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An Inventory Model with Time-Varying Demand and Comprehensive Cost

Vivek Vijay

Swami Keshvanand Institute of Technology Management & Gramothan, Jaipur

Abstract:
In this inventory model, we study the system for items with time varying demand. The deterioration of items is not considered here. The inventory level rate of change is given by the time and demand. Shortages are not allowed in this model. Holding cost, ordering cost are considered in this model. Our objective in this model is to evaluation of the total cost to maximize the objective of any industry. For validation of the model, the numerical example is also available which show the sensitivity of the cost with respect to variations in parameters.

Keywords: Inventory, demand, holding cost, ordering cost

1. Introduction

Everyone knows that in any industry or investing area, the investment and management of inventory are the crucial parts of business. Effective management fosters a clear vision, strategic leadership, and open communication, guiding businesses towards growth even in challenging scenarios. By prioritizing resource allocation, customer satisfaction, and talent development, management optimizes efficiency and innovation. Adaptability and sound financial practices ensure resilience, while a culture of continuous improvement drives ongoing success. With these principles, businesses can navigate uncertainties, capitalize on opportunities, and achieve remarkable development, even amidst dynamic environments.

The inventory level significantly impacts a business's economy and the market value of its industry. While storing goods is crucial, optimizing inventory for sales and profitability poses challenges. Market demand for items is influenced by usability and necessity, but strategic measures can enhance demand, showcasing advantages across disciplines. Product display and approach play pivotal roles in stimulating market interest. Notably, not everyone possesses every product, and products have finite lifetimes. Certain goods like medicine and electronics endure longer, whereas perishables such as fruits and vegetables have shorter lifespans. Managing perishable inventory demands careful attention to prevent deterioration and ensure optimal use. Effective inventory management involves balancing supply and demand dynamics, anticipating market trends, and minimizing wastage. Moreover, adapting inventory strategies to evolving consumer preferences and market conditions is essential for sustained success. Businesses must deploy efficient inventory management systems and adopt innovative approaches to meet demand, maximize profitability, and maintain competitiveness in dynamic market environments.

Numerous academics have created various inventory models that either include deterioration or do not. Deterioration plays a critical role in factory inventory management. Any item's demand can fluctuate depending on the inventory's presentation time.

Covert and Philip[14] have developed an EOQ model with weibull distribution deterioration. Goyal[17] has published and economic order quantity inventory model. Delay in payment is also useful to increase the profit so they consider it in this model. An economic order quantity inventory model is given by Goswami and Chaudhuri [1] in which they considered deteriorating of items with shortages and a linear trend in demand. Aggarwal and Jaggi[18] generated an optimum ordering policy for deteriorating items with permissible delay in payments. Hariga[10] has published an optimal economic order quantity inventory models for deteriorating items, he also consider the time dependent demand. Jamal, Sarkaer and Wang [2] have given an ordering policy model for deteriorating items with allowable shortage and permissible delay in payments. Teng, Chang, Dye and Hung[7] analyze and inventory model and established and replenishment policy for deteriorating items with time varying demand and partial backlogging. There is a note on an EOQ model for items. Dye[5] analysis an inventory system having the deterioration follow the weibull distributed. Shortages of inventory is also considered. Demand follow the power pattern law. Soni and Shah[6] has developed an optimal order policy for stock dependent demand under progressive payment scheme. Mishra and Mishra [12] evaluated the price for an EOQ model for deteriorating items under perfect competition. Sakaguchi[20] has given a study of inventory system, in which demand follow the varying nature. Singh and Shrivastava[24] have proposed an economic order quantity inventory model for perishable items with stock dependent selling rate and permissible delay in payments is also allowed and partial backlogging is also considered Sharma, Aggarwal, Vijay and Sharma[21] have published an article on optimum ordering interval with selling price dependent demand rate for items with random deterioration and shortages. An article on an EOQ model for perishable items is given by Singh, Singh and Dutt[25], in this article they considered demand with power pattern and partial back ordering. Mishra and Singh[26] have developed and deteriorating inventory model with time dependent demand and partial backlogging. Sharma and Preeti[22] have published an optinmm ordering inventory model, in which deterioration is random, demand depends on selling price and stock level in showcase. Rajeswari and Vanjikkodi[15] have published a deteriorating inventory model with power demand and partial backlogging An inventory control model with consideration of remanufacturing and product life cycle by Hsueh[4]. Teng, Krommyda, Skouri and Lou[8] have written an article having the comprehensive extension of optimal ordering policy for stock dependent demand under progressive payment scheme. Teng, Min and Pan[9] have published a study on economic order quantity inventory model with trade credit financing for non-decreasing demand. Rajeswari and Vanjikkodi[16] have proposed an inventory model, they consider deterioratin with two parameter weibull distribution and backlogging is also considered in this model. Sarkar[3] has published an economic order quantity inventory model, they considered delay in payments and they also study the analysis of time varying demand. Sharma and Vijay[23] have analyzed an EOQ Model for Deteriorating Items with Price Dependent Demand, Varying Holding Cost and Shortages under Trade Credit. Mishra and Singh[19] have presented an partial backlogging EOQ model, they consider the queuing system of customers with power demand and quadratic deterioration. Ghasemi and Damghani[13] study A robust simulation-optimization approach for pre-disaster multi-period location–allocation–inventory planning. An EOQ inventory model is given by Tripathy, Sharma and Sharma[11], in the inventory model deterioration is allowed with constant demand under progressive financial trade credit facility.

Many of inventory models have constant demand and no deterioration. The length of cycle time is fixed. Now here, we developed an economic production quantity inventory model. In which deterioration of inventory is not allowed. The demand of items is not taken as constant. It depends on time. In this model we calculate the total cost in cycle time and study the effect of all parameters on total cost with some numerical significance.

2. Assumption and Notation

1. C_0 = Setup cost per unit item.
2. C_h = Holding cost per unit item per unit time.
3. D(t) = demand = $\begin{cases} t^2 d, & 0 < t < t_1 \\ td, & t_1 \le t \le T \end{cases}$.
4. d = parameter for demand.
5. T = Length of cycle time.
6. Q = Initially inventory quantity.
7. I(t) = Inventory level at time t.
8. $C(t_1, T)$ = Total cost per cycle time.
9. TD = total demand in cycle time.
10. O_c = ordering cost per order per unit.
11. H_c = holding cost in cycle time.
12. T_c = total cost in cycle time.

3. Mathematical Model

We are examining a perpetual inventory system that operates indefinitely and involves demand that varies throughout time. The system does not permit inventory deterioration. Let I(t) represent the quantity of units at any given time t. The manufacture of inventory begins at time t = 0. The sale, management, and procurement activities commence at time t = 0. During a cycle time, if production ceases, the inventory level rapidly declines and reaches zero at time T. The mathematical model for such a system is as follows:

$$\frac{d}{dt} I(t) = t^2 d, \qquad 0 \le t \le t_1 \qquad \ldots(1)$$

$$\frac{d}{dt} I(t) = td + I(t), \qquad t_1 \le t \le T \qquad \ldots(2)$$

The boundary conditions are $I(0) = 0,\ I(T) = Q$.

Solution of differential equation (1) is $I(t) = \frac{t^3}{3} d$

Solution of differential equation (2) is $I(t) = \left(\frac{d}{2} t^2 + c_2\right)\left(1 + t + t^2\right)$

After using the boundary condition $I(t) = -\frac{d}{2} t^2 + Q\left(1 - T + \frac{T^2}{2}\right)$

From the above two solutions, we find $Q = \frac{d}{6} t_1^2 (3 + 2t_1)\left(2 + 2T + T^2\right)$

The total demand in cycle time [0, T] is given as D= $\int_0^T D(t) dt \quad = \frac{d}{6}\left(3T^2 - t_1^2\right)$

Number of items in duration [0, T]

$$= \int_0^T I(t) dt = \frac{d}{12}\left\{\left(t_1^2 - T^2\right)\left(t_1^3 - 2t_1^2 + T + t_1^2 T\right) - Tt_1\left(T^2 - t_1^2\right) + 4Tt_1^2(1 - T)\right\}$$

Now we calculate the different cost related to this inventory model

Ordering cost = QC_0 = $C_0 \frac{d}{6} t_1^2 (3+2t_1)(2+2T+T^2)$

Holding cost = HC = $C_h \int_0^T I(t)dt$

$$= \frac{C_h d}{12} \left\{ (t_1^2 - T^2)(t_1^3 - 2t_1^2 + T + t_1^2 T) - Tt_1 (T^2 - t_1^2) + 4Tt_1^2 (1-T) \right\}$$

The total cost per unit time per unit item is given as

$$C(t_1,T) = \frac{1}{T} \text{(Ordering cost + Holding cost)}$$

$$= \frac{C_0 d}{6T} t_1^2 (3+2t_1)(2+2T+T^2) + \frac{C_h d}{12T} \left\{ (t_1^2 - T^2)(t_1^3 - 2t_1^2 + T + t_1^2 T) - Tt_1 (T^2 - t_1^2) + 4Tt_1^2 (1-T) \right\}$$

Now to minimize the total cost we take $\frac{\partial C(t_1,T)}{\partial t_1} = 0 = \frac{\partial C(t_1,T)}{\partial T}$. And calculate stationary values of t_1 and T as t_1^*, T^*. At the above calculated point we find $\frac{\partial^2 C(t_1,T)}{\partial t_1^2}, \frac{\partial^2 C(t_1,T)}{\partial T^2}, \frac{\partial^2 C(t_1,T)}{\partial t_1 \partial T}$.

If $\frac{\partial^2 C(t_1,T)}{\partial t_1^2} \times \frac{\partial^2 C(t_1,T)}{\partial T^2} > \left(\frac{\partial^2 C(t_1,T)}{\partial t_1 \partial T} \right)^2$ then the profit is maximum.

4. Numerical Example with Graph

Effect of different parameters on Total cost are given in table and their graphical representation is also as follow

Table 1

d	t_1	T	cost
100	2	5	237.08
120	2	5	254.81
140	2	5	272.61
160	2	5	300.50
180	2	5	328.47
200	2	5	355.35
220	2	5	393.42
240	2	5	430.46
260	2	5	478.25
280	2	5	505.26

Tabel 2

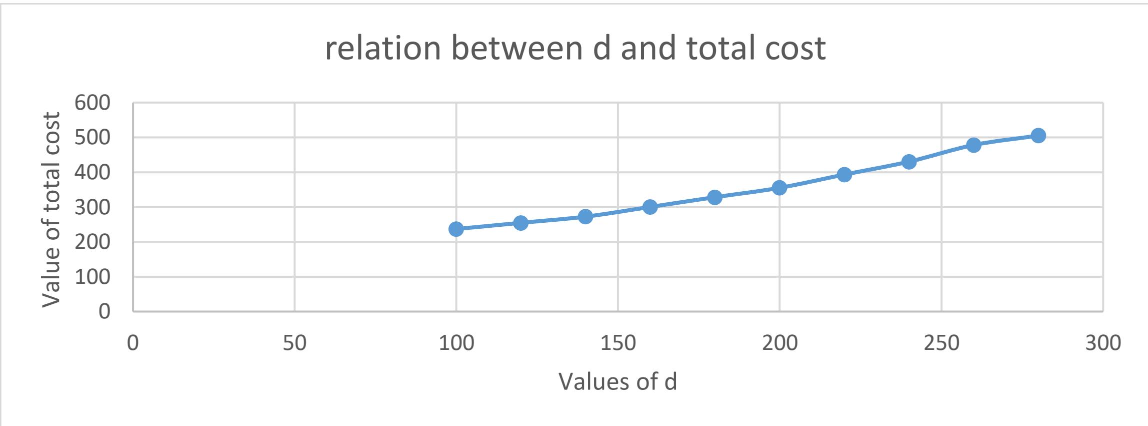
d	t_1	T	cost
100	2	5	237.08
100	2.1	5	244.55
100	2.2	5	256.65
100	2.3	5	267.09
100	2.4	5	285.11
100	2.5	5	303.28

100	2.6	5	325.79
100	2.7	5	350.90
100	2.8	5	380.64
100	2.9	5	415.21

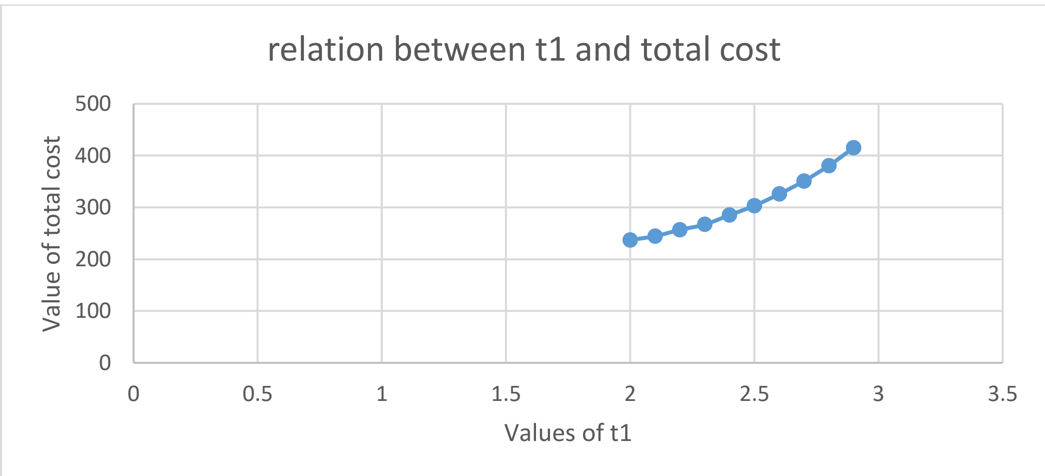
Table 3

d	t_1	T	cost
100	2	5.5	246.21
100	2	6	300.25
100	2	6.5	351.52
100	2	7	412.77
100	2	7.5	477.22
100	2	8	529.66
100	2	8.5	581.45
100	2	9	642.32
100	2	9.5	713.61
100	2	10	788.09

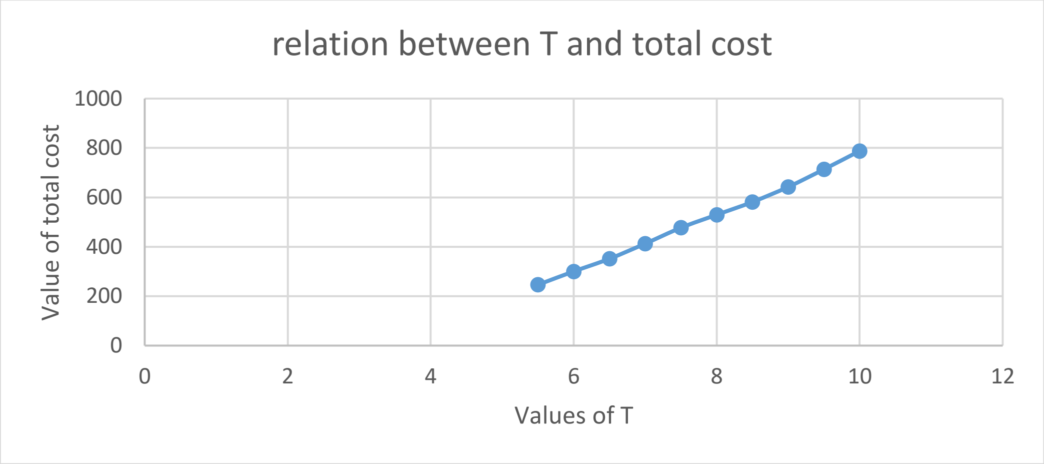
Graph 1



Graph 2



Graph 3



5. Conclusions and Remarks

In this research paper the production rate of items to replenish the goods is proportional to demand that varying as time. It is very clear that production rate directly depends on demand satisfied in the market. Demand means customer satisfaction is must. From the above numerical calculation and study of graph it is clear that the increasing value of d, t_1 and T increase the total cost, these value in the numerical example are ate the minimum level. These minimize the total cost. Based on the aforementioned data, we may enhance our inventory system and generate substantial profits. These factors have a significant impact on production, cost, sales, and profitability in real-world scenarios. The productivity of our operations is contingent upon factors such as labor expenses, workforce capacity, machinery infrastructure, and government levies. There may be more factors that can impact our inventory system. These will be incorporated into our forthcoming studies.

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