International Journal for Multidisciplinary Research (IJFMR)
E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

A Study of Multi-Criterion Fuzzy Decision-Making Methods: Recent Trends and Directions

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ABSTRACT:
A plausible soft computing model for addressing ambiguity and vagueness in decision-making circumstances is the concept of intuitionistic fuzzy sets (IFSs). Utilizing similarity-distance metrics, cases like diagnostic analysis, Finance and Investment, pattern recognition, Risk Analysis and Assessment, etc. have been investigated. Numerous methods of distance and similarity have been suggested and utilized to solve deciding circumstances. Although the existing similarity measures and their distance counterparts are fairly significant, they have certain accuracy and conceptual alignment issues that need to be addressed in order to improve output reliability. As a result, a unique similarity-distance technique is introduced in this paper. To demonstrate the benefits of the innovative similarity-distance strategy over related current approaches, a comparative analysis is presented**.**
The theoretical and philosophical aspects of the approach are set aside, and the analysis is solely focused on the algorithmic (technical) point of view. Applying the ranking algorithm yields a solution. A comparative study is presented to illustrate the advantages of the suggested measures. The outcomes of applying the suggested similarity metrics are confirmed by a technique known as Topsis. The outcomes are more logical, consistent, and productive in a skeptical setting.

Keywords: Multi-Attribute Decision Making, Similarity measure, Distance measure, TOPSIS

INTRODUCTION

Decision making is a fundamental process in various fields, ranging from business and finance to engineering and healthcare. Historically, approaches to making decisions have relied on accurate and clear data, presuming a distinct separation between membership and non-membership in choice criteria. However, it can be difficult to model and analyses choice problems effectively since real-world scenarios frequently entail ambiguity, uncertainty, and inaccurate information.

Fuzzy set theory gives decision-makers an effective tool to deal with uncertainty and imprecision in these situations. By allowing elements to have different degrees of membership, fuzzy set theory expands on traditional set theory and allows for a more adaptable representation of ambiguous or imprecise information. Incorporating human-like reasoning into decision-making, this theory offers a mathematical framework to model and reason using language variables and qualitative criteria. Fuzzy set theory offers a framework for incorporating linguistic variables, fuzzy logic operators, and fuzzy reasoning methods in the context of decision-making. As a result, complicated decision issues with criteria that are not strictly binary but instead have a range of membership are now possible to model and analyses. Fuzzy set theory can help decision-makers accept ambiguity and imprecision, resulting in more adaptable and practical choice outputs. They can consider judgements that are subjective, deal with insufficient information, and capture the inherent ambiguity in decision criteria using this method.

As a fundamental characteristic of fuzzy sets, vagueness has been the subject of intense literary attention for many years. The idea of a fuzzy set and the importance of vagueness in simulating real-world problems were first introduced in the earlier work of (Lotti Zadeh, I 965) in an effort to better simulate human reasoning [11]. An overview of fuzzy set theory and its uses in numerous domains, such as pattern recognition, decision-making, data analysis, and control systems, was prepared by (Li-Xin Wang, 1997). The term "fuzzy set" was expanded to include "intuitive fuzzy set" (IFS) by (Krassimir Atanassov, 1986) [3,8]. IFS is capable of expressing deviation. IFS defined the degree of reluctance towards an element in a set as well as the degree of membership and non-membership. An intuitionistic fuzzy clustering algorithm emphasizing the concept of density was addressed by (Xu and Yager, 2016). The approach performs better than the conventional fuzzy clustering algorithm at handling cluster data that contains ambiguities and uncertainties. A technique to multi-criteria decision making that produces more significant, accurate, and exact results was discussed by (Wang et al., 2017). After IFS, there was a lot of study done on the following topics: the fundamental hypothesis that IFS constitute operational rules, similarity measurements between IFS, and distance measure between IFS[19].

Finding similarities between any types of data is a fact. The quantification of similarity is used to see how closely related the data are. Information retrieval, medical diagnosis, pattern recognition, knowledge discovery in data, robotics, natural language processing, and clustering all use similarity as a key component in their decision-making processes.

Despite the wide range of MCDA Methods available, no method can be used in every case or be considered perfect or ideal. It is imperative that those making decisions select the appropriate approaches. Selecting the appropriate multicriteria strategy is crucial for achieving the intended outcome, as distinct approaches may provide distinct outcomes. Taking the decision maker's preferences into account, the right solution can be obtained from a method that is suited for a certain problem involving decision-making. Selecting the best course of action in these situations is a difficult issue. Many approaches have already been described to support the fundamental requirements for a particular multi-criteria decision-making problem, in accordance with the MCDA methods.

Because of their specificity, the approaches can be selected individually for a given decision-making scenario. Thus, it is important to investigate the applicability and limitations of various multicriteria decision making techniques in order to gain a sufficient understanding of them. Using several methodologies, the findings may vary depending on a variety of circumstances, most notably the fact that the methods' algorithms differ completely or that different people used different weights for the criterion in the calculation.

Belonging to the same coherent group with the American MCDM school approach is the motivation behind using the TOPSIS approach. It addresses reference point ideology. When comparing, it should be highlighted that TOPSIS offers a quantitative final ranking of the possibilities in the multi-criteria decision-making problem, in contrast to other exiting approaches.

The paper is bifurcated into subsequent sections: The purpose of Section 2 is to provide in-depth information on FSs and IFS. Section 3 discusses decision measures and similarity; Section 4 details proposed similarity measures and examines their properties; Section 5 describes situations in which the suggested equation is used and the results are compared; Section 6 discusses the TOPSIS method; Section

7 describes a case study in which the TOPSIS method is applied and a comparative analysis is conducted; and Section 8 wraps up the study with a conclusion.

2 PRELIMINARIES

Definition 2. 1: Let U be Universal Set, then a fuzzy set A in U is a set ordered pair that is

$$A = \{(x, \mu_A(x) : x \in X\}$$

Each element is assigned a number between 0 and 1 by the membership- function indicating how much it belongs to the set.

Definition 2.2 A defined on the universe of discourse U as

$$A = \{< x, \mu_A(x), v_A(x) > | x \in U\}$$

$\mu_A(x)$, $v_A(x)$: $U \rightarrow [0,1]$ represent the degree of membership and degree of non-membership of x value lies between 0 and 1.

3 Similarity – distance measure used for decision making

Similarity and distance measures play a crucial role in decision making by providing quantitative assessments of the relationships and differences between data points or objects. These measures help decision makers evaluate the similarity or dissimilarity between various entities and make informed choices based on the computed measures.

Definition 3.1

For M and N as IFSs in X, the similarity measure of M and N signified by S (M, N)is a mapping S: IFS ×IFS ⟶ [0,1] satisfying

i. Boundedness :$0 \leq S(M, N) \leq 1$

ii. Separability :S M, N) = 1 iff M=N

iii. Symmetric :S (M, N) = S (N, M)

iv. Inequality: if $P \subseteq Q \subseteq R$ then $S(M, O) \leq S(M, N)$ and $S(M, O) \leq S(N, O)$.

When S (M, N) approaches 1, it indicates that M and N are more similar (i.e., there is ahigh similarity rate), and when S (M, N) approaches 0, it indicates that M and N are notcomparable (i.e., there is a low similarity/resemblance rate).

Definition 3.2

For M and N as IFSs in X, the distance measure of M and N signified by D (M, N)is a mapping S: IFS × IFS ⟶ [0,1] satisfying

i. Boundedness :$0 \leq D(M, N) \leq 1$

ii. Separability :D (M, N) = 0 if M = N

iii. Symmetric :D (M, N) = D (N, M)

iv. Inequality: if $P \subseteq Q \subseteq R$ then D (M, O) ≤ D (M, N) and D (M, O) ≤ D (N, O).

If D (M, N) approaches 0, it indicates that M and N are closer together, and if itapproaches 1, M and N are farther apart.

Hong and Kim (1999)

$$S_1(M, N) = 1 - \frac{1}{2n} \sum_{i=1}^{n} [|u_M(x_i) - u_N(x_i)| + | v_M(x_i) - v_N(x_i) |$$

$$D_1(M, N) = \frac{1}{2n} \sum_{i=1}^{n} [|u_M(x_i) - u_N(x_i)| + | v_M(x_i) - v_N(x_i) |$$

Li et al. (2007)

$$S_2(M, N) = 1 -(\frac{1}{2n} \sum_{i=1}^{n} [(u_M(x_i) - u_N(x_i))^2 + (v_M(x_i) - v_N(x_i))^2]) ^{½}$$

$$D_2(M, N) = (\frac{1}{2n} \sum_{i=1}^{n} [(u_M(x_i) - u_N(x_i))^2 + (v_M(x_i) - v_N(x_i))^2]) ^{½}$$

Ye (2011)

$$S_3(M,N) = \frac{1}{n} \sum_{i=1}^{n} \frac{Um(xi)Un(xi)+Vm(xi)Vn(xi)}{\sqrt{U^2m(xi)+V^2m(xi)}\ \sqrt{U^2n(xi)+V^2n(xi)}}$$

$$D_3(M,N) = 1- \frac{1}{n} \sum_{i=1}^{n} \frac{Um(xi)Un(xi)+Vm(xi)Vn(xi)}{\sqrt{U^2m(xi)+V^2m(xi)}\ \sqrt{U^2n(xi)+V^2n(xi)}}$$

Shi and Ye (2013)

$$S_4(M,N) = \frac{1}{n} \sum_{i=1}^{n} \frac{Um(xi)Un(xi)+Vm(xi)Vn(xi)+Wm(xi)Wn(xi)}{\sqrt{U^2m(xi)+V^2m(xi)+W^2m(xi)}\ \sqrt{U^2n(xi)+V^2n(xi)+W^2n(xi)}}$$

$$D_4(M,N) = 1- \frac{1}{n} \sum_{i=1}^{n} \frac{Um(xi)Un(xi)+Vm(xi)Vn(xi)+Wm(xi)Wn(xi)}{\sqrt{U^2m(xi)+V^2m(xi)+W^2m(xi)}\ \sqrt{U^2n(xi)+V^2n(xi)+W^2n(xi)}}$$

4. Proposed Similarity and Distance Measure

In order to meet the axioms for the similarity measure as described above, we will now formulate some similarity and distance measure

$$S(M,N) = 1- \frac{1}{n} \sum_{i=1}^{n} \frac{|Um(xi)-Un(xi)| + |Vm(xi)-Vn(xi)| + |Wm(xi)-Wn(xi)|}{\sqrt{Um(xi)^2+Un(xi)^2} + \sqrt{Vm(xi)^2+Vn(xi)^2} + \sqrt{Wm(xi)^2+Wn(xi)^2}}$$

$$D(M,N) = \frac{1}{n} \sum_{i=1}^{n} \frac{|Um(xi)-Un(xi)| + |Vm(xi)-Vn(xi)| + |Wm(xi)-Wn(xi)|}{\sqrt{Um(xi)^2+Un(xi)^2} + \sqrt{Vm(xi)^2+Vn(xi)^2} + \sqrt{Wm(xi)^2+Wn(xi)^2}}$$

U_m, U_n: Membership degree

V_m, V_n: Non - Membership degree

W_m, W_n: Indeterminacy membership degree

Theorem 4.1: The proposed similarity measure satisfies the stated properties
Proof:

i. Boundedness : $0 \leq S(M, N) \leq 1$

Since $0 \leq |Um(xi) - Un(xi)| \leq 1$, $0 \leq |Vm(xi) - Vn(xi)| \leq 1$, $0 \leq |Wm(xi) - Wn(xi)| \leq 1$

Therefore, $0 \leq \sqrt{U^2m(xi) + U^2n(xi)} \leq 1$, $0 \leq \sqrt{V^2m(xi) + V^2n(xi)} \leq 1$,

$0 \leq \sqrt{W^2m(xi)+W^2n(xi)} \leq 1$

Then

$$0 \leq \frac{|Um(xi)-Un(xi)| + |Vm(xi)-Vn(xi)| + |Wm(xi)-Wn(xi)|}{\sqrt{U^2m(xi)+U^2n(xi)} + \sqrt{V^2m(xi)+V^2n(xi)} + \sqrt{W^2m(xi)+W^2n(xi)}} \leq 1$$

$$0 \leq 1 - \frac{1}{n} \sum_{i=1}^{n} \frac{|Um(xi)-Un(xi)| + |Vm(xi)-Vn(xi)| + |Wm(xi)-Wn(xi)|}{\sqrt{U^2m(xi)+U^2n(xi)} + \sqrt{V^2m(xi)+V^2n(xi)} + \sqrt{W^2m(xi)+W^2n(xi)}} \leq 1$$

$0 \leq S(M, N) \leq 1$

ii. Separability : $S(M, N) = 1$ if $M = N$

$$S(M,N) = 1 - \frac{1}{n} \sum_{i=1}^{n} \frac{|Um(xi)-Un(xi)| + |Vm(xi)-Vn(xi)| + |Wm(xi)-Wn(xi)|}{\sqrt{U^2m(xi)+U^2n(xi)} + \sqrt{V^2m(xi)+V^2n(xi)} + \sqrt{W^2m(xi)+W^2n(xi)}}$$

If M = N

Then Um(xi) = Un(xi), Vm(xi) = Vn(xi), Wm(xi) = Wn(xi)

$$S(M, N) = 1 - \frac{1}{n} \sum_{i=1}^{n} \frac{0+0+0}{\sqrt{2}(Um(xi)+Vm(xi)+Wm(xi))} = 1$$

iii. Symmetric : $S(M, N) = S(N, M)$

$$S(M,N) = 1 - \frac{1}{n} \sum_{i=1}^{n} \frac{|Um(xi)-Un(xi)| + |Vm(xi)-Vn(xi)| + |Wm(xi)-Wn(xi)|}{\sqrt{U^2m(xi)+U^2n(xi)} + \sqrt{V^2m(xi)+V^2n(xi)} + \sqrt{W^2m(xi)+W^2n(xi)}}$$

$$= 1 - \frac{1}{n} \sum_{i=1}^{n} \frac{|Un(xi)-Um(xi)| + |Vn(xi)-Vm(xi)| + |Wn(xi)-Wm(xi)|}{\sqrt{U^2n(xi)+U^2m(xi)} + \sqrt{V^2n(xi)+V^2m(xi)} + \sqrt{W^2n(xi)+W^2m(xi)}} = S(N,M)$$

$S(M, N) = S(N, M)$

iv. Inequality: if $P \subseteq Q \subseteq R$ then $S(M, O) \leq S(M, N)$ and $S(M, O) \leq S(N, O)$.

If $P \subseteq Q \subseteq R$, then

$U_m(x) \leq U_n(x) \leq U_o(x)$, $V_m(x) \leq V_n(x) \leq V_o(x)$, $W_m(x) \geq W_n(x) \geq W_o(x)$

We have,

$|Um(xi) - Un(xi)| \leq |Um(xi) - Uo(xi)|$, $|Vm(xi) - Vn(xi)| \leq |Vm(xi) - V_o(xi)|$

$|Wm(xi) - Wn(xi)| \leq |Wm(xi) - W_o(xi)|$

From the above condition

$$\frac{1}{n} \sum_{i=1}^{n} \frac{|Um(xi)-Un(xi)| + |Vm(xi)-Vn(xi)| + |Wm(xi)-Wn(xi)|}{\sqrt{U^2m(xi)+U^2n(xi)} + \sqrt{V^2m(xi)+V^2n(xi)} + \sqrt{W^2m(xi)+W^2n(xi)}} \leq \frac{1}{n} \sum_{i=1}^{n} \frac{|Um(xi)-Uo(xi)| + |Vm(xi)-Vo(xi)| + |Wm(xi)-Wo(xi)|}{\sqrt{U^2m(xi)+U^2o(xi)} + \sqrt{V^2m(xi)+V^20(xi)} + \sqrt{W^2m(xi)+W^2(xi)}}$$

$$\mathbf{1} - \frac{1}{n} \sum_{i=1}^{n} \frac{|Um(xi)-Uo(xi)| + |Vm(xi)-Vo(xi)| + |Wm(xi)-Wo(xi)|}{\sqrt{U^2m(xi)+U^2o(xi)} + \sqrt{V^2m(xi)+V^20(xi)} + \sqrt{W^2m(xi)+W^2(xi)}} \leq 1 - \frac{1}{n} \sum_{i=1}^{n} \frac{|Um(xi)-Un(xi)| + |Vm(xi)-Vn(xi)| + |Wm(xi)-Wn(xi)|}{\sqrt{U^2m(xi)+U^2n(xi)} + \sqrt{V^2m(xi)+V^2n(xi)} + \sqrt{W^2m(xi)+W^2n(xi)}}$$

$S(M, O) \leq S(M, N)$

Similarly, from the below condition,

$|Un(xi) - Uo(xi)| \leq |Um(xi) - Uo(xi)|$, $|Vn(xi) - Vo(xi)| \leq |Vm(xi) - V_o(xi)|$,

$|Wn(xi) - Wo(xi)| \leq |Wm(xi) - W_o(xi)|$

We obtain S (M, O) ≤ S (N, O)

5. Examples and comparison

Example 5. 1: Here, we demonstrate the similarity metrics numerically and do a comparison study to determine the new similarity method

Assume there are nine known patterns and each with the class labels, and IFS can express each pattern in the following ways:

$P\ 1 = \{\langle y_1, 0,1) \rangle, \langle y_2, 0,0\rangle, \langle y_3, 0,1 \rangle, \langle y_4, 0,1 \rangle, \langle y_5, 0.85,0 \rangle, \langle y_6, 0.04,0.94 \rangle, \langle y_7, 0.04,0.93\rangle, \langle y_8, 0,1 \rangle, \langle y_3, 0,1 \rangle\}$,

$P\ 2 = \langle y_1, 0,1) \rangle, \langle y_2, 0.28,0.69\rangle, \langle y_3, 0.09,0.88 \rangle, \langle y_4, 0.55,0.3\rangle, \langle y_5, 0,1 \rangle, \langle y_6, 0,1\rangle, \langle y_7, 0,1\rangle, \langle y_8, 0,1 \rangle, \langle y_9,0.08,0.87\}$,

$P3= \{\langle y_1, 0,1) \rangle, \langle y_2, 0,1\rangle, \langle y_3, 0,1 \rangle, \langle y_4, 0,1 \rangle, \langle y_5, 0.3,0.42 \rangle, \langle y_6, 0.4,0.38 \rangle, \langle y_7, 0.08,0.87 \rangle, \langle y_8, 0,1 \rangle, \langle y_3, 0,1 \rangle\}$

The following is the sample Q that needs to be recognized:

$Q = \{\langle y_1, 0,1) \rangle, \langle y_2, 0,1\rangle, \langle y_3, 0.1,0.9\rangle, \langle y_4, 0.9,0.1 \rangle, \langle y_5, 0,1\rangle, \langle y_6, 0,1\rangle, \langle y_7, 0,1 \rangle, \langle y_8, 0,1 \rangle, \langle y_3, 0,1 \rangle,\}$,

The degree of similarity between P_i where i = (1,2,3) and Q computed by

$S_N (P_1, Q) = 0.63727$

$S_N (P_2, Q) = 0.879156$

$S_N (P_3, Q) = 0.700824$

It is evident that pattern Q should be assigned the P2 classification with the class label C2. This result is consistent with that found in when using the highest degree of similarity between IFSs as a recognition principle.

Example 5.2 - Pattern Recognition

In this instance, a situation of choosing a company is described utilizing a similarity strategy and an intuitionistic fuzzy decision-making approach. Assume that M, a recent graduate, is looking for a job with a company that uses IFPs. The following characteristics, where d1 = salary ability, d2 = repute, d3 =brand image, serve as the freshest guiding principles. These language variables underwent fresher's transformation into IFPs.

The fresher found three companies with the appropriate traits reflected by IFPs after careful consultations and searching:

$B_1 = [(d_1,0.06,0.1); (d_2, 0.02,0.5); (d_3,0.05,0.1)$

$B_2= [(d_1, 0.5, 0.01); (d_2, 0.3, 0.47); (d_3,0.4,0.03)$

$B_3= [(d_1, 0.4, 0.25); (d_2, 0.7, 0.1); (d_3,0.1,0.04)$

Assume that pattern Z is an unclassified pattern and that patterns B1, B2, and B3 are each represented by an IFP,

$Z = [(d_1, 0.8, 0.02); (d_2, 0.5, 0.1); (d_3,0.15,0.7)]$

By comparing the similarity of the unidentified pattern to the categorized patterns, we use the pattern approach in the new similarity measure technique to ascertain which class the unknown pattern belongs to. The results are listed below.

$S_N (B_1, Z) = 0.254511$

$S_N (B_2, Z) = 0.439238$

$S_N (B_3, Z) = 0.464881$

The following outcomes are attained by using the fresh similarity technique to determine which organization is consistent with the fresher's requirements:

$S_N (B_3, Z)$ is greater than $S_N (B_2\ Z)$ is greater than $S_N (B_1, Z)$.

6 TOPSIS METHOD

STEP 1: First creates a Q-Rung Orthopair Fuzzy Decision Matrix N, which is defined as

$N = [s_{ij}]_{m\times n} = (u_{ij}, v_{ij}); (1 \le i \le m), (1 \le i \le m)$ where s_{ij} depicts the assessment of i^{th} alternatives to the j^{th} criteria.

STEP 2 For the ease of computation, the normalized Q-Rung Orthopair Fuzzy Decision Matrix (N) was constructed by converting the cost benefit (non-benefit) criteria into the benefit criteria for the decision-making procedure.

$$s_{ij} \quad \begin{cases} (u_{ij}, v_{ij}), \text{ if } c_j \in C_B \\ (v_{ij}, uij), \text{ if } c_j \in C_C \end{cases}$$

Where C_B and C_C, denote the benefit and cost criteria respectively.

STEP 3 The normalized q-rung orthopair fuzzy decision matrix is used to evaluate the relative position ideal solution (RPOS) and the relative negative ideal solution (RNOS)

defined as

$$z+ = \{s_j^+ = (\max_{i=1}^{m} u_{ij}, \min_{i=1}^{m} v_{ij})\};\ 1 \le j \le n$$

$$z+ = \{s_j^- = (\min_{i=1}^{m} u_{ij}), \max_{i=1}^{m} v_{ij}\};\ 1 \le j \le n$$

STEP 4 The similarity of each alternative is computed to the RPOS and RNOS using the proposed measure which is depicted as $S_{w1}(z_i, z^+)$, S_{w1}, (z_i, z^-) for similarity measure 1 and S_{w2} (z_i, z^+), S_{w2} (z_i, z^-) for similarity measure 2 respectively.

STEP 5 The proximity index is determined by the formula

$$p_i = \frac{S(zi\ ,z+)}{S(zi,z+)+S(zi,z-)} \qquad 1 \le i \le m$$

The options are ranked using the proximity index. The higher the value the close is the alternative to the ideal option

7 CASE STUDY

Air is the most essential resource for human survival in this world, yet in recent decades, industrial emissions of pollutants have significantly impacted the quality of the air we breathe, relative to other dynamic and local sources of pollution. The difficult job facing the industries is deciding which of the many APM (air pollution mitigation) techniques now in use is best after assessing how well each approach satisfies the necessary requirements.

Intuitionistic fuzzy sets (IFS) are utilized in order to improve consistency in the decision-making process. The APM techniques ranking results are determined by utilizing multiple major IFS measures to identify the most practical APM approach for reducing the obstacles faced by industrial sectors in maintaining environmental sustainability.

Currently, major Indian industrial towns including Delhi, Mumbai, Bangalore, and Kolkata are monitoring the air quality index to reduce pollution levels and prevent the possibility of air becoming toxic. With environmental sustainability becoming more and more important every day, industrial sectors should use APM techniques to control pollution removal. The four main techniques of APM are the use of vegetation, source rectification, pollution diffusion, and equipment.

METHOD	DESCRIPTION
Method of employing equipment	Utilization of special devices to control the emission of pollutants
Pollutant diffusion method	Dilution of pollutants in the atmosphere
Source rectification method	Prevents pollutants at the source level
Vegetation	Use of plant species for pollutant absorption

Each of the four APM techniques has advantages and disadvantages of its own yet are all effective in reducing air pollution. However, identifying a workable APM approach is really quite challenging. To decide in such a situation, the environmental specialists are supposed to reflect the extent to which each approach satisfies the APM requirements in terms of IFS.

Thus, we compare it with a few criteria to determine which of the four APMs is the best; these are several options that need to be considered. The criteria stand for many elements or characteristics that are important in making the choice.

CRITERIAS	ALTERNATIVES
C1 Economically Feasible C2 Durability C3 Compatibility C4 Flexibilit C5 Consistency C6 Efficiency in reducing pollutants C7 Commercially beneficial C8 Eco- friendly C9 Abatement of toxins C10 Prevention of secondary pollution	M1 Method of employing equipment M2 Pollutant diffusion method M3 Source rectification method M4 Vegetation

7.1 EVALUATION BY TOPSIS METHOD

STEP 1 FORMATION OF DECISION MATRIX

The decision maker compares the alternatives based on the necessary parameters in order to make the pick. Table 7.1.1 presents the decision-maker's assessment of the workable options.

Table 7.1.1 Data set in the form of decision matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
M1	(0.6,0.3)	(0.5,0.4)	(0.6,0.2)	(0.7,0.1)	(0.6,0.2)	(0.8,0.1)	(0.7,0.3)	(0.6,0.2)	(0.7,0.2)	(0.8,0.1)
M2	(0.2,0.7)	(0.6,0.2)	(0.5,0.3)	(0.6,0.3)	(0.7,0.2)	(0.7,0.2)	(0.6,0.2)	(0.4,0.5)	(0.6,0.2)	(0.7,0.2)
M3	(0.1,0.8)	(0.4,0.5)	(0.6,0.3)	(0.7,0.2)	(0.5,0.3)	(0.6,0.2)	(0.5,0.4)	(0.5,0.3)	(0.8,0.1)	(0.3,0.6)
M4	(0.7,0.2)	(0.3,0.6)	(0.4,0.5)	(0.5,0.4)	(0.5,0.2)	(0.4,0.3)	(0.4,0.5)	(0.8,0.1)	(0.5,0.4)	(0.4,0.5)

STEP 2 NORMALIZATION OF DECISION MATRIX

By transforming each criterion into a cost or benefit criterion, the normalized matrix is created. Since all of the criteria are benefit criteria in the decision maker's vision, the following table:

Table 7.1.2 Data set in the form of Normalized decision matrix

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
M1	(0.6,0.3)	(0.5,0.4)	(0.6,0.2)	(0.7,0.1)	(0.6,0.2)	(0.8,0.1)	(0.7,0.3)	(0.6,0.2)	(0.7,0.2)	(0.8,0.1)
M2	(0.2,0.7)	(0.6,0.2)	(0.5,0.3)	(0.6,0.3)	(0.7,0.2)	(0.7,0.2)	(0.6,0.2)	(0.4,0.5)	(0.6,0.2)	(0.7,0.2)
M3	(0.1,0.8)	(0.4,0.5)	(0.6,0.3)	(0.7,0.2)	(0.5,0.3)	(0.6,0.2)	(0.5,0.4)	(0.5,0.3)	(0.8,0.1)	(0.3,0.6)
M4	(0.7,0.2)	(0.3,0.6)	(0.4,0.5)	(0.5,0.4)	(0.5,0.2)	(0.4,0.3)	(0.4,0.5)	(0.8,0.1)	(0.5,0.4)	(0.4,0.5)

STEP 3 EVALUATING RELATIVE POSITIVE IDEAL AND RELATIVE NEGATIVE IDEAL

The optimal option is identified by FPOS, and the non-ideal solution is identified by NOS. The FNOS and FPOS are assessed as

Table 7.1.3 Relative positive (RPOS) and negative ideal (RNOS) data sets

RPOS AND RNOS	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
z^+	(0.7,0.2)	(0.6,0.2)	(0.6,0.2)	(0.7,0.1)	(0.7,0.2)	(0.8,0.1)	(0.7,0.2)	(0.8,0.1)	(0.8,0.1)	(0.8,0.1)
z^-	(0.1,0.8)	(0.3,0.6)	(0.4,0.5)	(0.5,0.4)	(0.5,0.3)	(0.4,0.3)	(0.4,0.5)	(0.4,0.5)	(0.5,0.4)	(0.3,0.6)

STEP 4 COMPUTING THE SIMILARITY OF EVERY ALTERNATIVE

Using the suggested measure, the similarity of each alternative is calculated to the RPOS and RNOS, as shown in the table below:

$$S_{w1}(z_i,z^+) = 1-\frac{1}{n}\sum_{i=1}^{n}\frac{|Um(xi)-Un(xi)|+|Vm(xi)-Vn(xi)|+|Wm(xi)-Wn(xi)|}{\sqrt{Um(xi)^2+Un(xi)^2}+\sqrt{Vm(xi)^2+Vn(xi)^2}+\sqrt{Wm(xi)^2+Wn(xi)^2}}$$

$$S_{w1}(z_i,z^-) = 1-\frac{1}{n}\sum_{i=1}^{n}\frac{|Um(xi)-Un(xi)|+|Vm(xi)-Vn(xi)|+|Wm(xi)-Wn(xi)|}{\sqrt{Um(xi)^2+Un(xi)^2}+\sqrt{Vm(xi)^2+Vn(xi)^2}+\sqrt{Wm(xi)^2+Wn(xi)^2}}$$

Table 7.1.4 Similarity measure calculation

ALTERNATIVES	S(Z,Z+)	S(Z,Z-)
M1	0.899115044	0.592155917
M2	0.7866624404	0.7041493043
M3	0.6863233902	0.8129557999
M4	0.6381092478	0.8357456593

STEP`-5 RANKING THE ALTERNATIVES AND COMPUTING THE PROXIMITY INDEX

The formula determines the closeness index.

$$p_i = \frac{S(zi\,,z+)}{S(zi,z+)+S(zi,z-)} \qquad 1 \leq i \leq m$$

The proximity of an alternative to the best option is determined by its proximity index value. The closer an option is to the ideal one and, therefore, performs best when its value is higher. The options are ranked in descending order of preference based on the proximity index.

Table 7.1.5 Ranking obtained by Similarity measure

ALTERNATIVES	PROXIMITY INDEX	RANK
M1	0.6029186295	1
M2	0.5276738952	2
M3	0.457768903	3
M4	0.4329525551	4

Result: By taking the similarity measure into account, the TOPSIS Method finds the best answer by working with the relative ideal solution. The ranking that the decision maker obtained after using the suggested similarity metrics is clearly shown in Table 7.1.5. Based on similarity measurements, M1 turns out to be the best option among the alternatives that were taken into consideration. The alternative M1 is thought to be the most effective APM in lowering air pollution.

7.2 COMPARATIVE ANALYSIS

It is essential to conduct a comprehensive analysis that compares the proposed measure to a collection of carefully chosen current measures. Selecting appropriate, well-established models that have been applied to international research over many years is necessary to achieve this. The analysis is done in order to confirm that the innovative measure is reliable and that the results are accurate. Verifying if the results of a newly developed approach are comparable to those of current methods is crucial.

1. Similarity Measure proposed by Hong and Kim (1999)

$$S_1(M, N) = 1 - \frac{1}{2n}\sum_{i=1}^{n} [|u_M(x_i) - u_N(x_i)| + | v_M(x_i) - v_N(x_i) |]$$

2. Similarity Measure proposed by Li .et al (2007)

$$S_2 (M, N) = 1 -(\frac{1}{2n}\sum_{i=1}^{n}[(u_M(x_i) - u_N(x_i))^2 + (v_M(x_i) - v_N(x_i))^2])^{½}$$

3. Similarity Measure proposed by Ye (2011)

$$S_3(M,N) = \frac{1}{n}\sum_{i=1}^{n}\frac{Um(xi)Un(xi)+Vm(xi)Vn(xi)}{\sqrt{U^2m(xi)+V^2m(xi)}\ \sqrt{U^2n(xi)+V^2n(xi)}}$$

4. Similarity Measure proposed by Shi and Ye (2013)

$$S_4(M,N) = \frac{1}{n}\sum_{i=1}^{n}\frac{Um(xi)Un(xi)+Vm(xi)Vn(xi)+Wm(xi)Wn(xi)}{\sqrt{U^2m(xi)+V^2m(xi)+W^2m(xi)}\ \sqrt{U^2n(xi)+V^2n(xi)+W^2n(xi)}}$$

Tables 7.2.1 and 7.2.2, respectively, show the ranking of alternatives using the existing and suggested methods while considering the IFS Similarity matrix mentioned with each criterion.

Table 7.2.1 Proximity index of proposed and existing measures

METHOD	M1	M2	M3	M4
Proposed	0.6029	0.5276	0.4577	0.4329
Hong and Kim (1999)	0.5662	0.5091	0.4698	0.4668
Li .et al (2007)	0.5682	0.5	0.4566	0.4879
Ye (2011)	0.5462	0.5044	0.4731	0.4882
Shi and Ye(2013)	0.5391	0.5009	0.463	0.4728

We employed an actual case of air pollution to illustrate the behavior of our proposed Similarity metric. To make it theoretically sound and practically acceptable, we provide an example by looking at 10 different criteria under 4 alternatives. We also compare analysis with the most recently created measure to select superior similarity measures. We contrast our data with current measurements in order to demonstrate the advantages of our setup.

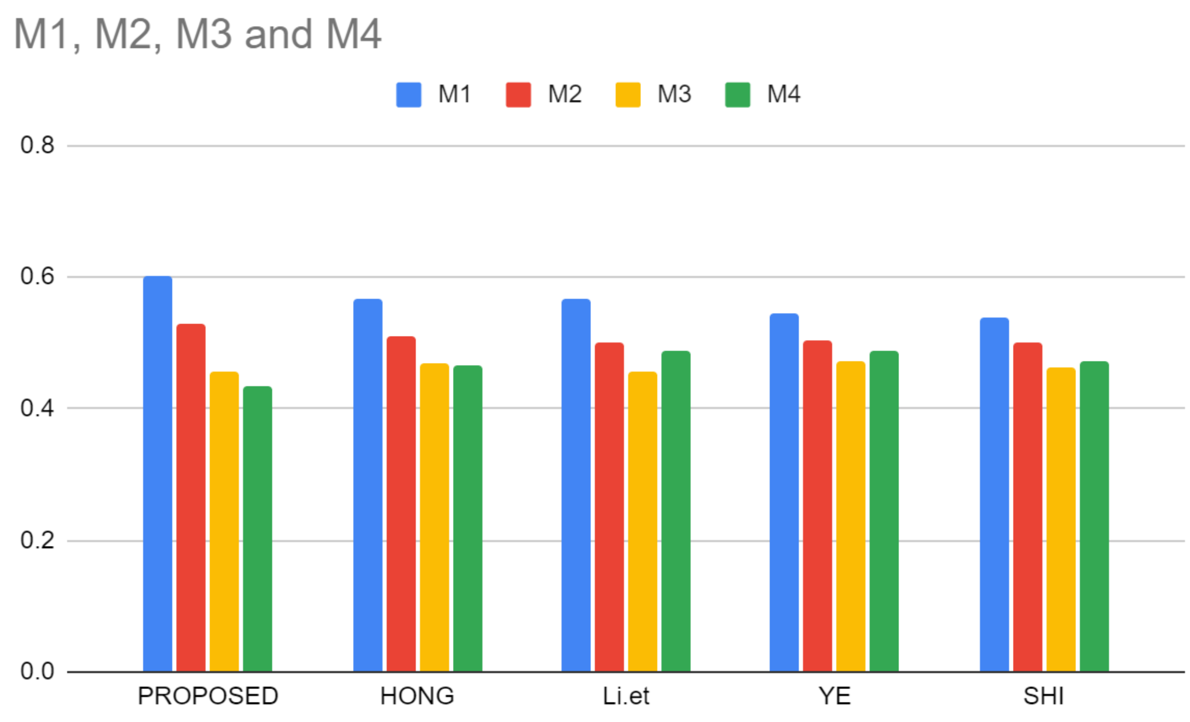
Table 7.2.2 Comparison of ranking with other existing methods

METHODS	RANKING
Proposed	M1>M2>M3>M4
Hong and Kim (1999)	M1>M2>M3>M4
Li .et al (2007)	M1>M2>M4>M3
Ye (2011)	M1>M2>M4>M3

Shi and Ye (2013)	M1>M2>M4>M3

Figure 7.2.1 Graphical representation

Proximity index of four alternatives



METHODS	RANK OF ALTERNATIVES	BEST ALTERNATIVE
Proposed	M1 > M2 > M3 > M4	M1

The comparative results obtained from the similarity measure have been assessed and examined. We deduced from the table 7.2.1 that the most effective alternative strategy for mitigating air pollution is M1.

Conclusion

Making decisions has always been a crucial part of life in the real world. A wide range of fields, including business, environmental science, public policy, engineering, and more, use decision-making. When using MCDM methodologies, the decision maker considers all relevant aspects that are acceptable to the decision, such as risk, cost, time, quality check, environmental impact, and health. It is seen to be the most obvious area of study to convey the ambiguities, lack of knowledge, and uncertainty that exist in the field. In this research work, we have addressed the most prominent fuzzy set that addresses the ambiguity and vagueness of the data; for this kind of hazy information, fuzzy set modification and intuitionistic fuzzy set are preferable.

In the same mind, we have put forth some innovative similarity metrics. By giving more accurate answers for multi-criteria decision-making situations, the proposed solutions seek to improve decision-making. We have proven the validity of the suggested metrics using the similarity measure theorem.

It was described how to use the suggested similarity metric in the classic TOPSIS approach. The suggested similarity metric, along with the well-established TOPSIS approach, allowed for a more aggressive and efficient ranking of the given alternative that demonstrated the decision maker's choice. Because it considers all relevant considerations, the outcome is commonly acceptable in decision-making.

As previously stated, the MCDM approach is extensively used in real-world situations. An analysis of a case study has been conducted from the perspective of pollution. Certain steps must be done to protect clean air and prevent air pollution. Finding the most effective air pollution mitigation strategy is the case study's main goal in order to reduce air pollution. The TOPSIS approach was used to rank the alternatives, and it was discovered that M1 i.e the method of employing equipment is the best way to mitigate air pollution.

It is necessary to accept the limits of both the MCDM approach and the suggested similarity measure. Since the data is entirely hypothetical and focuses on a particular research topic, the case study is the technique used to rate the option that is being investigated. The suggested method can be applied in a variety of contexts and real-world scenarios. But in some complicated cases, different MCDM frameworks or other uncertainty models may be more suited to provide the necessary rankings. In the future, the performance and application of the suggested unique similarity measure can be investigated within the context of complicated scenarios.

In summary, this study contributes to the realm of decision-making expertise. It has been suggested that a similarity measure be used to assess the similarity. TOPSIS, an MCDM technique, has been discussed and used to the case study. This general framework provides decision makers with effective tools to cope with ambiguity, impressions, lack of information, and uncertainty in order to arrive at the best possible conclusion. The experiment is verified by comparing the results of the various MCDM techniques.

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