs}(i\omega) = -\omega^2 m_{rs} + k_{rs}} \quad (30)$$

So equation (iii) can be written as

$$[\boldsymbol{Z_{rs}(i\omega)}]\vec{\boldsymbol{X}} = \overrightarrow{\boldsymbol{F_0}} \quad (31)$$

$$[\boldsymbol{Z(i\omega)}] = \begin{bmatrix} \boldsymbol{Z_{11}(i\omega)} & \boldsymbol{Z_{12}(i\omega)} \\ \boldsymbol{Z_{21}(i\omega)} & \boldsymbol{Z_{22}(i\omega)} \end{bmatrix} \quad (32)$$

$$\vec{\boldsymbol{X}} = \overrightarrow{\boldsymbol{F_0}}[\boldsymbol{Z(i\omega)}]^{-1} \quad (33)$$

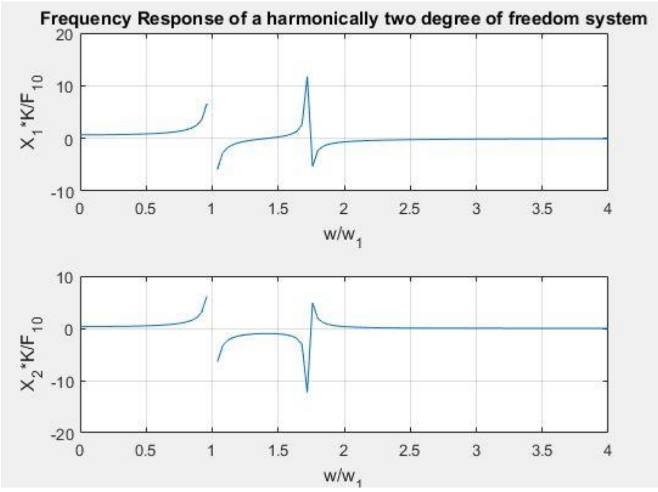
The solution will be;

$$\boldsymbol{X_1(i\omega) = \frac{Z_{22}(i\omega)F_{10}}{Z_{11}(i\omega)Z_{22}(i\omega) - Z_{12}{}^2(i\omega)} = \frac{-\omega^2 m_2 + 3k}{(-\omega^2 m_2 + 2k) - \omega^2 m_2 + 3k - 4k^2}} \quad (34)$$

$$\boldsymbol{X_2(i\omega) = \frac{-Z_{12}(i\omega)F_{10}}{Z_{11}(i\omega)Z_{22}(i\omega) - Z_{12}{}^2(i\omega)} = \frac{-2kF_{10}}{(-\omega^2 m_2 + 2k) - \omega^2 m_2 + 3k - 4k^2}} \quad (35)$$

With the data, MATLAB was used to plot frequency response curves.

Figure 12: Results of Forced Vibration of Undamped Two Degrees of Freedom System



4. Non Linear Vibration

4.1 Lindstedt's perturbation method

In the case of non linear vibration analysis, we considered the pertubation. The perturbation method is applicable to problems in which a small parameter associated with the nonlinear term of the differential equation[4]. The solution is formed in terms of a series of the perturbation parameter,$\boldsymbol{\varepsilon}$, the result being a development in the neighbourhood of the solution of the linearized problem. If the solution of the linearized problem is periodic, and if $\boldsymbol{\varepsilon}$, is small, we can expect the perturbed solution to be periodic also. We can reason from the phase plane that the periodic solution must represent a closed trajectory. The period, which depends on the initial conditions, is then a function of the amplitude of vibration. Consider the free oscillation of a mass on a nonlinear spring, which is defined by the equation;

$$\ddot{x} = \omega_n{}^2 x + \varepsilon x^3 \quad (36)$$

With initial conditions $x(0) = A, \dot{x}(0) = 0$ When $\boldsymbol{\varepsilon}$ = 0, the frequency of oscillation is that of the linear system,$\omega_n = \sqrt{\frac{k}{m}}$. We seek a solution in the form of an infinite series of the perturbation parameter p. as folIows:

$$x = x_0 t + \varepsilon x_1 t + \varepsilon^2 x_2 t \quad (37)$$

Furthermore, we know that the frequency of the nonlinear oscillation will depend on the amplitude of oscillation as well as $\boldsymbol{\varepsilon}$ on. We express this fact also in terms of a series in$\boldsymbol{\varepsilon}$;

$$\omega^2 = \omega_n{}^2 + \varepsilon\alpha_1 + \varepsilon^2\alpha_2 \quad (38)$$

Where the $\alpha_1$are as yet undefined functions of the amplitude, and ω is the frequency of the nonlinear oscillations. We consider only the first two terms of Equations (37) and (38), which will adequately illustrate the procedure. Substituting these into Eq. (i), we obtain;

$$\ddot{x}_0 + \varepsilon\ddot{x}_1 + (\omega^2 - \varepsilon\alpha_1)(x_0 + \varepsilon x_1) + \varepsilon(x_0{}^3 + 3\varepsilon x_0{}^2 x_1 + \cdots) \quad (39)$$

Because the perturbation parameter $\boldsymbol{\varepsilon}$ could have been chosen arbitrarily, the coefficients of the various powers of $\boldsymbol{\varepsilon}$ must be equated to zero. This leads to a system of equations that can be solved successively:

$$\ddot{x}_0 + \omega^2 x_0 = 0 \quad (40)$$

$$\ddot{x}_1 + \omega^2 x_1 = \alpha_1 x_0 - x_0{}^3 \quad (41)$$

The solution to the first equation, subject to the initial conditions, $x(0) = A, \dot{x}(0) = 0$ is;

$$x_0 = A cos\omega t \quad (42)$$

By substituting this into the equation above we get;

$$\ddot{x}_1 + \omega^2 x_1 = \alpha_1 A cos\omega t - A^3 cos^3\omega t \quad (43)$$

$$= \left(\alpha_1 - \frac{3}{4}A^2\right) A cos\omega t - \frac{A^3}{4} cos3\omega t \quad (44)$$

Where $cos^3\omega t = \frac{3}{4} cos\omega t + \frac{1}{4} cos3\omega t$ has been used. We note he re that the forcing term $cos\omega t$ would lead to a secular term $tcos3\omega t$ in the solution for x_1 (i.e., we have a condition of resonance). Such terms violate the initial stipulation that the motion is to be periodic; hence, we impose the condition;

$$\alpha_1 - \frac{3}{4}A^2 = 0 \quad (45)$$

With the forcing term $cos\omega t$ eliminated from the right side of the equation, the general solution for is x_1

$$x_1 = C_1 sin\omega t + C_2 cos\omega t + \frac{A^3}{32\omega^2} cos3\omega t \quad (46)$$

$$\omega^2 = {\omega_n}^2 + \frac{3}{4}\varepsilon A^2 \quad (47)$$

By imposing the initial conditions $x(0) = A, \dot{x}(0) = 0$, constants $C_1 and\ C_2$ are

$$C_1 = 0\ and\ C_2 = -\frac{A^3}{32\omega^2} \quad (48)$$

$$x_1 = \frac{A^3}{32\omega^2}(cos3\omega t - cos\omega t) \quad (49)$$

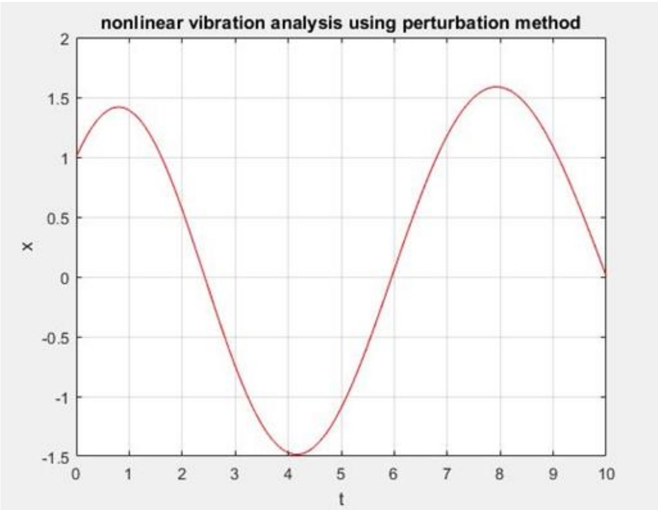
The non linear vibration response equation can therefore be obtain by;

$$x = A\ cos\omega t + \varepsilon \frac{A^3}{32\omega^2}(cos3\omega t - cos\omega t) \quad (50)$$

$$\omega = \omega_n \sqrt{1 + \frac{3}{4}\frac{\varepsilon A^2}{{\omega_0}^2}} \quad (51)$$

Below figure was obtained after analysis.

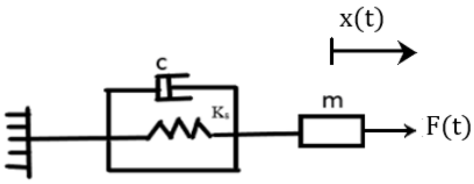
Figure 13: Results of Nonlinear Vibration by Perturbation Method



5. Random Vibration of a Single Degree Of Freedom

The type of functions that were considered to this point can be classified as determinisic i:e mathematical expressions can be written that will determine their instantaneous values at any time t. However, in real life situation, there are a number physical phenomena that results in non deterministic sense like in the case we consider forces acting on aircraft as it flies in air**.** Random vibration of a viscously damped system by arbitrary force was determined.

Figure 14: Random vibration of a viscously damped system by arbitrary force



Equation of motion:

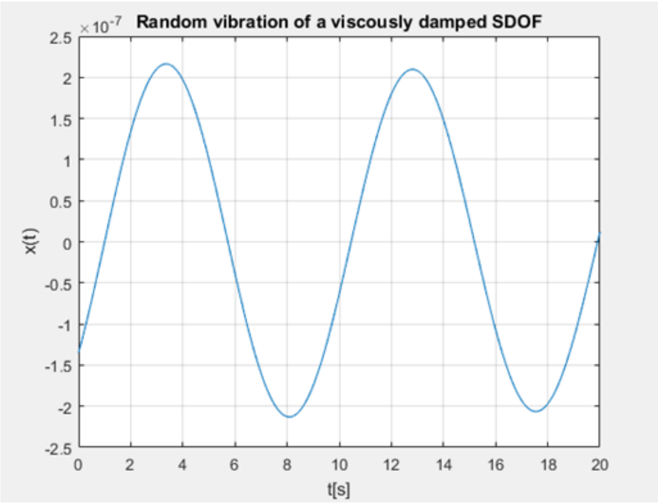
$$\ddot{x} + 2z\omega_n\dot{x} + \omega_n^2 x = \frac{F(t)}{m} \quad (52)$$

$$\omega_n = \sqrt{\frac{k}{m}}\ ,\ z = \frac{c}{c_c}\ , c_c = 2mk \quad (53)$$

In any statistical mehod a large amount of data is necessarryto establish reliability example in this case forces acting on the aircraft, so many records are collected. Each record is a sample and the total collection of samples is called ensemble. We can compute the ensemble average of instantaneous forces in each sample at time t_1. We can also multiply the instantaneous forces in each sample t_1 *and* $t_1 + \tau$.

$$x(t) = \int_0^t x(\tau)h(t-\tau)d\tau \quad (54)$$

Figure 15: Results of Random Vibration in a Single Degree Of Freedom System



6. Continuous Vibration Analysis of a Wing Spar

Since the wing spar is a cantalever beam, analysis of lateral vibration of beams was considered. Since it’s a uniform beam its classified as euler-bernoulli beam.

The equation of motion of Euler-Bernoulli Beam is

$$m(x)\frac{\partial^2\omega}{\partial t^2} + \varepsilon\frac{\partial\omega}{\partial t} + EI\frac{\partial^4\omega}{\partial x^4} = f(x,t) \quad (55)$$

Where m is mass per unit length of beam defined as m = ρA. There is no damping and no external force is applied so that c = 0, f(x, t) = 0, and EI(x) and m(x) are assumed to be constant, equation simplified to;

$$\frac{\partial^2 \omega}{\partial t^2} + \frac{EI}{m}\frac{\partial^4 \omega}{\partial x^4} = f(x,t) \qquad (56)$$

For the eigenvalue problem, assume the product solution as

$$\omega(x, t) = W(x)F(t) \qquad (57)$$

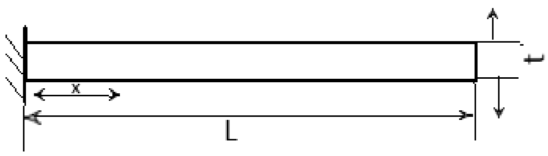
Where W(x) depends on the spatial position alone and F (t) depends on time alone. Introducing equation (58) into equation (57), we can obtain the following equation as;

$$\frac{d^4W(x)}{dx^4} - \beta^4 W(x) = 0 \qquad (58)$$

Where

$$\beta^4 = \frac{\omega^2 m}{EI}, 0 < x < l. \qquad (59)$$

Figure 16: Cantilever beam Boundary conditions[5]



The boundary conditions for the clamped-free case is

$$W(0) = 0,\ \left.\frac{dW(x)}{dx}\right|_{x=o} = 0, \left.\frac{d^2W(x)}{dx^2}\right|_{x=L} = 0 \qquad (60)$$

$$\left.\frac{d^2W(x)}{dx^2}\right|_{x=L} = 0 \qquad (61)$$

6.1 Results

The solution obtained is;

W(x) = C_1 sin βx + C_2 cos βx + C_3 sinh βx + C_4 cosh βx

Where C_1, C_2 ,C_3 and C_4 can be obtained by the boundary conditions

The characteristic equation is;

CosβL cosh βL = −1 (61)

From the numerical analysis, $\beta_1 L$ = 1.875, $\beta_2 L$ = 4.694, β_3 L = 7.855.

From the specification of a wing spar and characteristics of the aluminium alloy we obtain the data below (ASM aerospace specification Metal Inc.);

E= 7.31Gpa, EI= 1.44x10^6Nm^2, m= 62.096kg, L=3.6m

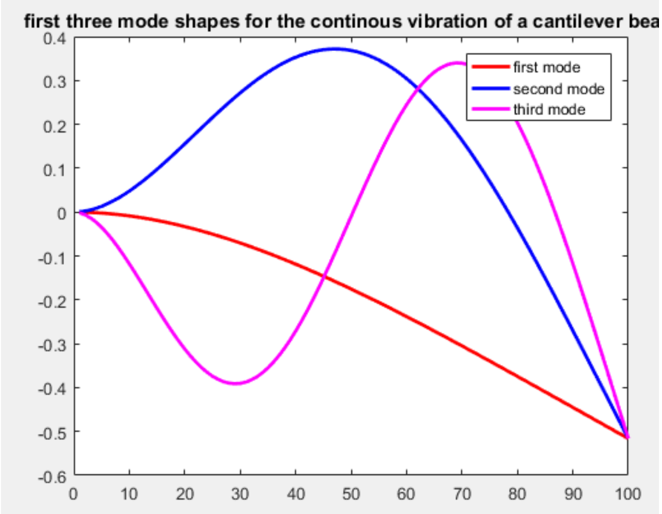
$$\omega_1 = 1.875^2\sqrt{\frac{EI}{mL^4}} = 41.31 rad/sec \qquad (62)$$

$$\omega_2 = 4.694^2\sqrt{\frac{EI}{mL^4}} = 258.9 rad/sec \qquad (63)$$

$$\omega_3 = 7.855^2\sqrt{\frac{EI}{mL^4}} = 730.5 rad/sec \qquad (64)$$

The above three modes frequencies of the Continuous Vibration Analysis of a Wing Spar were plotted in MATLAB.

Figure 17: Results of the first three modes of the Continuous Vibration Analysis of a Wing Spar



7. Conclusion and Summary

This study was performed for an UAV aircraft wing spar modeled to analyse different vibration patterns. The study was done for linear, nonlinear and random vibration considering the wing spar as a cantilever beam for both damping and undamped scenarios.

Responses for all the vibration analysis were plotted using MATLAB software to analyse the behaviour of the beam when subjected to different vibration phenomenon. The natural frequency for all single of freedom vibration analysis is constant meanwhile the frequencies for the two degree of freedom and continuous vibration showing some variations.

For future study, other forces acting on the aircraft due to the speed will be incorporated in the analysis to better observe conditions of vibration.

8. Authors' Biography

Ms. Merina Amon Mwasandube is an Assistant Lecturer, Researcher and Consultant in Aeronautical Engineering at the National Institute of Transport (NIT). She holds a Master's degree in Aircraft Design, from Nanjing University of Aeronautics and Astronautics and a Bachelor's degree in Aeronautical Engineering (Aircraft Manufacturing) from Shenyang Aerospace University both from China (2022). Also, she is a registered Graduate Engineer at the Engineers Registration Board (ERB). She has published three research articles in peer-reviewed journals. Her research interest is on aeronautical engineering. She is currently a head of Department at the National institute of Transport in Tanzania.

9. References

1. V. Saran, V. Jayakumar, G. Bharathiraja, K. S. Jaseem, and G. Ragul, "Analysis of natural frequency for an aircraft wing structure under pre-stress condition," *Int. J. Mech. Eng. Technol.*, vol. 8, no. 8, pp. 1118–1123, 2017.
2. "Theory of Vibration with Applications (5th).pdf."
3. S. A. Q. Siddiqui, M. F. Golnaraghi, and G. R. Heppler, "Large free vibrations of a beam carrying a

moving mass," *Int. J. Non. Linear. Mech.*, vol. 38, no. 10, pp. 1481–1493, Dec. 2003, doi: 10.1016/S0020-7462(02)00084-7.

4. E. Carrera, A. Pagani, and R. Augello, "Effect of large displacements on the linearized vibration of composite beams," *Int. J. Non. Linear. Mech.*, vol. 120, p. 103390, Apr. 2020, doi: 10.1016/j.ijnonlinmec.2019.103390.
5. R. H. L. Chen *et al.*, "Impacts Of Various Boundary Conditions On beam variations" 2015.