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Modelling and Optimizing Sorghum Yield for Food Security Using Central Composite Design Through Application of Organic and Inorganic Fertilizers

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ABSTRACT

The Response surface methodology has offered great solutions to day-to-day life issues, through its outstanding characteristics in optimization of response variables. The demand for sorghum for industrial use necessitated the study. The objective of this study was to model and optimize sorghum yield by varying the amount of organic and inorganic fertilizers applied on experimental plots measuring 2 metres by 1.25 metres. A 5-level-3-factor Central Composite Design (CCD) a Response Surface Methodology (RSM) was applied with the independent variables being nitrogen fertilizer (X_1) supplied in form of Calcium Ammonium Nitrate (CAN) 26%, goat manure (X_2) and VEGIMAX folia fertilizer (X_3). The p-values for nitrogen fertilizer, goat manure and VEGIMAX folia were less than 0.05, an implication that the effect of the three variables on sorghum yield was significant with the nitrogen fertilizer being highly significant ($p\text{-value} < 0.0001$). The quadratic effects were also significant ($p\text{-values} < 0.05$) but the effect of factor interactions were all insignificant ($p\text{-value} > 0.05$). The response surface plots revealed that high sorghum yield was favoured by applying moderately high levels of the three independent variables. The R-squared and Adjusted R-squared for the fitted model were 0.9277 and 0.8627 respectively, indicating that the fitted model was valid and explained 86.3% of the variations in the sorghum yield at a confidence level of 0.95. The optimal settings for the (X_1), (X_2) and (X_3) were 112 Kg / ha, 12 tons / ha and 5.5 ml / 20 litres of water, resulting to an optimal yield of 4.95 tons per hectare.

Keywords: Organic and Inorganic fertilizer, Central Composite Design, Sorghum, Optimization, Response Surface Methodology

1.0 INTRODUCTION

The Response Surface Methodology (RSM) is a collection of statistical experiment designs that are used for modeling and issue analysis when a response of interest is affected by a wide range of variables. The goal of RSM is to find the independent variables' ideal settings in order to maximize the response (Myers, 2004). In order to properly conduct experiments through adequate design and to establish operating conditions on a set of controllable variables that yield an ideal response, Box and Wilson (1951) invented the Response Surface Methodology.

The RSM enables the creation of an empirical model that makes a response surface a function of the input variables and enables the determination of a more precise approximation of the process' ideal operating conditions. This distinguishes RSM from best guess and the one factor at a time approach.

If the response in RSM can be approximated by linear function of independent variables, then the approximation function is a first-order model represented as;

$$y = \beta_0 + \sum_i^s \beta_i x_i + \varepsilon \qquad (1.1)$$

In case of the presence of curvature in the response, a higher degree polynomial such as the RSM model given as (1.2) is employed in order to obtain the curvature.

$$y = \beta_0 + \sum_{i=1}^s \beta_i x_i + \sum_{i=1}^s \beta_{ii} x_i^2 + \sum\sum_{i\neq j}^s \beta_{ij} x_i x_j + \varepsilon\,; \quad (i = 1,2,\dots,s; j = 2,3,\dots,s \text{ and } i < j) \qquad (1.2)$$

Where β_0 is a constant, β_i is the linear coefficient, β_{ii} is the coefficient of pure quadratic terms and β_{ij} is the coefficient of cross-product terms (interaction)

To establish a relationship between the input factors and the corresponding responses, RSM has been widely applied in a variety of fields, including industrial, biological, clinical, social, food, engineering, and agricultural sciences (Chelule, 2014).

Research studies have revealed the effect of different input factors like fertilizers, rate of irrigation, seed density etc. on sorghum yield (Muui *et al*., 2013; Kagwiria, 2019; Mwadalu *et al*., 2022). However, the studies neither determined the optimal settings for maximum sorghum yield nor developed an empirical model making the yield of the sorghum a function of the input factors. Factors like organic and inorganic fertilizers when added into soil creates optimal growing environment for the crops through improvement of its fertility as sited by (Titirmare *et al*., 2023) and consequently promote the crop performance. For instance, a study by Kagwiria, (2019) showed that application of fertilizers affects positively sorghum production. However, this study did not attempt to optimize levels of fertilizers applied for maximum production of sorghum grains.

Local industrial and domestic demand for sorghum has increased as a result of the current increase in sorghum consumption as food and a raw material for industry (Wambugu, 2011). This is undoubtedly promoting higher sorghum grain output and improving the livelihood and standard of living of farmers. This means that sorghum production has the potential to promote regional development, as well as the eradication of poverty, the creation of jobs, the fight against hunger, and undoubtedly an effort to realize Vision 2030 for the agricultural sector as well as the Sustainable Development Goals on achieving zero hunger, good health, and wellbeing.

The general objective of study was to model and optimize sorghum yield (*Sorghum bicolor (L) moench)* for food security using central composite design through application of organic and inorganic fertilizers.

The study was guided by the specific objectives given;

1. To determine the effect of inorganic and organic fertilizer on the yield sorghum using CCD
2. To determine the optimal values of organic and inorganic fertilizers that leads maximum sorghum yield.

2.0 MATERIALS, EQUIPMENTS AND METHOD

2.1 Materials and Equipments

SC-sila- a hybrid sorghum variety, vigimax foliar, Pesticide, Escort 50 EC, Integra, goat manure. 20 plots of land

2.2.1 Experiment

The experiment used twenty plots of 2 metres by 1.25 metres each with no blocking and each plot randomly assigned to each run for the respective treatment. Each plot had two rows with ten seed holes per row and seed density of two per seed hole. In the experiment, rows were spaced 75 cm apart, and seed holes were spaced 20 cm apart. Before the seed was covered, the soil and goat dung were completely combined. First weeding was done at day ten after emergence and at day fifteen, thinning was done to ensure a uniform seed density of two per hole. Application of vegimax folia and first half dose of nitrogen fertilizer were administered the same day. After the second weeding, a second administration of vegimax folia and the second half dose of nitrogen fertilizer was done at the thirtieth day after emergence.

Pesticide such as Escort 50 EC at the rate of 20-25 ml plus Integra 3 ml in 20 litres of water were used to manage pests such as fall armyworms. SC-sila- a hybrid sorghum variety from a certified seed company was also used in the experiment. Since the experiment was carried out in only one ecological zone, the study assumed homogeneity of soils in the twenty plots with other factors such as temperatures and humidity considered constant.

2.2.2 Experimental design

This experimental investigation was designed as a CCD with 20 experimental runs and six replications. The variance of the fitted response at different points of interest was kept constant and stable via a rotatable and orthogonal CCD. CAN 26% nitrogen fertilizer, goat manure, and vegimax folia were utilized as the independent variables (s=3), with their effects on sorghum output recorded as the response surface. The current application setting for nitrogen fertilizer, goat manure and vegimax folia is 100 kg / ha, 10 tonnes ha^{-1} and 5 ml / 20 litres of water respectively. Therefore the region of exploration for this study was (80-120) kg ha^{-1} of nitrogen fertilizer, (5-15) tonnes ha^{-1} of goat manure and (4-6) ml / 20 litres of water of vegimax folia.

2.2.3 Data collection

Data on the weight of sorghum yield per run was collected after harvesting and sun drying the harvested sorghum heads at maturity. This was done by measuring the weight of total sorghum grain yield (TSGY) and the weight of thousand grains (WTG) per run separately in grams using a laboratory electronic balance and recorded to one decimal point. Table 3.1 presents the CCD moda data on sorghum yield per run collected from the experiment. The weight of the thousand grains of sorghum in grams and the weight of the total yield of sorghum grains per run are the two coded and actual values for the three independent variables that make up the CCD design matrix.

2.2.4 Three Factor Central Composite Design

The CCD was centered about the current operating conditions of the system and to simplify the arithmetic operations, the independent variables were coded to (-1,1) interval. The respective coded variables were obtained using equation (2.1).

$$x_i = \frac{X_i - X_0}{\omega} \qquad (2.1)$$

Letting X_i be the i^{th} actual variable, X_0 be the average of the high and low level of the i^{th} variable and ω be half of the difference between the levels of the i^{th} variable, then in reference to equation (2.1) the coded variable x_i for $i = 1,2,3$ were

$$x_1 = \frac{X_1 - 100}{20}, \quad x_2 = \frac{X_2 - 10}{5} \quad \text{and} \quad x_3 = \frac{X_3 - 5}{1}$$

The i^{th} actual variables X_i for $i = 1,2,3$ corresponding to the axial variable coded $\pm\alpha$ were obtained through equation (2.2)

$$X_i|_{\pm\alpha} = \tau_i \pm \alpha\omega \quad (2.2)$$

Where τ_i is the current operating condition for the i^{th} variable and ω being half of the difference between the levels of the i^{th} variable. In reference to equation (2.2), the actual variables X_i corresponding to the axial variable coded $\pm\alpha$ for this study were

$$X_1|_{\pm\alpha} = 100 \pm 1.682(20), \; X_2|_{\pm\alpha} = 10 \pm 1.682(5) \text{ and } X_3|_{\pm\alpha} = 5 \pm 1.682(1)$$

The experimental actual variables and their coded factor levels used in this study are shown in Table 2.1.

Table 2.1: Actual variables and the coded factor levels

Independent variable	Coded factor levels				
	-1.682	-1	0	1	1.682
Nitrogen fertilizer (x_1) Kg ha^{-1}	66.36	80	100	120	133.64
Goat manure (x_2) tons ha^{-1}	1.59	5	10	15	18.41
VEGIMAX folia (x_3) ml/20L	3.318	4	5	6	6.682

Source: Author conceptualization

The independent variables that correlate to each of the CCD coded factors are shown in Table 2.1 at five levels. The variables' low and high levels are represented by the factors with the codes -1 and 1, respectively. The low and high axial levels for the three variables are represented by factors with the codes -1.682 and 1.682, respectively.

2.2.5 Optimization

The optimum levels of organic and inorganic fertilizers that optimized the response (sorghum grain yield) is a set of x_1, x_2, x_3 for which the partial derivatives of the fitted response with respect to x_1, x_2, x_3 are zero.

$$\frac{\partial \hat{y}}{\partial x_1} = \frac{\partial \hat{y}}{\partial x_2} = \frac{\partial \hat{y}}{\partial x_3} = 0 \quad (2.3)$$

The derivative of $\hat{y}$ with respect to the elements of vector X is

$$\frac{\partial \hat{y}}{\partial X} = b + 2\beta X = 0 \quad (2.4)$$

This circumstance arises at the stationary point where the set of x_1, x_2, x_3 mathematical solution corresponds to the optimal values for the input variables that maximized the response. The mathematical solution for the set of x_1, x_2, x_3at the stationary point that produces the optimal values was deduced from equation (2.5), and it is represented as

$$x_t = -\frac{1}{2}\beta^{-1}b \quad (2.5)$$

Once the set of levels for the optimum point is established, say, $\boldsymbol{x_t} = (x_{1t}, x_{2t},, x_{3t})^T$ the actual variable for optimal values of nitrogen fertilizer (X_1) supplied in form of CAN 26%, goat manure (X_2) and VEGIMAX foliar (X_3) that optimized the sorghum grain yield were obtained in reference to equation (2.6) as

$$x_{1t} = \frac{X_1 - 100}{20}, x_{2t} = \frac{X_2 - 10}{5} \text{ and } x_{3t} = \frac{X_3 - 5}{1} \quad (2.6)$$

The optimum sorghum grain yield at the stationary point was therefore predicted using the equation (2.7)

$$\hat{y} = \hat{\beta}_0 + \frac{1}{2} x_t{}^T b \qquad (2.7)$$

Through the response surfaces and related contour plots, this study determined whether the stationary point was a maximum, a minimum, or a saddle point (a point of inflection).

3.0 RESULTS AND DISCUSSION

3.1 Full Factorial Central Composite Design Matrix and Experimental Results for Sorghum Yield

The table 3.1 presents the results from the experiment

Table 3.1: CCD Model Data on Sorghum Yield per Run

	Coded values			Actual values			Main crop Yields (g/run)	
Runs	x1	x2	x3	N (Kg/ha)	GM (tons/ha)	VF (ml/20 L)	WTG (Y1)	TSGY (Y2)
1	-1	-1	-1	80	5	4	19.4	905.6
2	1	-1	-1	120	5	4	21.3	1088.2
3	-1	1	-1	80	15	4	22.4	931.2
4	1	1	-1	120	15	4	25.3	1103.7
5	-1	-1	1	80	5	6	20.6	970.3
6	1	-1	1	120	5	6	22.7	1115.5
7	-1	1	1	80	15	6	25.3	1005.2
8	1	1	1	120	15	6	26.2	1255.1
9	-1.682	0	0	66.36	10	5	18.2	897.4
10	1.682	0	0	133.64	10	5	21.7	1064.7
11	0	-1.682	0	100	1.59	5	20.8	992.5
12	0	1.682	0	100	18.41	5	22.1	1072.7
13	0	0	-1.682	100	10	3.318	22.4	1001.3
14	0	0	1.682	100	10	6.682	25.6	1108.7
15	0	0	0	100	10	5	24.8	1198.2
16	0	0	0	100	10	5	25.2	1223.4
17	0	0	0	100	10	5	24.9	1250.3
18	0	0	0	100	10	5	25.3	1206.6
19	0	0	0	100	10	5	24.4	1150.3
20	0	0	0	100	10	5	25.1	1189.6

Where, N is Nitrogen fertilizer supplied as CAN 26%, GM is the goat manure, VF is the Vegimax Folia, WTG (Y1) is the weight of 1000 grains of sorghum and TSGY (Y2) is the weight of total sorghum grain yield per run

3.2 Summary Statistics for the Weight of Total Sorghum Grain Yield (TSGY) and Thousand Grains *(WTG) Per Run*

Descriptive and normality test statistics for the weight of thousand grains (WTG) and total sorghum grain yield (TSGY) per run were carried out and results presented in the Table 3.2. The study used values of skewness and kurtosis to describe the distribution of the experimental outcomes around the mean.

Table 3.2: Summary Statistics for the Weight of Sorghum Grain Yield and 1000 Grains of Sorghum in Grams Per Run

Response	Mean	Standard deviation	Median	Min	Max	Skew	Kurtosis
WTG (Y1)	23.18	2.33	23.55	18.20	26.20	-0.50	-1.06
TSGY (Y2)	1086.53	113.95	1095.95	897.40	1255.10	-0.13	-1.32

Where WTG is the weight of 1000 grains of sorghum in grams and TSGY is the weight of total sorghum grain yield in grams per run.

According to the study's findings, each run generated 23.18 grams of WTG sorghum grains on average, with a median of 23.55 grams and a standard deviation of 2.33. The weight of TSGY produced overall, with a median of 1095.95 grams per run and a standard deviation of 113.95, was 1086.5 grams on average. For WTG and TSGY, the skewness values were -0.50 and -0.13, respectively. While WTG and TSGY had kurtosis scores of 1.06 and 1.32, respectively. Since the weight of TSGY and WTG of sorghum per run's skewness and kurtosis values fell within the range of 3, this showed that the data had a normal distribution. The results of this study concur with those of Aczel and Sounderpadian (2002), who suggested that for data to be considered normal, skewness should be within a range of ± 3

3.3 Effect of Nitrogen Fertilizer, Goat Manure and Vegimax Folia on Sorghum Yields

The Central Composite Design (CCD) comprised of a total of 20 experimental runs: with a full factorial design of eight experimental points, six axial points, and central points replicated six times.

3.1.1 Experimental Results of Oil Yield from Mango Seeds

The experimental, predicted and deviations results for the oil extraction from the mango seed are presented in Table 2. The difference between the actual value and predicted value ranged between -1.3 to 0.6 with a percentage error ranging between -14.13 to 22.82. pared t-test was carried out to establish whether there was any statistical significant difference between the actual value and predicted value. The t-statistical value for the test was found to be 0.473381 with two sided critical value of ± 2.12 i.e. $(-2.12 < 0.473381 < 2.12)$. this shows that there was no statistical significant difference between the actual value and predicted value suggesting that the response surface approach was appropriate for predicting the oil production from mango seeds.

The collected data in Table 3.1 in subsection 3.3 indicated that the treatment given to the 8th run had an average yield of 26.2 grams and 1255.1 grams being the WTG and TSGY respectively. Similarly, the results indicated that treatment given to the 9th run had an average yield of 18.2 grams and 897.4 grams being the WTG and TSGY respectively. The results showed that the treatment combination with 66.36 Kg/ ha of nitrogen, 10 tonnes / ha of goat manure and 5 ml per 20 litres of vegimax had the lowest sorghum grain yield. At the same time, the combination with 120 Kg / ha of nitrogen fertilizer, 15 tonnes / ha of goat manure and the 6 ml per 20 litres of vegimax folia produced the highest sorghum yield. The results showed that varying the quantity of nitrogen fertilizer, goat manure and vegimax folia applied on sorghum from low to moderately high level led to increase in sorghum yield. The treatment combination with 120 Kg / ha of nitrogen fertilizer, 15 tonnes / ha of goat manure and the 6 ml per 20 litres of vegimax folia was the most effective treatment among the range of treatments applied. These results reveals that integration of nitrogen fertilizer, goat manure and vegimax folia influence sorghum yield.

3.4 Second-order RSM Regression Model for Total Sorghum Grain Yield (TSGY)

The study used only the data collected on Total Sorghum Grain Yield (TSGY) to develop a regression model useful for determining a functional relationship between nitrogen fertilizer, goat manure, vegimax folia and sorghum grain yields within the area of study.In reference to equation (2.12), the $\boldsymbol{X}$ matrix and $\boldsymbol{y}$ vector needed for estimation of the model parameters of the study were obtained as

$$X = \begin{bmatrix}
1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1.682 & 0 & 0 & 0 & 0 & 0 & 2.829 & 0 & 0 \\
1 & 1.682 & 0 & 0 & 0 & 0 & 0 & 2.829 & 0 & 0 \\
1 & 0 & -1.682 & 0 & 0 & 0 & 0 & 0 & 2.829 & 0 \\
1 & 0 & 1.682 & 0 & 0 & 0 & 0 & 0 & 2.829 & 0 \\
1 & 0 & 0 & -1.682 & 0 & 0 & 0 & 0 & 0 & 2.829 \\
1 & 0 & 0 & 1.682 & 0 & 0 & 0 & 0 & 0 & 2.829 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$y = \begin{bmatrix}
905.6 \\ 1088.2 \\ 931.2 \\ 1103.7 \\ 970.3 \\ 1115.5 \\ 1005.2 \\ 1255.1 \\ 897.4 \\ 1064.7 \\ 992.5 \\ 1072.7 \\ 1001.3 \\ 1108.7 \\ 1198.2 \\ 1223.4 \\ 1250.3 \\ 1206.6 \\ 1150.3 \\ 1189.6
\end{bmatrix}$$

The R-statistical software and least squares method were used in the regression analysis to determine the parameter estimates of the polynomial model. The R output for the second order RSM model parameter estimates for TSGY per run were presented in Table 3.3.

Table 3.3: Second Order RSM Model for TSGY (weight in gram)

Factors	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	1201.946	17.223	69.7857	8.899e-15 ***
x1	75.529	11.427	6.6099	6.003e-05 ***
x2	25.662	11.427	3.1912	0.0485249 *
x3	36.465	11.427	3.1912	0.0096342 **
x1:x2	11.825	14.930	0.7920	0.4467363
x1:x3	5.000	14.930	0.3349	0.7446286
x2:x3	16.675	14.930	1.1168	0.2901743
x1^2	-71.124	11.122	-6.3949	7.884e-05 ***
x2^2	-52.903	11.122	-4.7566	0.0023434 **
x3^2	-44.985	11.122	-4.0447	0.0023434 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1; Multiple R-squared: 0.9277, Adjusted R-squared: 0.8627; F-statistic: 14.26 on 9 and 10 DF, p-value: 0.0001358

The output (Table 3.3) of the second order regression model was presented using a the optimization model (3.1)

$$y = 1201.946 + 75.529x_1 + 25.662x_2 + 36.465x_3 + 11.825x_1x_2 + 5.000x_1x_3 + 16.675x_2x_3 - 71.124x_1^2 - 52.903x_2^2 - 44.985x_3^2 \qquad (3.1)$$

This is the model useful for determining a functional relationship between nitrogen fertilizer, goat manure, vegimax folia and sorghum grain yields within Gakumbo village in Meru County. Where y is the total sorghum grain yield in grams; x_1 is nitrogen fertilizer supplied inform of CAN 26%; x_2 is goat manure and x_3 being vegimax folia.

The model output (Table 3.3) showed that the three predictors were significant at varied levels of significance with the p-values for nitrogen fertilizer, goat manure and vegimax folia being < 0.0001, <0.05, and <0.01 respectively. The model output shows that any unit increase in nitrogen fertilizer, goat manure and vegimax folia leads to a corresponding increase of75.529, 25.662 and 36.465 in the weight of TSGY. As a result, higher quantities of nitrogen fertilizer, goat dung, and vegimax folia would boost sorghum grain output. The model's F-statistic of 14.26 on degrees of freedom 9 and 10 was higher at 5% level of significance than the critical F value of 3.02 on the same degrees of freedom. This indicated that there was a correlation between the sorghum grain yield and the three independent factors. An assertion that a major portion of the fluctuation in the sorghum grain production was explained by nitrogen fertilizer, goat dung, and vegimax folia.

With their p- values all over 0.05, the interaction effects of the three predictors on the sorghum grain yield were determined to be insignificant. The model's output also showed that the quadratic effect of nitrogen fertilizer was very significant (p-value 0.001) in the data. With p-values less than 0.05, the quadratic effects of goat dung (p-value 0.001) and vegimax folia (p-value 0.01) were also statistically significant. This meant that any unit increase in the nitrogen fertilizer, goat manure, or vegimax folia quadratic impact resulted in a proportional drop of 71.124 g, 52.903 g, or 44.985 g in the weight of sorghum grain production, respectively. The results of this study are consistent with those of Masai (2019), who found that organic manure improved common bean performance ($p < 0.05$). The research (Muriithi, 2015) showed that fertilizer applications of nitrogen and phosphorus boosted the production of potato tubers. The research (Mwadalu et al., 2022) also showed that the sorghum response was significantly influenced by the application of both organic and inorganic fertilizers.

3.5 Model Adequacy Check and Validation

To study used the Analysis of variance (ANOVA) for the regression significance at 95% confidence level to check the adequacy of the model for the response. The ANOVA output was presented in Table 10.

Table 1: ANOVA for the Second Order model for TSGY per run

Sources of variance	Df	Sum Sq	Mean Sq	F value	Pr(>F)
FO(x1, x2, x3)	3	105072	35024	19.6396	0.0001632
TWI(x1, x2, x3)	3	3543	1181	0.6623	0.5938769
PQ(x1, x2, x3)	3	120258	40086	22.4781	9.185e-05
Residuals	10	17833	1783		
Lack of fit	5	12187	2437	2.1584	0.2091840
Pure error	5	5646	1129		

The results are provided in Table 10 and include the first order model (FO), a study of the response surface with two-way interactions (TWI), and pure quadratic terms (PQ). The outcome also demonstrates the second-order model's poor fit and obvious error. The importance of the second-order model was shown by the results, which showed that the lack of fit was not significant (p-value = 0.2091840 > 0.05). The F-statistic for 5 and 5 degrees of freedom was 2.16 whereas the important F value for the same degrees of freedom was 5.05. This demonstrated once more that the importance of the model was not dependent on the model's lack of fit. The study came to the conclusion that the model is sufficient and significant for predicting sorghum grain output when nitrogen fertilizer, goat dung, and vegimax folia are applied. The results (Table 3.3) showed that, at a confidence level of 0.95, the fitted model explained at least 86.3% of the variability in the sorghum yield with multiple R-squared and adjusted R-squared values of 0.9277 and 0.8627, respectively.

3.5 Optimization

3.5.1 Stationary Points

The stationary point of the response surface was computed using the R-statistical software and the output presented in Table 3.4.

Table 3.4: Stationary point of the response surface

x_1	x_2	x_3
0.5811102	0.3877729	0.5094602

The response surface stationary points were in coded form and were transformed to actual values through the equations:

$$x_1 = \frac{X_1 - 100}{20}, \qquad x_2 = \frac{X_2 - 10}{5} \quad \text{and} \qquad x_3 = \frac{X_3 - 5}{1}$$

Such that $X_1 = 112$ Kg / ha, $X_2 = 12$ tonnes / ha and $X_3 = 5.5$ ml / 20 litres of water.

The stationary points could also be calculated manually in reference to equation (2.20) as discussed below. The $(s \times 1)$ vector b and the $(s \times s)$ symmetric matrix β^{-1} needed for the analysis of the stationary point were given as

$$\beta = \begin{bmatrix} -71.124 & 5.913 & 2.5 \\ 5.913 & -52.903 & 8.338 \\ 2.5 & 8.338 & -44.985 \end{bmatrix}$$

$$\det \beta = -162169.14$$

$$C^T = \begin{bmatrix} 2310.32 & 286.84 & 181.56 \\ 286.84 & 3193.26 & 607.81 \\ 181.56 & 607.81 & 3727.71 \end{bmatrix}$$ where C^T is the transpose for matrix of cofactors.

$$\beta^{-1} = \frac{1}{\det \beta} C^T = \frac{1}{-162169.14} \begin{bmatrix} 2310.34 & 286.84 & 181.56 \\ 286.84 & 3193.26 & 607.81 \\ 181.56 & 607.81 & 3727.71 \end{bmatrix}$$

$$\beta^{-1} = \begin{bmatrix} -0.01425 & -0.001769 & -0.0011196 \\ -0.001769 & -0.01969 & -0.003748 \\ -0.0011196 & -0.003748 & -0.02299 \end{bmatrix}$$

and

$$b = \begin{bmatrix} 75.529 \\ 25.662 \\ 36.465 \end{bmatrix}$$

Therefore the stationary point was obtained as

$$X_t = -\frac{1}{2}\beta^{-1}b = -\frac{1}{2} \begin{bmatrix} -0.01425 & -0.001769 & -0.0011196 \\ -0.001769 & -0.01969 & -0.003748 \\ -0.0011196 & -0.003748 & -0.02299 \end{bmatrix} \begin{bmatrix} 75.529 \\ 25.662 \\ 36.465 \end{bmatrix} = \begin{bmatrix} 0.5813 \\ 0.3878 \\ 0.5095 \end{bmatrix}$$

The stationary points were in coded form and in reference to equation (3.2), they were transformed into their actual values as

$$x_1 = \frac{X_1 - 100}{20} \Rightarrow 0.5813 = \frac{X_1 - 100}{20} \therefore X_1 = 111.63$$

$$x_2 = \frac{X_2 - 10}{5} \Rightarrow 0.3878 = \frac{X_2 - 10}{5} \therefore X_2 = 11.9 \quad \text{and}$$

$$x_3 = \frac{X_3 - 5}{1} \Rightarrow 0.5095 = \frac{X_3 - 5}{1} \therefore X_3 = 5.5$$

Where X_1, X_2 and X_3 are actual variables for nitrogen fertilizer, goat manure and vegimax folia respectively. This implies that at the stationary point the values of nitrogen fertilizer, goat manure and vegimax folia were 111.63 Kg / ha, 11.9 tonnes / ha and 5.5 ml / 20 litres of water respectively. At the stationary point, the surface response could be predicated through the use of equation (3.2) such that

$$\hat{y} = \hat{\beta}_0 + \frac{1}{2} x_t{}^T b \qquad (3.2)$$

$$\hat{y} = 1201.946 + \frac{1}{2}\begin{bmatrix} 0.5813 & 0.3878 & 0.5095 \end{bmatrix}\begin{bmatrix} 75.529 \\ 25.662 \\ 36.465 \end{bmatrix}$$

$$\hat{y} = 1201.946 + 36.218$$

$$\hat{y} = 1238.16 \text{ grams}$$

A summary of the optimal settings for nitrogen fertilizer, goat manure and vegimax folia leading to optimal sorghum yield was presented in Table 3.5.

Table 3.52: Optimal Conditions for Optimum Sorghum Grain Yield.

Variables	Description	Actual value
X_1	Nitrogen fertilizer	112 Kg / ha
X_2	Goat manure	12 tonnes / ha
X_3	Vegimax folia	5.5 ml / 20 litres
$\hat{y}$	Grain yield	4.95 tonnes / ha

Table 3.5 shows that 112 kg / ha of nitrogen fertilizer supplied as CAN 26%, 12 tonnes / ha of goat manure and 5.5 ml / 20 litres of vegimax folia were the optimum values that led to maximum sorghum grain yield (TSGY) of 1238.16 grams per plot measuring 2 metres by 1.25 metres. This was translated to 4.95 tons/ha of sorghum grain yield.

3.5.2 Nature of the Stationary Point

This study determined the stationary point and then assessed whether it was a saddle point (i.e., a point of inflection), a maximum, a minimum, or both. R-statistical software was used to automatically produce the eigenvalues for this investigation, and the results are shown in table 3.6.

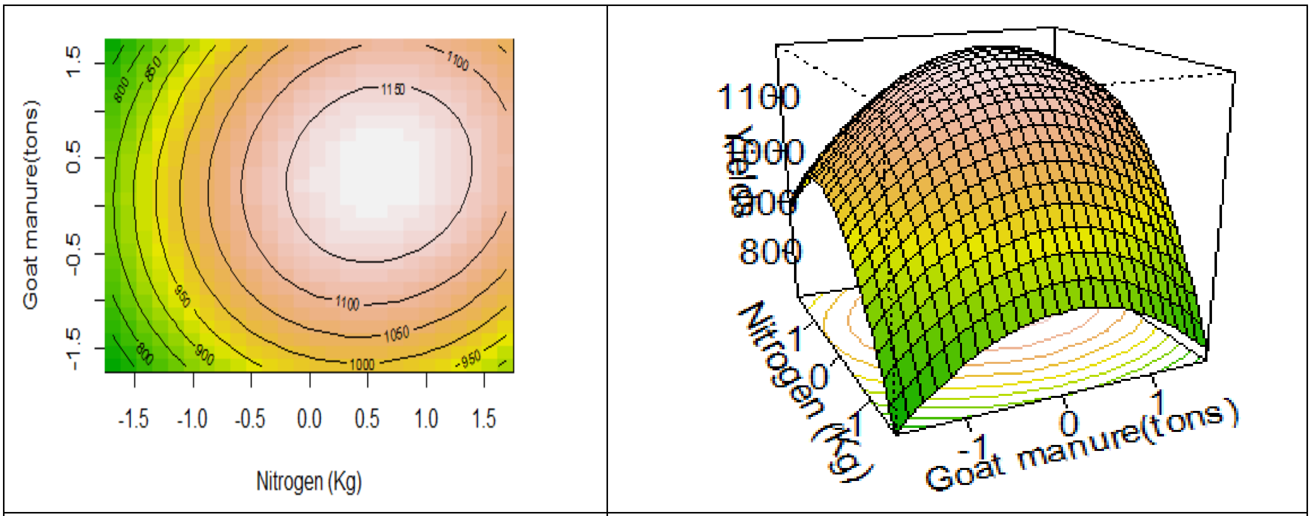
Table:3.6 Eigenvalues for the Stationary Point

$values		
-38.83478	-57.30332	-72.87467

The features of the stationary point can be determined from the signs of the respective eigenvalues. If all of the eigenvalues have positive signs, the stationary point is a location of least reaction; if all of them have negative signs, a location of maximum reaction; and if their signs differ, an inflection point. The stationary point must have been where there was the greatest response because the eigenvalues were negative.

3.5.3 Response Surface and Contour Plots

Contour plots give the researcher a graphical representation of the response surface that makes it easier to describe the surface's form and pinpoint the optimum with good accuracy. The response surfaces and associated contour plots were produced by the study using the R-Statistical programme. Plots for various combinations of variables (nitrogen fertiliser, goat manure, and vegimax folia) are shown in Figures 1, 2, and 3, together with the accompanying trend of variance in the response (sorghum grain yield) within the chosen range of input variables.

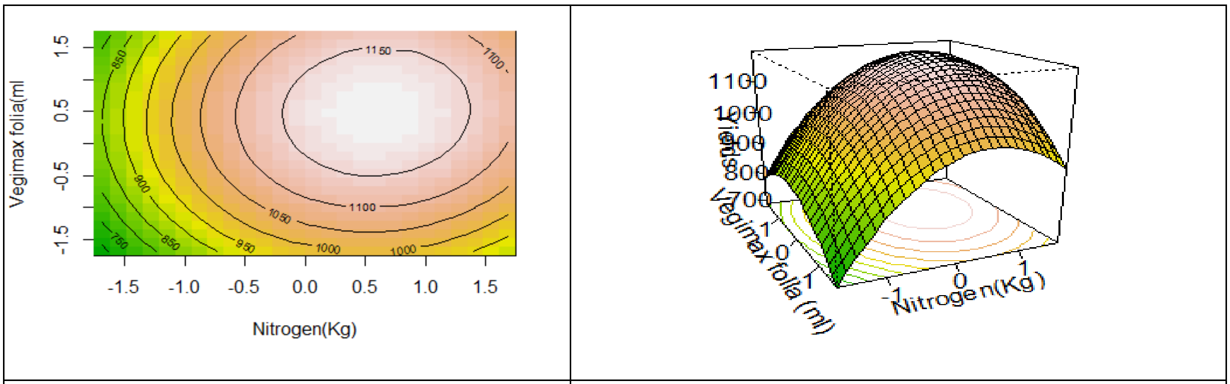


(a) Contour plot with Vegimax folia kept constant

(b) Response surface plot with Vegimax folia kept constant

Figure1. A Contour Plot and a Response Surface Illustrating a Surface with Vegimax Folia Kept Constant

Figure 1, depicts the relationship between goat dung and nitrogen fertilizer on sorghum grain output. The figure indicates that the production of sorghum grain yield was positively impacted by goat dung and nitrogen fertilizer. This shows that raising the amount of goat dung and nitrogen fertilizer from low to high will boost the yield of sorghum grain up to a certain point. This is so because both goat dung and nitrogen fertilizer are abundant in crucial plant nutrients, particularly nitrogen, which promotes sorghum growth and production.

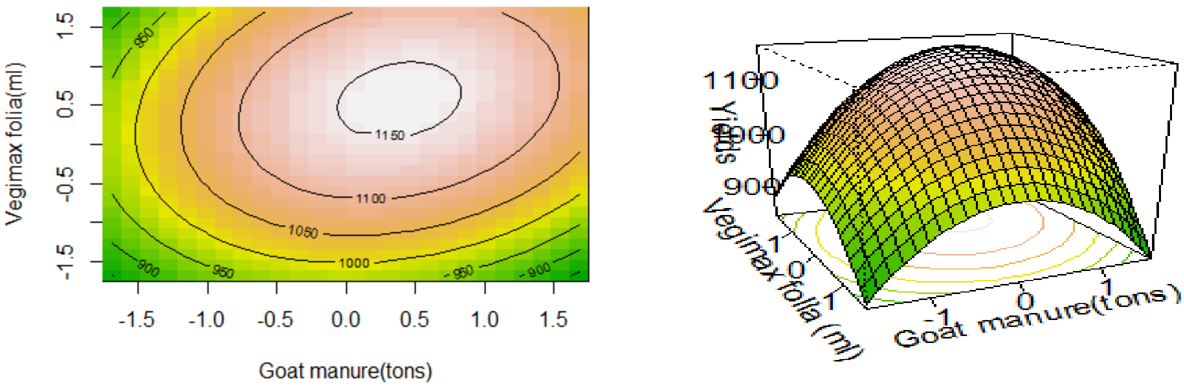


(a) Contour plot with Goat manure kept constant

(b) Response surface with Goat manure kept constant

Figure 2. A Contour Plot and a Response Surface Illustrating a Surface With Goat Manure Kept Constant

Figure 2 shows sorghum grain yield as a function of nitrogen fertilizer and vegimax folia. It was noted that increasing nitrogen fertilizer and vegimax folia to moderate levels increased the production of sorghum grain yield. Further increase of both nitrogen fertilizer and vegimax folia leads to decrease in the production of sorghum grain. Just like nitrogen fertilizer, vegimax folia are rich in nitrogen, phosphorous and potassium that are absorbed directly by the plant through the surface. Therefore the study attributed the increased sorghum grain production to the essential elements in nitrogen fertilizer and vegimax folia.



(a) Contour plot with nitrogen fertilizer kept constant

(b) Response surface with nitrogen fertilizer kept constant

Figure 3. A Contour Plot and a Response Surface Illustrating a Surface with Nitrogen Fertilizer Kept Constant

Figure 3 shows a surface plot of sorghum grain yield as a function of goat manure and vegimax folia. It was noted that goat manure and vegimax folia have a direct effect on the production of sorghum grain yield up to a certain level and then the sorghum grain yield decreases with further increase of both goat manure and vegimax folia. The three response surface plots reveal that high production of sorghum grain yield is favoured by applying moderately high levels of nitrogen fertilizer, goat manure and vegimax foli

4.0 Conclusion and Recommendations

4.1 Conclusion

The following conclusions were drawn from the study:

There is significant evidence (p-values < 0.05) of a predictive relationship between nitrogen fertilizer, goat manure, VEGIMAX folia and the sorghum yield. The second-order model developed was valid and at a confidence level of 0.95, the model adequately explain at least 86.3 % of the variations on the sorghum response to nitrogen fertilizer, goat manure and VEGIMAX folia. The optimal settings for nitrogen fertilizer, goat manure and VEGIMAX folia that lead to a maximum sorghum yield of 4.95 tons per hectare were 112 Kg / ha, 12 tons / ha and 5.5 ml / 20 litres respectively.

4.2 Recommendations

Based on the findings the study made the following recommendations:

1. Given the significance evidence of the predictive relationship between nitrogen fertilizer, goat manure and vegimax folia with sorghum yield, the study recommended continued research for further optimization of sorghum cultivation.
2. The validation of the second-order model and its ability to explain at least 86.3% of the variations in the sorghum response to nitrogen fertilizer, goat manure and vegimax folia is a significant finding hence the study recommend that the model be applied by farmers in predicting the sorghum response to nitrogen fertilizer, goat manure and vegimax folia.
3. With the optimal settings for nitrogen fertilizer, goat manure and vegimax folia identified for maximum sorghum yield, the study recommend that farmers and agricultural practitioners adopt these settings in their sorghum cultivation practices. The study also recommends that outreach and education

programs be designed to disseminate the information to farmers, ensuring they understand and can apply these optimal settings effectively.

CONFLICT OF INTEREST

The authors declare no conflict of interest

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REFFERENCES

1. Aczel, A. D., & Sounderpandian, J. (2002). *Complete business statistics*. Boston, MA: Irwin/McGraw Hill.
2. Box, G. E. P., and K. B. Wilson. "On the experimental designs for exploring response surfaces." *Ann Math Stat* 13 (1951): 1-45.
3. Chelule, J. C. C. (2014). *Application of Response Surface Methodology in Modelling a Farm Production Process in the Presence of Random-Effects* (Doctoral dissertation).
4. Kagwiria, D. (2019). *Effect of organic and inorganic fertilizer regimes on grain sorghum growth and yield* (Doctoral dissertation, University of Nairobi)
5. Masai, K. L. (2019). *Application of Response Surface Methodology in Modelling and Optimization of The Yields of Common Bean (Phaseolus Vulgaris L.) using Animal Organic Manures* (Doctoral dissertation)..
6. Muriithi, D. K. (2015). Application of response surface methodology for optimization of potato tuber yield. *American Journal of Theoretical and Applied Statistics*, *4*(4), 300-304.
7. Muui, C. W., Muasya, R. M., & Kirubi, D. T. (2013). Baseline survey on factors affecting sorghum production and use in eastern Kenya. *African journal of food, agriculture, nutrition and development*, *13*(1), 7339-7353
8. Mwadalu, R., Mochoge, B., Mwangi, M., Maitra, S., & Gitari, H. (2022). Response of Gadam sorghum (Sorghum bicolor) to farmyard manure and inorganic fertilizer application. *International Journal of Agriculture, Environment and Biotechnology*, *15*(1), 51-60.
9. Myers, R. H., Montgomery, D. C., Vining, G. G., Borror, C. M., & Kowalski, S. M. (2004). Response surface methodology: a retrospective and literature survey. *Journal of quality technology*, *36*(1), 53-77
10. Titirmare, N. S., Ranshur, N. J., Patil, A. H., Patil, S. R., & Margal, P. B. (2023). Effect of Inorganic Fertilizers and Organic Manures on Physical Properties of Soil: A Review. *International Journal of Plant & Soil Science*, *35*(19), 1015-1023.
11. Magomere, T., Obukosia, S., Albertsen, M., Wambugu, F., Kamanga, D., Njuguna, M., ... & Mwasame, E. (2016). Evaluation of fitness in F2 generations of Africa Biofortified Sorghum event 188 and weedy Sorghum bicolor ssp. drummondii. *Electronic Journal of Biotechnology*, *22*, 52-61