

# Vector Autoregressive Model Using Non-Normal Copula Errors

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## Abstract

This article focuses on a bivariate vector autoregressive model of lag 1 with non-normal errors. We propose a bivariate error distribution using two identical marginals through a copula function. The copula used for the study is the Farlie-Gumbel-Morgenstern copula, which is considered to be one of the efficient class of copula in describing the dependence between two random variables. The model parameters are estimated using the method of inference functions for margins, and the finite sample properties of the model is illustrated through simulation studies. A real life example is considered to illustrate the applications of the proposed model. The adequacy of the copula model is examined using the two sample version of kolmogorov smirnov test statistic.

**Keywords:** Vector autoregression, copula function, FGM copula, inference function for margins.

## 1. Introduction

A vector autoregression model is mainly used in multivariate time series analysis, it is the generalisation of a univariate autoregressive model. A VAR model is generally a linear function of past lags where each variable depends on its own past lags and also on the past lags of other variables. It is considered to be an efficient model for forecasting the future of particular data. A bivariate VAR model of lag 1 consists of two target variables  $Y_1$  and  $Y_2$ , in order to predict  $Y_1$  at time  $t$  the methodology of VAR works as using the past lags of  $Y_1$  and  $Y_2$  and similarly to predict  $Y_2$  at time  $t$ . One main criteria to perform forecasting techniques using VAR is that the model should be stationary in nature. The error terms in a time series model play a very vital role in order to obtain accurate forecast values but in most of the scenarios the error terms are considered to follow normal distribution or if it is a multivariate case (VAR) all the error terms follow one particular distribution with the help of copula there are possibilities of overcoming this situation wherein a copula helps in combining non-identical distributions together. Thus, in a VAR model there might exist more than one target variables that is a function of past lags of itself and other variables along with the error terms that might have non-identical distribution, a copula for a VAR model is useful and non-ambiguous for further analysis and forecast.

The modelling of time series using a vector autoregressive model with the help of copulas is considered to be very limited. Sklar (1973) justifies the importance of modelling a multivariate random variable through a copula. Copulas help in modelling the dependence structure and the marginal behaviour of each non-identical distribution that is coupled together. I'ene and Jan (1989) deal with the estimation of the density of a copula function in building a bivariate distribution function with respect to the product of its marginal distribution functions. Thus, using a copula, there is a high probability of constructing a wide

variety of copula-based time series models, but in the literature, most of the models derived using copulas are based on either normal, exponential, and so on. Scaillet and Fermanian (2002) presented a non-parametric method for estimating copulas over time. As there are many copulas considered in the literature, Ferhan and Hulya (2020) have proved that, among all the families of copulas, the Farlie-Gumbel-Morgenstern (FGM) copula provides optimum results and is non-ambiguous due to its simple approach to algebraic techniques. Helen, Marie-Pier, Etienne, and Khouzeima (2013) have illustrated the possibilities of constructing a multivariate distribution using an FGM copula. There are certain assumptions that errors have to follow a normal distribution to construct a multivariate distribution, but Nimitha and Balakrishna (2019) have discussed the possibilities of constructing a bivariate distribution using errors that do not follow normal distribution. L.K. Hotta, E. C. Lucas, and H. P. Palaro (2008) propose a method for estimating a VAR model using a copula and extreme value under the assumption that the model follows normal distribution Markku Lanne and Helmut Lutkepohl (2012) argue that considering a vector autoregressive model with normal residuals appears to be limiting in a practical approach and that a VAR model can be built with non-normal residuals. Gediminas et.al

(2012) have suggested a method of analysing the stationarity regions for a vector autoregressive model. Joe and Xu (2016) proposed a method for estimating the parameters separately by maximising the marginal likelihoods and then estimating the dependence parameter from the joint likelihood function. Christian et.al (2013) suggest the easier methodology of using the Kolmogorov-Smirnov test to test the goodness of fit of the copula.

To the best of our knowledge, the literature has not yet studied the construction of a bivariate vector autoregressive model with errors following a logistic distribution using the FGM copula. Hence, this article takes advantage of formulating a forecasting structure using these methods.

The article is organised as follows: In Section 2, we introduce the VAR model for lag 1 with errors generated using the FGM copula with marginals following logistic distribution. In Section 3, we discuss the inference procedure of the model. In Section 4, we discuss the simulation study and the results. Section 5 illustrates the application of the model using a real data set. Section 6 summarises the conclusions.

## 2. Model and properties

Here, we focus on vector autoregressive model with copula errors. The explicit bivariate vector autoregressive model is given as

$$Y1_t = \beta_{11}Y1_{t-1} + \beta_{12}Y2_{t-1} + \varepsilon_{1t} \tag{1}$$

$$Y2_t = \beta_{21}Y1_{t-1} + \beta_{22}Y2_{t-1} + \varepsilon_{2t} \tag{2}$$

where the above equations are vector autoregressive model with lag one. Now we assume that in this bivariate case the random variables  $\varepsilon_{1t}$  and  $\varepsilon_{2t}$  specified in Equation (1) and (2) follow identical marginals specified as standard logistic distribution at  $\mu = 0$  and  $s = 1$  defined as:

$$f(\varepsilon_{1t}) = \frac{e^{-\varepsilon_{1t}}}{(1 + e^{-\varepsilon_{1t}})^2}, -\infty < \varepsilon_{1t} < \infty \tag{3}$$

$$f(\varepsilon_{2t}) = \frac{e^{-\varepsilon_{2t}}}{(1 + e^{-\varepsilon_{2t}})^2}, -\infty < \varepsilon_{2t} < \infty \tag{4}$$

Thus, a bivariate distribution or a density function can be constructed using a dependence structure,  $C : [0,1] \times [0,1] \Rightarrow [0,1]$  called as copula with the general structure specified as :

$$H(X1, X2) (x1, x2) = C (FX1 (x1), FX2 (x2))$$

Let us consider the Farlie-Gumbel-Morgenstern (FGM) copula in order to construct the density function.

$$C(u,v) = uv + \theta uv(1-u)(1-v), -1 < \theta < 1 \quad (5)$$

The corresponding conditional copula densities are given by:

$$\hat{C}_{v|u}(u,v) = \frac{\partial}{\partial u} \hat{C}(u,v) = v + v\theta(1-v)(1-2u)$$

$$\hat{C}_{u|v}(u,v) = \frac{\partial}{\partial v} \hat{C}(u,v) = u + u\theta(1-u)(1-2v)$$

Note that the aforementioned copula is a member of one parameter families of Archimedean copula which is useful for modelling positive-dependent data Remark 1: The generator of the FGM copula given in Equation(4) is

$$\phi(t) = -\log t$$

The generator is useful for finding the cumulative density function and useful for checking the goodness of fit. From the Sklar theorem, the joint density function of  $\varepsilon_{1t}$  and  $\varepsilon_{2t}$  using the marginals of

$$F(\varepsilon_{1t}) = \frac{1}{1+e^{-\varepsilon_{1t}}}, -\infty < \varepsilon_{1t} <$$

$\infty, s > 0 F(\varepsilon_{2t}) = \frac{1}{1+e^{-\varepsilon_{2t}}}, -\infty < \varepsilon_{2t} < \infty, s > 0$  and the FGM copula defined in Equation(4) can be defined as:

$$H(\varepsilon_{1t}, \varepsilon_{2t}) = C(F(\varepsilon_{1t}), F(\varepsilon_{2t}))$$

$$= \frac{1}{(1+e^{-\varepsilon_{1t}})} \frac{1}{(1+e^{-\varepsilon_{2t}})} + \frac{\theta e^{-\varepsilon_{1t}} e^{-\varepsilon_{2t}}}{(1+e^{-\varepsilon_{1t}})(1+e^{-\varepsilon_{2t}})} \quad (6)$$

It can be shown that the FGM copula has uniform marginals given by

$$C(u, 1) = u = \frac{1}{1+e^{-\varepsilon_{1t}}} C(1, v) = v = \frac{1}{1+e^{-\varepsilon_{2t}}}$$

The properties of the joint density function are verified as

$$F(\varepsilon_{1t}, \infty) = \frac{1}{1+e^{-\varepsilon_{1t}}} F(\infty, \varepsilon_{2t},) = \frac{1}{1+e^{-\varepsilon_{2t}}}$$

The cumulative density function of the FGM copula  $T^* = C(u,v)$  is given as

$$K_c(t) = t - \frac{\phi(t)}{\phi'(t)} = t(1 - \log(t)), 0 \leq t \leq 1$$

where  $\phi(t)$  is the generator of the defined copula and the  $K_c(t)$  value helps in analysing if the proposed copula is a good fit for the data by plotting the cumulative distribution,  $K_c(w)$

For the given joint distribution function, a bivariate probability density function is obtained by differentiating the density function with respect to the variables  $\varepsilon_{1t}$  and  $\varepsilon_{2t}$

$$h(\varepsilon_{1t}, \varepsilon_{2t}) = (-1)^2 \frac{(\partial^2 H(\varepsilon_{1t}, \varepsilon_{2t}))}{\partial \varepsilon_{1t} \varepsilon_{2t}}$$

where  $\frac{(\partial^2 H(\varepsilon_{1t}, \varepsilon_{2t}))}{\partial \varepsilon_{1t} \varepsilon_{2t}}$  can be further decomposed to

$$\frac{(\partial^2 H(\varepsilon_{1t}, \varepsilon_{2t}))}{\partial \varepsilon_{1t} \varepsilon_{2t}} = \hat{c}_\theta(u, v) \cdot f(\varepsilon_{1t}) f(\varepsilon_{2t})$$

Thus, the bivariate density function constructed using logistic marginals is of the form,

$$h(\varepsilon_{1t}, \varepsilon_{2t}) = \frac{e^{-\varepsilon_{1t}-\varepsilon_{2t}}}{(1+e^{-\varepsilon_{1t}})^2(1+e^{-\varepsilon_{2t}})^2} + \frac{(\theta e^{-2\varepsilon_{1t}-2\varepsilon_{2t}}(1-e^{\varepsilon_{1t}})(1-e^{\varepsilon_{2t}}))}{(1+e^{-\varepsilon_{1t}})^3(1+e^{-\varepsilon_{2t}})^3} \quad (7)$$

For a sample of T observations the conditional log-likelihood function is given as:

$$L(\Theta) = \sum_{t=1}^T \log [\hat{c}_\theta(F(\varepsilon_{1t}), F(\varepsilon_{2t})) \cdot f(\varepsilon_{1t}) f(\varepsilon_{2t})] \quad (8)$$

where  $\Theta$  is the set of all parameters in the model.

One of the best way to find the measure of dependence to the linear correlation coefficient is Kendall's Tau measure of dependence and it is given as

Remark 2 : For the given copula function , the population version of Kendall's tau is given by  $C^{\theta}(u,v) = uv + \theta uv(1 - u)(1 - v), -1 < \theta < 1$

$$\tau_{\varepsilon_{1t}, \varepsilon_{2t}} = \tau_C = 4 \int_0^1 \int_0^1 C(u, v) dC(u, v) - 1 = \frac{2\theta}{9}$$

Remark 3 : For the given copula function , the population version of Spearman's correlation is given by  $C^{\theta}(u,v) =$

$$uv + \theta uv(1 - u)(1 - v), -1 < \theta < 1$$

$$\rho_{\varepsilon_{1t}, \varepsilon_{2t}} = \rho_C = 12 \int_0^1 \int_0^1 C(u, v) dudv - 3 = \frac{\theta}{3}$$

### 3. Estimation of copula

This section deals with estimation of a copula-based vector autoregressive model using the method of inference functions for margins(IFM).

#### 3.1. Inference functions for margins(IFM).

This approach consists of obtaining the maximum likelihood estimates of model parameters based on marginal likelihood function and maximum likelihood estimate of dependency parameter based on the whole likelihood function. The conditional log-likelihood functions for the univariate marginals are given as:

$$L_1(\Theta_1) = \sum_{t=1}^T \log f(\varepsilon_{1t}; \Theta_1) \tag{9}$$

$$L_2(\Theta_2) = \sum_{t=1}^T \log f(\varepsilon_{2t}; \Theta_2) \tag{10}$$

where  $\Theta_1 = (\beta_{11}, \beta_{12})$  and  $\Theta_2 = (\beta_{21}, \beta_{22})$ . The corresponding log-likelihood function with respect to the joint distribution is:

$XT$

$$L(\theta, \beta_{11}, \beta_{12}, \beta_{21}, \beta_{22}) = \sum_{t=1}^T \log h(\varepsilon_{it}; \theta, \beta_{11}, \beta_{12}, \beta_{21}, \beta_{22})$$

$$= \sum_{t=1}^T \log [c_{\theta}(F(\varepsilon_{1t}), F(\varepsilon_{2t})) f(\varepsilon_{1t}) f(\varepsilon_{2t})]$$

The procedure of IFM works as follows: first we obtain the estimates of  $\Theta_1$  and  $\Theta_2$  by maximising the condition log-likelihood functions given the above equations. If  $\hat{\beta}_{11}, \hat{\beta}_{12}, \hat{\beta}_{21}, \hat{\beta}_{22}$  are the estimates obtained from the first step we obtain the estimate of  $\theta$  by maximising,

$$L(\theta, \hat{\beta}_{11}, \hat{\beta}_{12}, \hat{\beta}_{21}, \hat{\beta}_{22}) = \sum_{t=1}^T \log h(\varepsilon_{it}; \theta, \hat{\beta}_{11}, \hat{\beta}_{12}, \hat{\beta}_{21}, \hat{\beta}_{22})$$

with respect to  $\theta$  Now we apply the procedure to the models (1) and (2). The log-likelihood function based on the logistic errors  $\varepsilon_{1t}$  is given by: The log-likelihood equations are obtained as:

$$\frac{\partial \log L_1(\Theta_1)}{\partial \beta_{11}} = \sum_{t=1}^T Y_{1t-1} - 2 \frac{Y_{1t-1} e^{-Y_{1t} + \beta_{11} Y_{1t-1} + \beta_{12} Y_{2t-1}}}{1 + e^{-Y_{1t} + \beta_{11} Y_{1t-1} + \beta_{12} Y_{2t-1}}}$$

$$\frac{\partial \log L_1(\Theta_1)}{\partial \beta_{12}} = \sum_{t=1}^T Y_{2t-1} - 2 \frac{Y_{2t-1} e^{-Y_{1t} + \beta_{11} Y_{1t-1} + \beta_{12} Y_{2t-1}}}{1 + e^{-Y_{1t} + \beta_{11} Y_{1t-1} + \beta_{12} Y_{2t-1}}}$$

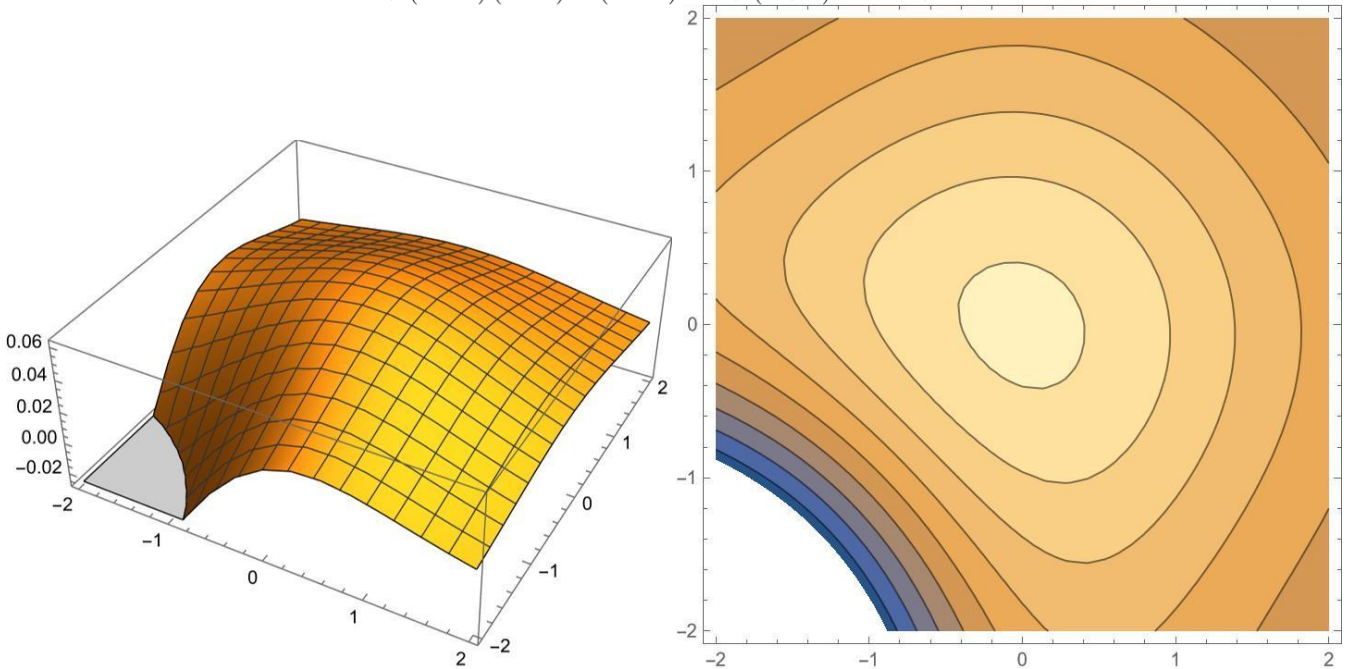
$$\frac{\partial \log L_2(\Theta_2)}{\partial \beta_{21}} = \sum_{t=1}^T Y_{1t-1} - 2 \frac{Y_{1t-1} e^{-Y_{2t} + \beta_{21} Y_{1t-1} + \beta_{22} Y_{2t-1}}}{1 + e^{-Y_{2t} + \beta_{21} Y_{1t-1} + \beta_{22} Y_{2t-1}}}$$

$$\frac{\partial \log L_2(\Theta_2)}{\partial \beta_{22}} = \sum_{t=1}^T Y_{2t-1} - 2 \frac{Y_{2t-1} e^{-Y_{2t} + \beta_{21} Y_{1t-1} + \beta_{22} Y_{2t-1}}}{1 + e^{-Y_{2t} + \beta_{21} Y_{1t-1} + \beta_{22} Y_{2t-1}}}$$

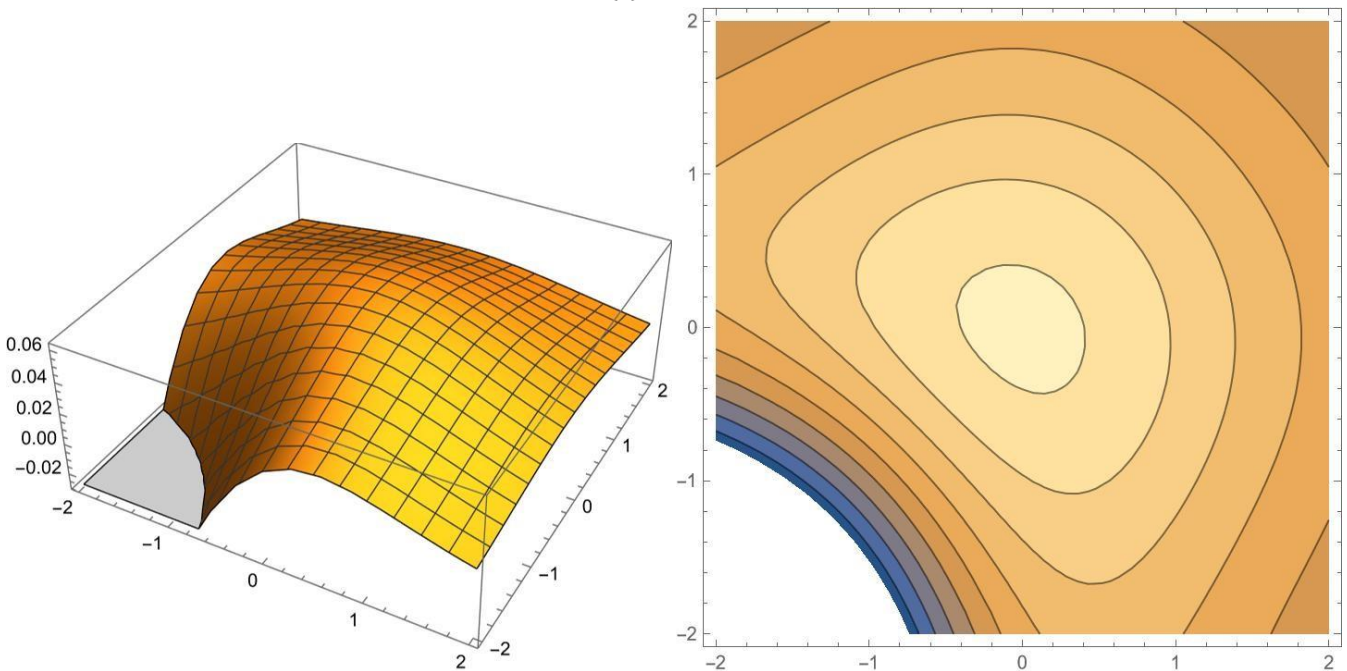
On solving the likelihood equations numerically, we will get the estimates of  $\beta_{11}, \beta_{12}, \beta_{21}, \beta_{22}$  and we call them as  $\hat{\beta}_{11}, \hat{\beta}_{12}, \hat{\beta}_{21}, \hat{\beta}_{22}$ . We do not have analytically closed form expressions for the estimators. The MLE's of the parameters  $\beta_{11}, \beta_{12}, \beta_{21}, \beta_{22}$  are obtained as: Once the marginal parameters are obtained, the function  $L(\theta, \hat{\beta}_{11}, \hat{\beta}_{12}, \hat{\beta}_{21}, \hat{\beta}_{22})$  is maximized over  $\theta$  to get  $\hat{\theta}$ . On differentiating the log-likelihood function over the maximized set of parameters with respect to  $\theta$  implies: where

$$\varepsilon_1 t = Y_{2t} - \beta_{21} Y_{1t-1} - \beta_{22} Y_{2t-1} - \frac{\partial \log L(\Theta)_1}{\partial \theta} = 0 \Rightarrow \frac{\varepsilon_1 t}{(e^{\varepsilon_1 t} - 1)(e^{\varepsilon_2 t} - 1)} = Y_{1t} - \beta_{11} Y_{1t-1} - \beta_{12} Y_{2t-1} - \beta_{22} Y_{2t-1}$$

$$\frac{\varepsilon_2 t}{\theta + (\theta - 1)(-e^{\varepsilon_1 t}) - (\theta - 1)e^{\varepsilon_2 t} + (\theta + 1)e^{\varepsilon_1 + \varepsilon_2 t} + 1}$$

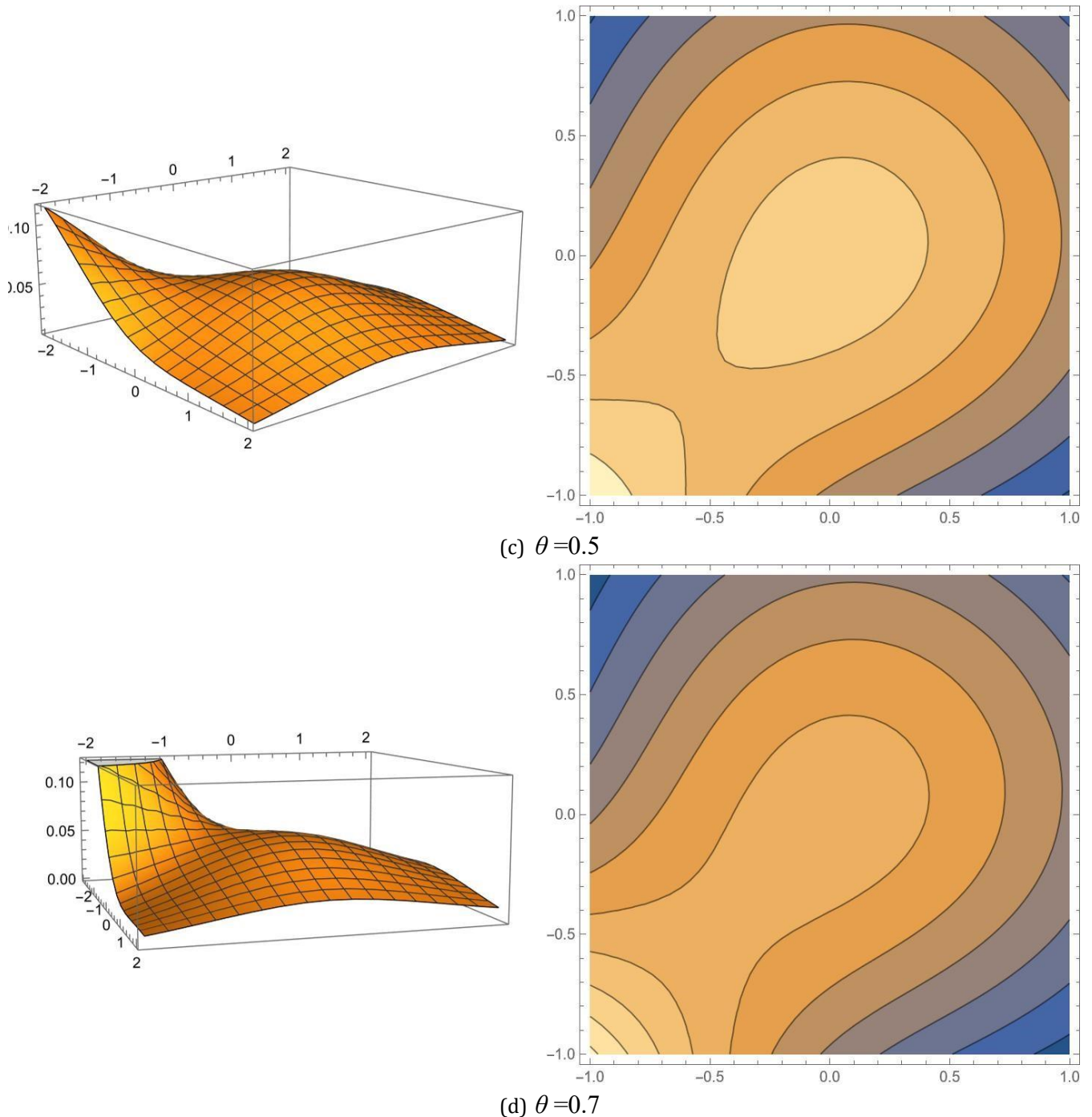


(a)  $\theta = -0.3$



(b)  $\theta = -0.5$





**Figure 1: Bivariate density plots and contours with Logistic marginals**

#### 4. Simulation

We carried out a simulation study to evaluate the efficiency of the estimates obtained by IFM method. For the simulation study, we obtain the bivariate data  $(\varepsilon_{1t}, \varepsilon_{2t})$  using the available packages in R for generating data based on FGM copula for a specified choice of dependent parameter,  $\theta$ . Then for different model parameter values, we simulated the bivariate data  $(Y_{1t}, Y_{2t})$  using the equations (1) and (2) using the IFM method mentioned in section 3.1. For each specified value of the parameter, we repeated the experiment 50 times. Alternatively, we calculated the dependency parameter  $\theta$  using the population version of Kendall's tau measure given in Remark 1,  $\hat{\theta}_\tau = \frac{9\tau}{2}$ .

The estimates of IFM along with their root mean squared error are reported in Tables 1-2.

**Table 1. Estimates and MSE of the model parameters based on IFM for  $\theta = 0.5$**

n	$\beta_{11}$	$\beta_{12}$	$\beta_{21}$	$\beta_{22}$	$\hat{\beta}^{*11}$	$\hat{\beta}^{*12}$	$\hat{\beta}^{*21}$	$\hat{\beta}^{*22}$
100	0.4	0.2	0.2	0.4	0.436 (0.001)	0.02 (0.032)	0.165 (0.001)	0.331 (0.005)
	0.6	0.3	0.3	0.6	0.592 (0.01)	0.188 (0.013)	0.297 (0.02)	0.572 (0.001)
	0.5	0.4	0.4	0.5	0.487 (0.03)	0.3 (0.01)	0.392 (0.02)	0.482 (0.01)
	0.2	0.1	0.1	0.2	0.253 (0.003)	0.081 (0.033)	0.059 (0.002)	0.134 (0.004)
	0.3	0.1	0.1	0.3	0.351 (0.003)	0.088 (0.035)	0.061 (0.001)	0.221 (0.006)
250	0.4	0.2	0.2	0.4	0.473 (0.005)	0.204 (0.02)	0.166 (0.001)	0.474 (0.005)
	0.6	0.3	0.3	0.6	0.629 (0.001)	0.292 (0.001)	0.323 (0.001)	0.636 (0.001)
	0.5	0.4	0.4	0.5	0.529 (0.001)	0.391 (0.001)	0.426 (0.001)	0.531 (0.001)
	0.2	0.1	0.1	0.2	0.291 (0.008)	0.113 (0.002)	0.039 (0.004)	0.271 (0.005)
	0.3	0.1	0.1	0.3	0.252 (0.002)	0.098 (0.001)	0.12 (0.001)	0.327 (0.001)
500	0.4	0.2	0.2	0.4	0.353 (0.002)	0.202 (0.002)	0.243 (0.002)	0.435 (0.001)
	0.6	0.3	0.3	0.6	0.583 (0.005)	0.31 (0.004)	0.352 (0.003)	0.612 (0.001)
	0.5	0.4	0.4	0.5	0.486 (0.003)	0.406 (0.003)	0.453 (0.003)	0.51 (0.003)
	0.2	0.1	0.1	0.2	0.155 (0.002)	0.091 (0.002)	0.113 (0.002)	0.217 (0.002)
	0.3	0.1	0.1	0.3	0.252 (0.002)	0.098 (0.001)	0.12 (0.001)	0.327 (0.001)
1000	0.4	0.2	0.2	0.4	0.374 (0.001)	0.33 (0.017)	0.059 (0.02)	0.486 (0.007)
	0.6	0.3	0.3	0.6	0.583 (0.003)	0.35 (0.002)	0.241 (0.004)	0.651 (0.003)

	0.5	0.4	0.4	0.5	0.482 (0.002)	0.453 (0.003)	0.341 (0.004)	0.552 (0.003)
	0.2	0.1	0.1	0.2	0.155 (0.002)	0.254 (0.024)	0.081 (0.033)	0.285 (0.007)
	0.3	0.1	0.1	0.3	0.26 (0.002)	0.243 (0.02)	0.072 (0.03)	0.384 (0.007)

**Table 2. . Average Values and MSE of Dependency Parameter Estimates based on IFM ( $\theta_{IFM}$ ) and Kendall's**

tau Measure ( $\hat{\theta}_\tau$ ) for  $\theta = 0.5$

n	$\beta_{11}$	$\beta_{12}$	$\beta_{21}$	$\beta_{22}$	$\theta_{IFM}$	$\hat{\theta}_\tau$
100	0.4	0.2	0.2	0.4	0.632 (0.005)	0.64 (0.005)
	0.6	0.3	0.3	0.6	0.501 (0.001)	0.511 (0.002)
	0.5	0.4	0.4	0.5	0.632 (0.005)	0.60 (0.002)
	0.2	0.1	0.1	0.2	0.632 (0.005)	0.60 (0.025)
	0.3	0.1	0.1	0.3	0.632 (0.005)	0.60 (0.002)
250	0.4	0.2	0.2	0.4	0.771 (0.005)	0.71 (0.002)
	0.6	0.3	0.3	0.6	0.514 (0.004)	0.54 (0.001)
	0.5	0.4	0.4	0.5	0.771 (0.005)	0.671 (0.002)
	0.2	0.1	0.1	0.2	0.771 (0.005)	0.671 (0.002)
500	0.4	0.2	0.2	0.4	0.697 (0.053)	0.655 (0.001)
	0.6	0.3	0.3	0.6	0.262 (0.057)	0.158 (0.003)
	0.5	0.4	0.4	0.5	0.697 (0.005)	0.655 (0.003)

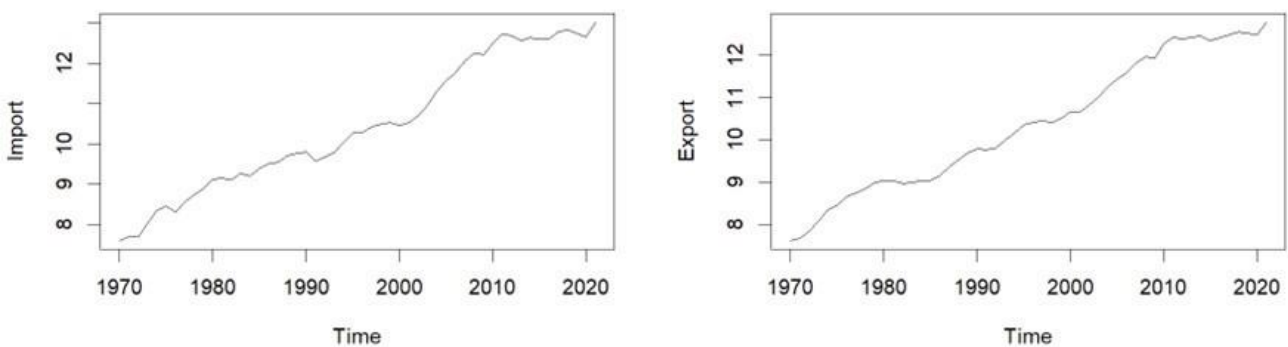


	0.2	0.1	0.1	0.2	0.697 (0.005)	0.655 (0.003)
	0.3	0.1	0.1	0.3	0.697 (0.005)	0.655 (0.003)
1000	0.4	0.2	0.2	0.4	0.681 (0.057)	0.651 (0.002)
	0.6	0.3	0.3	0.6	0.501 (0.005)	0.511 (0.002)
	0.5	0.4	0.4	0.5	0.681 (0.005)	0.621 (0.002)
	0.2	0.1	0.1	0.2	0.681 (0.057)	0.621 (0.003)
	0.3	0.1	0.1	0.3	0.681 (0.005)	0.621 (0.002)

### 5. Data Analysis

In order to illustrate the developed model we consider some real life data sets. The data set consists of 52 observations for the import and export of non-oil products for the period 1970-2022. All the variables have been transformed into their natural logarithm. The data set is downloaded from the website of Reserve Bank of India.

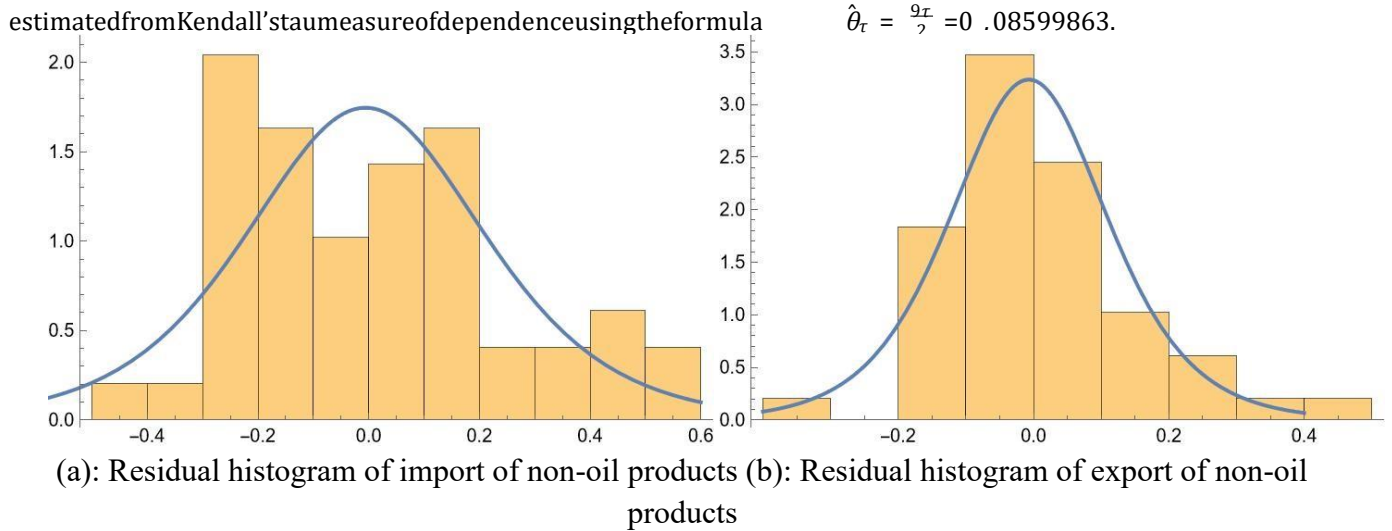
The plot obtained from the log-transformed data indicates that the time series is non-stationary. Using the Augmented-Dickey Fuller test we also confirm that the time series is non stationary as the p-value for both imports and exports is 0.4811 and 0.1535 respectively, indicating that both the variables are non-stationary in nature.



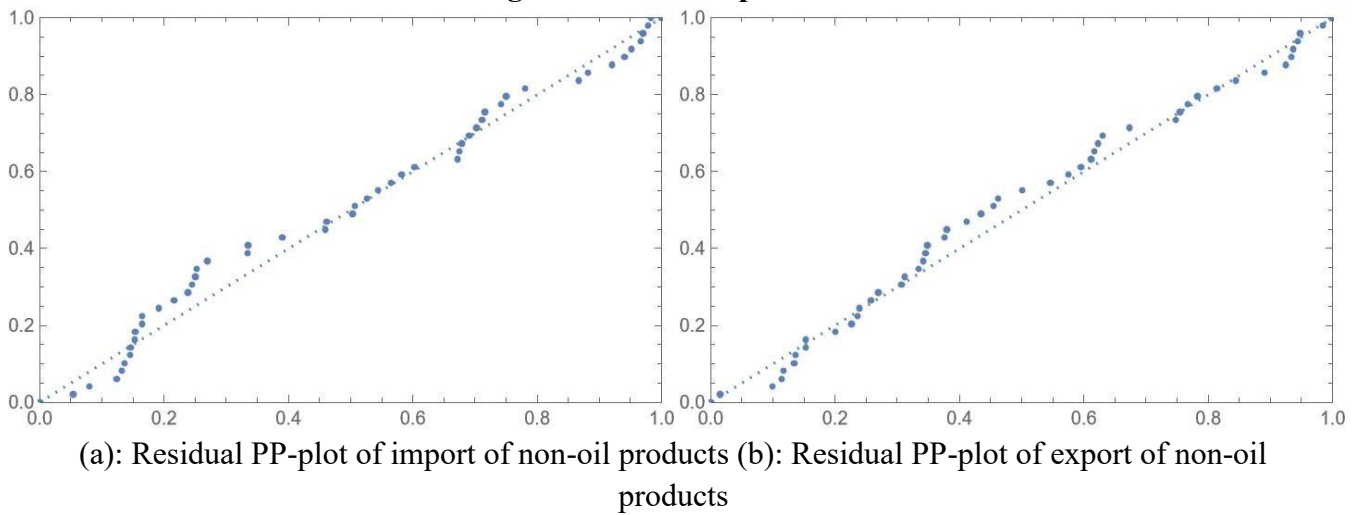
(a): Time series plot of import of non-oil products (b): Time series plot of export of non-oil products

**Figure 2: Time series plot**

By performing a second order differencing we make the data stationary and by again performing The Augmented Dickey Fuller test the p-value is obtained as 0.01 for both the data. We perform the IFM method to obtain the parameter estimates of the VAR parameters and the copula parameters which is obtained as  $\hat{\theta} = 0.3869938$ ,  $\hat{\beta}_{11} = 0.690695$ ,  $\hat{\beta}_{12} = 0.2826458$ ,  $\hat{\beta}_{21} = 0.2972279$ ,  $\hat{\beta}_{22} = 0.1423185$ . Similarly the dependency parameter is also



**Figure 3: Residual plots**



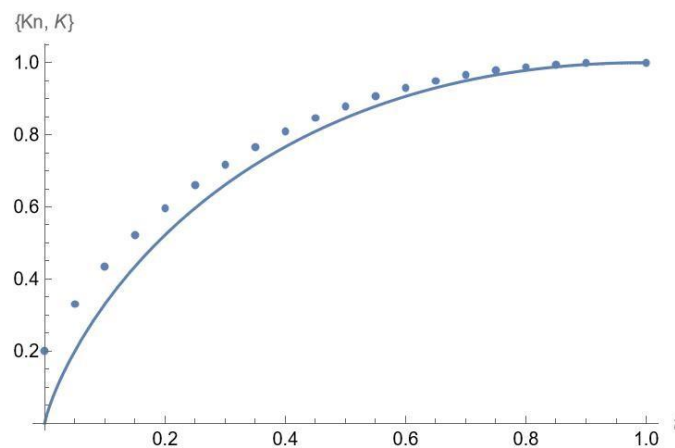
**Figure 4: Probability-Probability plot**

Now the residuals obtained from the vector autoregressive model as mentioned in (1) and (2) is given as:

$$\varepsilon_{1t} = Y_{1t} - \beta_{11}Y_{1t-1} - \beta_{12}Y_{2t-1}$$

$$\varepsilon_{2t} = Y_{2t} - \beta_{21}Y_{1t-1} - \beta_{22}Y_{2t-1}$$

Using all the above mentioned estimates, we tested whether the residuals follow logistic distribution using Kolmogrov-Smirnov test. The marginals of the data can be fitted by logistic [0.690695,0.2826458] and [0.2972279,0.1423185] with Kolmogrov-Smirnov test statistics values are 0.1635 and 0.218 respectively.



**Figure 5: Plots of  $K_c(t)$  and  $K_c(w)$**

The cdf  $K_c(t)$  with  $\theta = 0.3869938$  and the empirical distribution function  $K_c(w)$  from the data are plotted in

Figure 5. In order to test whether the FGM copula is suitable for the data, we test the hypothesis  $H_0: C(u, v) = C(u, v, \hat{\theta})$  using the Kolmogorov-Smirnov test statistic given by Franq et.al(2000), where  $D_n$  is the Kolmogorov statistic. For the 5 percent level of significance and sample size equal to 52, the limiting distribution is given as 1.358. The obtained  $D_n$  value, using Mathematica, is 0.714273 and as  $0.714273 < 1.358$ ; thus, we cannot reject the null hypothesis, and hence the given copula can be fitted by  $C(u, v, 0.3869938)$ . Hence, we can conclude that the defined distribution is better and suitable for the data.

## 6. Conclusion

In this article, we introduced a bivariate vector autoregressive model of lag 1 with errors following logistic distribution that is generated using the FGM copula. We have considered FGM copula to generate a bivariate distribution considering logistic marginals. Using the method of IFM we have estimated the model parameters and from the simulation study we have concluded that the estimates are efficient as there is a decrease in mean square errors. The applicability of the proposed model using the dependence structure is analyzed and is illustrated by using some financial variables like import and export of non-oil products.

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