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The Physical Non-Markovian Parameter and Its Statistical Spectrum

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Abstract:
To quantify the non-Markovian effect first was introduced the non-Markovian parameter [1] , [2], and later was introduced the idea of the statistical spectrum of the non-Markovian parameter for macroscopic systems [3] , [4]. Here we analyze in more details the non-Markovian properties of the relaxation process, and introduce a sequence of values (spectrum) of the non-Markovian parameter.

Keywords: non-Markovian, statistical spectrum, correlation, lifetime, memory, relaxation

Introduction

It was proposed [4] , [5] to describe the effect of memory in such concepts as the lifetime of correlations, the lifetime of memory and the depth of Markovity. Here we briefly outline the basic concepts of the physics of non-Markovian processes.

Theoretical Approach

Using [6] First we write the three equations of an infinite chain under the assumption that all natural frequencies $\omega_0^{(n)} = 0$

$$\begin{aligned} \frac{da(t)}{d(t)} &= -\Omega_1^2 \int_0^t M_1(t-\tau)a(\tau)d\tau, \\ \frac{dM_1(t)}{dt} &= -\Omega_2^2 \int_0^t M_2(t-\tau)M_1(\tau)d\tau, \\ \frac{dM_2(t)}{dt} &= -\Omega_3^2 \int_0^t M_3(t-\tau)M_2(\tau)d\tau \end{aligned} \tag{1}$$

Four cross correlation functions are introduced here: $M_0(t) = a(t)$, $M_1(t)$, $M_2(t)$, $M_3(t)$. and their corresponding frequencies Ω_1 , Ω_2 , Ω_3 .

The functions $M_i(t)$ where $i = 1, 2, 3$ are functions of memory $i-th$ order, and relaxation frequencies Ω_i are associated with even frequency moments of the spectral density

$$\Omega_1^2 = I_2 \ , \ \Omega_2^2 = I_4 I_2^{-1} - I_2 \ , \ \Omega_3^2 = \frac{I_3 I_2 - I_4^2}{I_4 I_2 - I_3^2} \ ,$$

$$I_{2n} = \frac{\int_{-\infty}^{\infty} \omega^{2n} \tilde{a}(\omega) d\omega}{\int_{-\infty}^{\infty} \tilde{a}(\omega) d\omega} \ , \ \tilde{a}(\omega) = Re \int_0^{\infty} e^{-i\omega t} a(t) dt \tag{2}$$

The relaxation time of an irreversible process is understood as the value

$$\tau_{rel} = Re \int_0^{\infty} a(t)dt \tag{3}$$

It is convenient to introduce two more time scales: correlation lifetime (τ_{ic}) and the memory lifetime (τ_{im}) by the following relations

$$\tau_{ic} = \frac{Re\int_0^\infty a(t)tdt}{Re\int_0^\infty a(t)dt}, \quad \tau_{im} = \frac{Re\int_0^\infty M_1(t)tdt}{Re\int_0^\infty M_1(t)dt} \tag{4}$$

Non-Markovian property (presence of memory) can be quantitatively characterized by a dimensionless parameter

$$\varepsilon = \frac{\tau_{ic}}{\tau_{im}} = \frac{Re\int_0^\infty a(t)tdt \quad Re\int_0^\infty M_1(t)dt}{Re\int_0^\infty a(t)dt \quad Re\int_0^\infty M_1(t)tdt} \tag{5}$$

Since the parameter ε is positive in its meaning, from the physical point of view there are 3 relaxation modes

$$1)\ \varepsilon \to \infty, \quad 2)\ \varepsilon \gg 1, \quad 3)\ \varepsilon \gtrsim 1 \tag{6}$$

The first mode corresponds to the Markovian process of real relaxation, when the lifetime of memory τ_{im} is much shorter than the lifetime of correlation τ_{ic} . Then one can proceed from the concept of instantaneous memory ($\tau_{im} \to 0$) in the system. The second mode can be qualified as quasi-Markovian (starting from $\varepsilon \sim 10$ and more). Then the time scales of memory and relaxation (correlation) are essentially different. Memory effects are significant and particularly noticeable, when the time scales of memory and correlation are the same (this is the third mode)($\tau_{ic} \gtrless \tau_{im}$) . Then in the time evolution of the system, it is necessary to take into account all the states that preceded that one, and the specific kinetics of the prehistory will significantly affect the chosen state of the system. Kinetics becomes nonlinear, and its specific details depend on the properties of interparticle interactions and external fields. It becomes possible in principle to control the statistical memory by an external action, when the system smoothly passes from the Markovian relaxation mode, first to the quasi-Markovian and then to the non-Markovian one and vice versa. Fundamentally, such a smooth transition is observed in the scattering of slow neutrons during coherent scattering in liquid metal and in liquid Argon.
A more accurate non-Markovian parameter is also proposed [1] , [2]

$$\varepsilon = \frac{\tau_{i0}}{\tau_{i1}} \tag{7}$$

where τ_{i0} is the lifetime of fluctuation correlations $\delta A_0(t)$, and τ_{i1} is the lifetime of the memory

$$\tau_{i0} = \left\{\int_0^\infty t^n W_0(t)dt\right\}^{\frac{1}{n}} \tag{8}$$

$$\tau_{i1} = \left\{\int_0^\infty t^n W_1(t\}dt\right\}^{\frac{1}{n}} \tag{9}$$

Here two different temporal normalized probability densities are introduced

$$W_0(t) = \frac{|a(t)|^2}{\int_0^\infty |a(t)|^2 dt}, \quad W_1(t) = \frac{|M_1(t)|^2}{\int_0^\infty |M_1(t)|^2 dt} \tag{10}$$

which are related to the initial cross correlation function $a(t)$ and the first memory function $M_1(t)$ and n – any positive integer. It is interesting to note that the case with $n = 2$ in (8) was used earlier in optics [1] to determine the coherence time.

Conclusion

To analyze in more detail the non-Makovian properties of the relaxation process, following [1] - [5] we introduce a sequence of values of the parameter ε_i $(i = 1, 2, 3, \dots)$

$$\varepsilon_1 = \frac{\tau_0}{\tau_1}, \ \varepsilon_2 = \frac{\tau_1}{\tau_2}, \ \varepsilon_3 = \frac{\tau_2}{\tau_3}, \dots$$

where $\tau_i -$ the relaxation (memory) time of $i -$ th order. The parameter $\varepsilon = (\varepsilon_1, \ \varepsilon_2, \ \varepsilon_3, \dots)$ is called the spectrum of the non-Markovian parameter. A certain n – th equation of an infinite chain [6] is put in correspondence with the n – th level of relaxation. Then the initial relaxation process corresponds to an infinite spectrum of relaxation processes.

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