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Somewhat mr-Continuous and Somewhat mr-Open Functions

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Abstract
The present article introduces and investigates new classes of functions, namely somewhat mr-continous and somewhat mr-open functions, by utilizing minimal regular open sets and minimal regular closed sets. The paper establishes the relationship between these new categories and other classes of functions, such as somewhat continuous, somewhat r-continuous and completely continuous functions, while also providing examples, counter examples, and various properties. The study of these classes of functions represents a significant contribution to the field of mathematics and provides new insights into the properties and behaviour of functions in various contexts.

Keywords. $somewhat\ mr-continuous, somewhat\ mr-open, mr-space, mr-seperable.$

1. Introduction

Karl R Gentry and Hughes B Hoyle [4] studied the concept of Somewhat continuous functions and somewhat open functions. Anuradha N and Baby Chacko [1] introduced and discussed minimal regular open sets and maximal regular open sets. In this paper a new type of somewhat continuous and open functions namely somewhat mr-continuous and somewhat mr-open functions are introduced. These types of functions were discussed in sections (3) and (4). In section (2) basic definitions are introduced. In section (3) we define somewhat mr-continuous functions and study its properties. Characterizations of somewhat mr-continuous functions are given and its relation with some other types of functions is also studied. Section (4) deals with the definition of somewhat mr-open function and its properties.

2. Preliminaries

Definition 2.1. [4] A function $g:(X,\gamma) \to (Z,\mu)$ is said to be Somewhat continuous if for $V \in \mu$ and $g^{-1}(V) \neq \emptyset$, there exists an open set W in X such that $W \neq \emptyset$ and $W \subset g^{-1}(V)$.
Definition 2.2. [2] A function $g:(X,\gamma) \to (Z,\mu)$ is said to be Somewhat r-continuous if for $V \in \mu$ and $g^{-1}(V) \neq \emptyset$, there exists a regular open set W in X such that $W \neq \emptyset$ and $W \subset g^{-1}(V)$.
Definition 2.3. A function $g:(X,\gamma) \to (Z,\mu)$ is said to be cl-super Continuous [8] (clopen continuous [7]) if for each $x \in X$ and each open set V containing $g(x)$ there exists clopen set W containing x such that $g(W) \subset V$.
Definition 2.4. A function $g:(X,\gamma) \to (Z,\mu)$ is said to be δ −Continuous [7] if for each $x \in X$ and for

each regular open set V containing $g(x)$ there exists a regular open set W containing x such that $g(W) \subset V$.

Definition 2.5. A function $g:(X,\gamma) \to (Z,\mu)$ is said to be completely Continuous [3] if $g^{-1}(W)$ is a regular open set in X, for every open set $W \subset Z$.

Definition 2.6. A function $g:(X,\gamma) \to (Z,\mu)$ is said to be almost completely Continuous [6] if $g^{-1}(W)$ is a regular open set in X, for every regular open set $W \subset Z$.

Definition 2.7. [9] A proper non-empty regular open subset U of a topological space (X, τ) is said to be minimal regular open, if any regular open set which is contained in U is ∅ or U. A proper non-empty regular closed subset F of a topological space (X, τ) is said to be minimal regular closed, if any regular closed set which is contained in F is ∅ or F.

Definition 2.8. [9] A proper non-empty regular open subset U of a topological space (X, τ) is said to be maximal regular open, if any regular open set which contains U is X or U. A proper non-empty regular closed subset F of a topological space (X, τ) is said to be maximal regular closed, if any regular closed set which contains F is X or F.

Definition 2.9. [5] If X is a set and τ_1 and τ_2 are topologies for X. Then τ_2 is said to be stranger than τ_1 (or τ_1 is weaker than τ_2) provided if $U \in \tau_1$ and $U \neq \emptyset$, then there is an open set V in (X,τ_2) such that $V \subset U$.

Definition 2.10. [5] A is dense in X if and only if the only closed subset of X containing A is X itself.

3. Somewhat mr- Continuous functions

Definition 3.1. Let (X,γ) and (Z,μ) be any two topological spaces. A function is said to be somewhat mr-continuous if for each $V \in \mu$ and $g^{-1}(V) \neq \emptyset$ there exists a minimal regular open set W in X such that $W \neq \emptyset$ and $W \subset g^{-1}(V)$.

Example 3.1. Let $X = Z = \{1,2,3\}$
$\gamma = \{X,\emptyset,\{2\},\{1,3\}\}$
$\mu = \{X,\emptyset,\{1,3\}\}$
Define $g:(X,\gamma) \to (Z,\mu)$ by $f(1) = 3, f(2) = 2, f(3) = 1$
Here $g^{-1}(\emptyset) = \emptyset, g^{-1}(X) = X, g^{-1}(\{1,3\}) = \{1,3\}$ and $\{1,3\}$ is a minimal regular open set, then g is somewhat mr- continuous.

Theorem 3.1. *Every somewhat* mr-continuous functins are somewhat r −continuous.

Remark: The opposite proposition doesnot holds.

Example 3.2. Let $X = Z = \mathcal{R}, \gamma = \mu =$ usual topology on $\mathcal{R}$.
Define $g:(X,\gamma) \to (Z,\mu)$ by $f(x) = x,\ x \in \mathcal{R}$, then g is somewhat r −continuous function but g is not somewhat mr −continuous function, since $g^{-1}(a,b) = (a,b)$, there doesnot exists minimal regular open set contained in (a,b).

Remark: Minimal regular open set need not be a minimal open set and Minimal open set need not be a minimal regular open set.

Let $X = \{p,q,r,s\}$ with $\gamma = \{X,\emptyset,\{p,q\},\{r,s\}\}$. Here $\{p,q\}$ is a minimal regular open set but not a minimal open.

Theorem3.2. Any function from a discrete space to any other space is somewhat $mr - continuous$.

Proof. Let $g:(X,\gamma) \to (Z,\mu)$ with $\gamma =$ discrete topology and μ be any topology. For each $V \in \mu$ with $g^{-1}(V) \neq \emptyset$. There exist some $x \in g^{-1}(V)$, then $\{x\}$ is a minimal regular open set contained in $g^{-1}(V)$.

Implies that g is a somewhat $mr -$ continuous function.

Definition 3.2. A space X is said to be mr −space if for each $x \in X$ and every r −neighbourhood (regular open set) V of x, there exist a minimal regular open set W such that $W \subset V$ and $x \in W$.

Example 3.3. 1). Every discrete space is $mr - space$.

2). $X = \{p, q, r\}$ with $\gamma = \{X, \emptyset, \{q\}, \{p, r\}\}$, (X, γ) is $mr - space$.

Theorem 3.3. Every finite space is $mr - space$**.**

Theorem 3.4. Every cl-super continuous function in mr-space is somewhat mr −continuous.

Proof. Let $g: (X, \gamma) \to (Z, \mu)$ is $cl - super$ continuous and X is $mr - space$. For each $x \in X$ and open set V containing $g(x)$, there exists a clopen set U containing x such that $g(U) \subset V$. Since X is $mr - space$ and U is regular open, there exist a minimal regular open set W containing x such that $W \subset U$. Then, there exists a minimal regular open set W such that $x \in W$ and $g(W) \subset V$. Then $W \subset g^{-1}(V)$, since $W \subset g^{-1}\big(g(W)\big) \subset g^{-1}(V)$. Implies that g is somewhat $mr - continuous$.

Example 3.4. $X = \{p, q, r, s\}, Z = \{p, q, r\}$

$\gamma = \{X, \emptyset, \{p, q\}, \{p\}, \{q\}, \{p, q, s\}\}$

$\mu = \{ Z, \emptyset, \{p\}, \{q\}\}$

Let $g: (X, \gamma) \to (Z, \mu)$ be the identity function. Then g is somewhat $mr - continuous$, but g is not $cl - super$ continuous.

Definition 3.3. [10] A topological space is locally indiscrete if every open set is closed.

Theorem 3.5. Let $g: (X, \gamma) \to (Z, \mu)$ be somewhat mr −continuous and X is locally in-discrete. Then f is cl −super continuous.

Theorem 3.6. Every completely continuous function in mr-space is somewhat $mr - continuos$.

Proof. Let $g: (X, \gamma) \to (Z, \mu)$ be a completely continuous function. For each $W \in \mu$ with $g^{-1}(W) \neq \emptyset$ is regular open. Since X is $mr - space$, there exist a minimal regular open set V such that $V \subset g^{-1}(W)$. Which implies g is somewhat $mr - continuous$.

Remark: The opposite proposition does not hold.

Example 3.5. Let $X = \{p, q, r\} = Z$ and $\gamma = \{X, \emptyset, \{p\}, \{p, q\}\}, \mu = \{Z, \emptyset, \{p, q\}\}$.

Let $g: (X, \gamma) \to (Z, \mu)$ be the identity function. Then g is somewhat mr −continuous. Since int cl($\{g^{-1}(\{p, q\}) =$ int cl $(\{p, q\}) = intX = X$ and $\{q, p\}$ is not regular. Then g is not completely continuous.

Theorem 3.7. If X is a discrete space and $g: (X, \gamma) \to (Z, \mu)$ is somewhat mr-continuous, then g is completely continuous.

Corollary 3.7.1 Let $g: (X, \gamma) \to (Z, \mu)$is somewhat mr −continuous, where X is finite and T_1, then g is completely continuous.

Theorem 3.8. Every somewhat mr-continuous function is $\delta - continuous$.

Proof. Let $g: (X, \gamma) \to (Z, \mu)$ be somewhat mr −contimuous. Let V be a non-empty regular open set in Z, then it is open. Since g is somewhat mr −continuous, there exists a minimal regular open set U such that $U \subset g^{-1}(V)$. Since every minimal regular open set is regular open, then g is δ −continuous.

Remark: The opposite proposition does not hold.

Example: Let $X = Z = \{p, q, r\}, \gamma = (X, \emptyset, \{p, q\}, \{r\}\}, \mu = \{Z, \emptyset, \{p\}, \{q\}, \{q, r\}\}$.

Define $g: (X, \gamma) \to (Z, \mu)$ by $g(p) = q, g(q) = r, g(r) = p$. Then g is δ −continuous, but not somewhat mr −continuous.

Theorem 3.9. Let W be any non-empty finite regular open set. Then there exists at least one $(finite)minim$al regular open set such that $V \in W$.

Theorem 3.10. Let X is finite, $g: (X, \gamma) \to (Z, \mu)$ is somewhat r −continuous if and only if g is somewhat mr −continuous.

$Proof$. Suppose $g: (X, \gamma) \to (Z, \mu)$ is somewhat $r - continuous$ and X is finite. Let $U \in \mu$, and $g^{-1}(U) \neq \emptyset \in \gamma$. Then there exist a regular open set V such that $V \subset f^{-1}(U)$. Since X is finite, V is a finite regular open set. Then by the Theorem 3.9, there exist a minimal regular open set W such that $W \subset V \subset g^{-1}(U)$. That implies g is somewhat mr −continuous. Converse part is follows from the definition.

Theorem 3.11. If $g: (X, \gamma) \to (Z, \mu)$ is almost completely continuous and Y is locally indiscrete and X is mr-space, then g is somewhat $mr - continuous$.

$Proof$. Let V be open in Z and $g^{-1}(V) \neq \emptyset$. Since Z is locally in-discrete, V is clopen and so is regular open. Since g is almost completely continuous $g^{-1}(V)$ is regular open. Since X is $mr - space$ there exist a minimal regular open set W such that $W \subset g^{-1}(V)$. There for g is somewhat mr −continuous.

Theorem 3.12. If $g: (X, \gamma) \to (Z, \mu)$ is somewhat mr-continuous and X is a discrete space, then g is almost completely continuous.

$Proof$. Let V is a regular open set in Z and $g^{-1}(V) \neq \emptyset$. Since g is somewhat mr −continuous, there exist a minimal regular open set W such that $W \subset g^{-1}(V)$. Since X is discrete $g^{-1}(V)$ is clopen, then it is regular open. Therefore g is almost completely continuous. It is follows from the definition, any function from discrete space is almost completely continuous.

Corollary 3.12.1. $If\ g: (X, \gamma) \to (Z, \mu)$ is somewhat mr −continuous and X is finite and T_1, then f is almost completely continuous.

Theorem 3.13. $g: (X, \gamma) \to (Z, \mu)$ somewhat continuous and X be locally indiscrete and finite, them g is somewhat mr −continuous function.

$Proof$. For each $U \in \mu$ with $g^{-1}(U) \neq \emptyset$ there exists open set $V \neq \emptyset$ and $V \subset g^{-1}(U). Since$ X is locally in-discrete, V is closed. Then, V is regular open. Since X is finite, then V is a nonempty finite regular open set. By the Theorem 3.9 there exist a minimal regular open set W such that $W \subset V$. Implies $W \subset V \subset g^{-1}(U)$, then $W \subset g^{-1}(U)$. Since $U \in \mu$ is arbitrary, therefore g is somewhat mr −continuous function.

Definition 3.4. A topological space is minimal space $(m - space)$ if every open set in X is minimal open.

Theorem 3.14 $g: (X, \gamma) \to (Z, \mu)$ is δ −continuous and open function and Z is m-space, then g is somewhat mr −continuous.

$Proof$. Let $U \in \mu$ and $g^{-1}(U) \neq \emptyset$, since Z is m −space, U is minimal open. Since g is $\delta - continuous$, for each $x \in g^{-1}(U)$, there exist a regular open set V containing x, such that $g(V) \subset U$, implies $V \subset g^{-1}(U), V \neq \emptyset$. If V is minimal regular open set, then g is somewhat mr −continuous. On contrary, suppose that V is not a minimal regular open set, then there exists a nonempty regular open W such that $W \subset g^{-1}(U)$. Then $g(W) \subset U$ and $g(W)$ is open, since g is an open function. Implies $g(W) \neq \emptyset, g(W) \subset U$. Which is a contradiction, since U is a minimal open set.

Theorem 3.15. Every $m - space$ contains at most two proper open set.

$Proof$. Suppose that X contains more than two open sets. Without loss of generality X contains three open sets A, B, C.

Let $D = A \cup B$, open set.
Case I: If $D \neq X$.
Since $D = A \cup B$, then $A \subset D$ and $B \subset D$. Since D is open, then it is minimal open. Which is contradiction.
Case II: If $D = X$
$C \subset A \cup B \Longrightarrow C \cap A \neq \emptyset$ and $C \cap B \neq \emptyset$. $C \cap A$ is open set and $C \cap A \subset C$, $C \cap B$ is open set and $C \cap B \subset C$. Which is a contradiction, since C is minimal open set.
If possible $C \cap B = \emptyset \Longrightarrow C \subset A$. Which is a contradiction, since A is a minimal open. Similarly we get a contradiction if $C \cap A = \emptyset$. Therefore we can conclude that every $m - space$ contains at most two proper open set.

Theorem 3.16. Let $g: (X,\gamma) \to (Y,\mu), g: (Y,\mu) \to (Z,\eta)$ be any two functions. If f is somewhat mr −continuous function, and g is continuous then $g \circ f$ is somewhat mr −continuous.

Proof. Let $U \in \eta$ and $(g \circ f)^{-1}(U) \neq \emptyset$. That is $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$. Since g is continuous $(g)^{-1}(U) \in \mu$ and $(g)^{-1}(U) \neq \emptyset$. Since f is somewhat mr −continuous, there exist a minimal regular open set V in X, $V \neq \emptyset$ such that $V \subset f^{-1}(g^{-1}(U))$. That is $V \subset (g \circ f)^{-1}(U), V \neq \emptyset$ and $V \in \gamma$. Therefore $g \circ f$ is a somewhat mr −continuous function.

Remark: Somewhat mr −continuous function need not be continuous.
$X = Z = \{a,b,c,d\}, \gamma = \{X, \emptyset, \{b\}, \{a,c\}, \{a,b,c\}\}, \mu = \{X, \emptyset, \{a,b\}, \{a,b,c\}\}$.
Define $g: (X,\gamma) \to (Y,\mu)$ by $g(a) = a, g(b) = b, g(c) = d, g(d) = c$.
Here $g^{-1}(\{a,b,c\}) = \{a,b,d\}, g^{-1}(\{a,b\}) = \{a,b\}$ and $\{b\}$ is minimal regular open set in (X,γ). Also $g^{-1}(\{a,b,c\}) = \{a,b,d\}$ and $g^{-1}\{a,b\} = \{a,b\}$ not open in (X,γ). Therefore g is somewhat mr −continuous function, but not continuous.

Theorem 3.17. If $f: (X,\gamma) \to (Y,\mu)$ is continuous $g: Y \to Z$ is somewhat mr −continuous, then $g \circ f: X \to Z$ is somewhat continuous.

Proof. Let $W \subset Z$ be open and $(g \circ f)^{-1}(W) \neq \emptyset, then\ g^{-1}(W) \neq \emptyset$. Since g is somewhat mr −continuous function, there exist minimal regular open set V such that $V \subset g^{-1}(W)$. Since $V \neq \emptyset$ is open and f is continuous, implies $f^{-1}(V) \subset f^{-1}(g^{-1}(W)) = (g \circ f)^{-1}(W)$ and $f^{-1}(V) \neq \emptyset$ is an open set. Therefore $g \circ f: X \to Z$ is somewhat continuous.

Theorem 3.18. Let (X,γ) be a topological space and B be a regular open set of X and W be a minimal regular open subset of $(B, \gamma / B)$, then W is a minimal regular open subset of X.

Proof. Let W is a regular open subset of $(B, \gamma/B)$ and $W = T \cap B, T$ is regular open subset of X and B is a regular open subset of X. Implies W is regular open subset of X.
If possible W is not minimal regular open subset of X, there exist a regular open set S of X such that $S \subset W$, $(S \cap B) \subset (W \cap B) = T \cap B = W$, $S \cap B \subset W$ and $S \cap B$ is a regular open subset of $(B, \gamma/B)$. But W is a minimal regular open subset of $(B, \gamma/B)$, which is a contradiction. Implies that W is a minimal regular open subset of X.

Theorem 3.19. Let $(X,\gamma)\ and\ (Z,\mu)$ be any two topological spaces. Let B be a regular open set of (X,γ) and $g: (X, \gamma/B) \to (Z,\mu)$ be somewhat mr-continuous such that $g(B)$is dense in Z. Then any extension G of g is somewhat $mr - continuous$.

Proof. Let U be any open set in (Z,μ) such that $G^{-1}(U) \neq \emptyset$. Since $g(B) \subset Z$ is dense in $Z, U \cap f(B) \neq \emptyset$. So $g^{-1}(U) \cap B \neq \emptyset$. Since g is somewhat mr −continuous, there exist a minimal regular open set V with respect to γ/B such that $V \subset g^{-1}(U)$. Since V is minimal regular open set with respect to γ/B and B is regular open subset of X, then V is a minimal regular open subset of X with respect to γ and $V \subset G^{-1}(U)$. Implies that G is somewhat mr −continuous.

Theorem 3.20. Let (X,γ) be a topological space and B be a regular open set of X and U be a minimal regular open subset of $(B,\gamma\,/\,B)$, then U is a minimal regular open set of X.

$Proof$. Let U is a regular open subset of $(B,\gamma/B)$ and $U = W \cap B, W$ is regular open subset of X and B is a regular open subset of X. That is, U is regular open subset of X.

If possible U is not minimal regular open subset of X, there exist a regular open set W_1 of X such that $W_1 \neq \emptyset$ and $W_1 \subset U$. Then $W_1 \cap B \subset U \cap B = U$. Since W_1 is regular open set in X, $W_1 \cap B \neq \emptyset$ is regular open set in B. That implies U is not minimal regular open subset of B, which is a contradiction. Therefore U is a minimal regular open subset of X.

Definition 3.5. Let N be a subset of a topological space (X,γ). Then N is said to be mr −dense in X if there does not exist a maximal regular closed set D in X such that $N \subset D \subset X$.

Theorem 3.21. Let X be a topological space and $E \subset X. E$ is a minimal regular closed set if and only if $X - E$ is a maximal regular open.

Theorem 3.22. Let X be a topological space and $T \subset X. T$ is a maximal regular open set if and only if $X - T$ is a maximal regular closed set.

Theorem 3.23. Let (X_1,τ_1) and (X_2,τ_2) be an injective function. Then the following are equivalent.

(1) g is somewhat mr-continuous.

(2) If B is a closed subset of X_2 such that $g^{-1}(B) \neq X_1$, then there is a minimal regular closed subset E of X_1 such that $E \supset g^{-1}(B)$.

(3) If N is an mr-dense in X_1, then $g(N) is\ a\ dense\ subset\ of\ X_2$.

$Proof$. (1) ⟹ (2)

Let B be a closed subset of X_2 such that $g^{-1}(B) \neq X_1$, then $X_2 - B$ is open in X_2, such that $g^{-1}(X_2 - B) = X_1 - g^{-1}(B) \neq \emptyset$. Since g is somewhat mr −continuous, there exist a minimal regular open set W such that $W \subset X_1 - g^{-1}(B) \Longrightarrow g^{-1}(B) \subset X_1 - W$. Since W is a minimal regular open, then $E = X_1 - W$ is maximal regular closed. That is there exists maximal regular closed set E such that $E \supset g^{-1}(B)$.

(2) ⟹ (3)

Let N be a mr −dense subset of X_1. Suppose $g(N)$ is not dense in X_2. Then there exists a proper closed set B in X_2 such that $g(N) \subset B \subset X_2$. Clearly $g^{-1}(B) \neq X_1$. Hence by ii), there exists a maximal regular closed set E such that $E \supset g^{-1}(B)$. That is $N \subset g^{-1}(B) \subset E \subset X_1$. This contradicts the fact that N is mr −dense in X_1. So $g(N)$ is dense in X_2.

(3) ⟹ (2)

Suppose ii) is not true, this means that there exists a closed set B in X_2 such that $g^{-1}(B) \neq X_1$, But there is no maximal regular closed set in E in X_1 such that $g^{-1}(B) \subset E$. This means $g^{-1}(B)$ is mr −dense in X_1. But by iii) $g\left(g^{-1}(B)\right) = B$ must be dense in X_2. Which is contradiction to the choice of B. So ii) is true.

(2) ⟹ (1)

Let $T \in \mu$ and $g^{-1}(T) \neq \emptyset$. Then $X_2 - T$ is closed in X_2 and $g^{-1}(X_2 - U) = X_1 - g^{-1}(T) \neq X_1$. So by ii) there exists a maximal regular closed subset E of X_1 such that $E \supset g^{-1}(X_2 - T) = X_1 - g^{-1}(T)$. That is $X_1 - E \subset g^{-1}(T)$ and $X_1 - E$ is a minimal regular open subset. So g is somewhat mr −continuous function.

Theorem 3.24. Every dense subset in topological space (X,γ) is mr −dense.
$Proof$. Let (X,γ) be any topological space and C be any dense subset of X. Then by the definition, there does not exist a proper closed set V such that $C \subset V \subset X$. That implies there does not exist any maximal regular closed set F satisfying this property $C \subset F \subset X$ since every maximal regular closed set is a closed set. Which implies C is mr −dense in X.
Remark: mr −denseness of a set in a space X does not imply the denseness of that set.
Example 3.6. $X = \{a,b,c,d\}$, $\gamma = \{\emptyset, \{a\}, \{a,b\}, X\}$, $\overline{\{a,b\}} = X$, $\{a,b\}$ is dense.
$\overline{\{a\}} = X$, $\{a\}$ is dense and $\overline{\{b,c\}} = \{b,c,d\}$, $\{b,c\}$ is not dense. But $\{b,c\}$ is mr −dense. Because each closed set is not regular closed, there is no maximal regular closed set containing $\{b,c\}$ and property contained in X.
Theorem 3.25. Let $(X_1,\gamma_1)$and (X_2,γ_2) be any two topological spaces. $X_1 = D_1 \cup D_2$ where D_1 and D_2 are regular open subsets of X_1. Let $g: (X_1,\gamma_1) \to g: (X_2,\gamma_2)$ be a function such that g/D_1 and g/D_2 are somewhat mr-continuous. Then g is a somewhat mr − continuous function
$Proof$. Let V be any open set in (X_2,γ_2) such that $g^{-1}(V) \neq \emptyset$. Then either $(g/D_1)^{-1}(V) \neq \emptyset$ or $(g/D_2)^{-1}(V) \neq \emptyset$.
<u>Case-1:</u> $(g/D_1)^{-1}(V) \neq \emptyset$
Since g/D_1 is somewhat mr −continuous, there exists minimal regular open set W in D_1 such that $W \neq \emptyset$ and $W \subset (g/D_1)^{-1}(V) \subset g^{-1}(V)$. Since W is a minimal regular open set in $D_1 and\ D_1$ is a regular open in X_1, then W is a minimal regular set. So g is somewhat mr −continuous.
<u>Case-2:</u> $(g/D_2)^{-1}(V) \neq \emptyset$.
This can be proved by using the same argument as in case-1
<u>Case-3:</u> $(g/D_1)^{-1}(V) \neq \emptyset$ and $(g/D_2)^{-1}(V) \neq \emptyset$.
The proof follows from the proofs of case-1 and case-2.
Definition 3.6. A topological space (Z,μ) is said to be mr −seperable if there exists a countable subset D of Z which is mr −dense in Z (or if there exists a countable mr −dense subset D of Z).
Example 3.7. Let $Z = \{a,b,c\}, \mu = \{Z, \emptyset, \{a\}, \{a,b\}\}$.
Then $\{a,b\}$ is mr −dense in Z and it is countable. So Z is mr −seperable.
Theorem 3.26. If g is somewhat mr-continuous function from Z_1 onto Z_2 and if Z_1 is mr-seperable, then Z_2 *is seperable*.
$Proof$. Let $g: Z_1 \to Z_2$ be somewhat mr −continuous function such that Z_1 is mr −seperable. Then there exists a countable set B of Z_1 which is mr −dense in Z_1. Then by theorem 3.23 $g(B)$ is dense in Z_2. Since B is countable and g is onto then $g(B)$ is countable, so Z_2 is seperable.
Definition 3.7. If X is a set and τ_1 and τ_2 are topologies for X. Then τ_2 is said to be mr −weakly stronger than τ_1(or τ_1 is mr − weakly weaker than τ_2) provided if $U \in \tau_1$ and $U \neq \emptyset$, then there is a minimal regular open set V in (X,τ_1) such that $V \subset C$.
Example 3.8. $X = \{a,b,c\}, \tau_1 = \{X, \emptyset, \{b,c\}, \{a\}\}$,
$intcl\{a\} = int\{a\} = \{a\}$, $\{a\}$ is minimal regular open
$intcl\{b,c\} = int\{b,c\} = \{b,c\}$, $\{b,c\}$ is a minimal regular open.
$\tau_2 = \{X, \emptyset, \{a,b\}\}$
$intcl\{a,b\} = intX = X$, $\{a,b\}$ is not regular open.
Here τ_1 is mr −weakly stronger than τ_2.

Theorem 3.27. Let τ and $\tau^\star$ be two topologies in X_1 and $\tau^\star$ is mr-weakly stronger than τ, If $g: (X_1, \tau) \to (X_2, \sigma)$ be a somewhat continuous function. Then the function $g: (X_1, \tau^\star) \to (X_2, \sigma)$ is somewhat $mr - continuous$.

$Proof$. Let V be any open set in X_2 such that $g^{-1}(V) \neq \emptyset$. Since g is somewhat continuous, there exists an open set W in X_1 such that $W \neq \emptyset$ and $W \subset g^{-1}(V)$

Since $\tau^\star$ is mr −stronger than τ, there exists a minimal regular open W_1 in $(X_1, \tau^\star)$ such that $W_1 \subset W$. That is $W_1 \subset W \subset g^{-1}(V)$. So g is somewhat mr −continuous.

Theorem 3.28. Let $g: (X_1, \tau) \to (X_2, \sigma)$ be a somewhat mr-continuous and onto function. Let $\sigma^\star$ be another topology for X_2 and σ is stronger than $\sigma^\star$, then $g: (X_1,, \tau) \to (X_2, \sigma^\star)$ is somewhat $mr - continuous$.

$Proof$. Let V_1be an open set in $(Y, \sigma^\star)$ with $g^{-1}(V_1) \neq \emptyset$, then $V_1 \neq \emptyset$. Since σ is stronger than $\sigma^\star$, there exists an open set V_2 in (Y, σ) such that $V_2 \subset V_1$ and $V_2 \neq \emptyset$. Since g is onto, then $g^{-1}(V_2) \neq \emptyset$. Since $g: (X_1, \tau) \to (X_2, \sigma)$ is a somewhat mr −continuous function, then there exists a minimal regular open set W such that $W \subset g^{-1}(V_2)$ and $g^{-1}(V_2) \subset g^{-1}(V_1)$. That is $W \subset g^{-1}(V_1)$ and W is a minimal regular open set. Therefore $g: (X_1, \tau) \to (X_2, \sigma^\star)$ is a somewhat mr −continuous.

4. Somewhat mr-open functions

Definition 4.1. A function $g: (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be somewhat mr −open provided for $V_1 \in \tau_1$ and $V_1 \neq \emptyset$, there exists a minimal ramge open set V_2 in X_2 such that $V_2 \subset g(V_1)$.

Example 4.1. Let $X_1 = X_2 = \{a, b, c\}, \tau_1 = \{X_1, \emptyset,, \{b\}\}, \tau_2 = \{X_2, \emptyset, \{a\}, \{c\}, \{a, c\}\}$.

Define a function $g: (X_1, \tau_1) \to (X_2, \tau_2)$ by $g(a) = b, g(b) = c, g(c) = a$. Then g is somewhat mr −open function.

Definition 4.2. (1). A function $g: (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be somewhat clopen provided for $V_1 \in \tau_1$ and $V_1 \neq \emptyset$, there exists a clopen set V_2 in X_2 and $V_2 \neq \emptyset$ such that $V_2 \subset g(V_1)$.

Theorem 4.1. Let $g: (X_1, \tau_1) \to (X_2, \tau_2)$ be a somewhat clopen function and X_2 is finite then g is somewhat $mr - open$.

$Proof$. Let $g: (X_1, \tau_1) \to (X_2, \tau_2)$ be a somewhat clopen function and let $V_1 \in \tau_1$ and $V_1 \neq \emptyset$. Since g is somewhat clopen, then there exist a clopen set V_2 such that $V_2 \subset g(V_1)$. Then V_2 is regular open set. Since X_2 is finite, then V_2 is finite regular open set. Then by Theorem 3.9, there exist a minimal regular open set W such that $W \subset V_2 \subset g(V_1)$. Implies $W \subset g(V_1)\, and\, W \neq \emptyset$. Therefore g is somewhat mr −open function.

Theorem 4.2. If $g: X_1 \to X_2$ is somewhat mr-open, where X_1 is locally indiscrete, then $g\, is\, somewhat\, clopen$.

$Proof.\, Let\, V_1 \neq \emptyset$ be open in X_1. Since g is somewhat mr −open, there exists a minimal regular open set V_2 in X_1 such that $V_2 \subset g(V_1)$. But minimal regular open set is open and open set in a locally indiscrete space is a clopen. So g is somewhat clopen.

Definition 4.3. (1) A function $g: (X_1, \tau_1) \to (X_2, \tau_2)$ is said to be somewhat open provided for $V_1 \in \tau_1$ and $V_1 \neq \emptyset$, there exists an open set V_2 in X_2 such that $V_2 \neq \emptyset$ and $V_2 \subset g(V_1)$.

Theorem 4.3. If $g: X_1 \to X_2$ is somewhat mr-open, then g is somewhat open.

Remark: The opposite proposition does not hold.

Example 4.2. $X_1 = X_2 = \{a, b, c\}, \tau_1 = (X_1, \emptyset, \{a\}, \{a, b\}\}, \tau_2 = \{X_2, \emptyset, \{b\}\}$.

Define $g:(X_1,\tau_1)\to(X_2,\tau_2)$ by $g(a)=b,\ g(b)=c, g(c)=a$ and $g(\{a\})=\{b\},\ g(\{a,b\})=\{b,c\}$. Therefore g is somewhat open. Also $intcl(\{b\})=intX_1=X_1$, then $\{b\}$ is not regular open. But g is not somewhat mr −open function.

Theorem 4.4. If $g:(X_1,\tau_1)\to(X_2,\tau_2)$ is an open map and $h:(X_2,\tau_2)\to(X_3,\tau_3)$ is a somewhat mr-open map, then $h\circ g:(X_1,\tau_1)\to(X_3,\tau_3)$ is a somewhat mr −open map.

Proof. Let $V_1\in\tau_1$ and $V_1\neq\emptyset$, since g is an open map, $g(V_1)$ is open. Also $g(V_1)\neq\emptyset$. Since h is somewhat mr −open and $g(V_1)\in\tau_2$ with $g(V_1)\neq\emptyset$, there exists a minimal regular open set V_2 in τ_3 such that $V_2\subset h\big(g(V_1)\big)$, ie, $V_2\subset(h\circ g)(V_1)$. So $h\circ g$ is somewhat mr −open.

Theorem 4.5. If $g:(X_1,\tau_1)\to(X_2,\tau_2)$ is a one-one and onto mapping, then the following are equivalent.

(1) g is somewhat mr-open map.

(2) If E is a closed subset of X_1 such that $g(E)\neq X_2$,then there is a maximal regular closed subset F of X_2 such that $F\neq X_2$ *and* $F\supset g(E)$.

Proof. (1) $\Longrightarrow$ (2)

Let Ebe a closed subset of X_1 such that $g(E)\neq X_2$. Then X_1-E is open in X_1 and $X_1-E\neq\emptyset$. Since g is somewhat mr −open there exist a minimal regular open set $U\neq\emptyset$ such that $U\subset g(X_1-E)$. Let $F=X_2-U$, clearly F is maximal regular closed set in X_2.

We claim that $F\neq X_2$. If possible $F=X_2$, then $U=\emptyset$, a contradiction.

$$U\subset g(X_1-E)\Longrightarrow X_2-U\supset X_2-g(X_1-E)\Longrightarrow F=X_2-U\supset g(E)\Longrightarrow F\supset g(E)$$

(2) $\Longrightarrow$ (1)

Let $U\in\tau_1$ and $U\neq\emptyset$ and let $E=X_1-U$. Then E is a closed subset of X_1 and $g(E)=g(X_1-U)=X_2-g(U)$. Also $g(E)\neq X_2$. So by (2), there exists aa maximal regular closed subset F of X_2 such that $F\neq X_2$ and $F\supset g(E)$. Let $W=X_2-F$, then W is minimal regular open and $W\neq\emptyset$. Also $W=(X_2-F)\subset\big(X_2-g(E)\big)=X_2-(X_2-g(U))$. Implies $W\subset g(U)$. So g is somewhat mr −open.

Theorem 4.6. Let $g:(X_1,\tau_1)\to(X_2,\tau_2)$ is a somewhat mr-open function. Let B be any open subset X_1. Then $g/B:(B,\tau_1/B)\to(X_2,\tau_2)$ is also Somewhat $mr-open$.

Proof. Let $U_1\in\tau_1/B$ and $U_1\neq\emptyset$. Since U_1 is open in B and B is open in X_1, U_1 is open in X_1, Since $g:(X_1,\tau_1)\to(X_2,\tau_2)$ is somewhat mr −open, there exists a minimal regular open set U_2 in X_2 such that $U_2\subset g(U_1)$. Thus for any open set U_1 in τ_1/B with $U_1\neq\emptyset$, there exists minimal regular open set U_2 in X_2 such that $U_2\subset(g/B(U_1))$. So g/B is somewhat mr −open.

Theorem 4.7. Let (X_1,τ_1) and (X_2,τ_2) be any two topological space and $X_1=B_1\cup B_2$, where B_1 and B_2 are open subsets of X_1. Let $g:(X_1,\tau_{1)}\to(X_2,\tau_2)$ be a function such that g/B_1 and g/B_2 are somewhat mr-open. Then g is also somewhat $mr-open$.

Proof. Let $U_1\in\tau_1$ and $U_1\neq\emptyset$. Then either $(g/B_1)(U_1)\neq\emptyset$ or $(g/B_2)(U_1)\neq\emptyset$ or both $(g/B_1)(U_1)\neq\emptyset$ and $(g/B_2)(U_1)\neq\emptyset$.

Case-1: $(g/B_1)(U_1)\neq\emptyset$

Since g/B_1 is a somewhat mr −open, there exists a minimal regular open set U_2 in B_1 such that $U_2\subset(g/B_1)(U_1)\subset g(U_1)$. Since U_2 is a minimal regular open in B_1 and B_1 is a regular open in X_1, then U_2 is a minimal regular open set in X_1. So g is somewhat mr −open function.

Case-2: $(g/B_2)(U_1)\neq\emptyset$

This can be proved by the same argument as in case-1.

Case-3: $(g/B_1)(U_1)\neq\emptyset$ and $(g/B_2)(U_1)\neq\emptyset$

$B_1\subseteq X_{1.}$ U_2 is minimal regular open in X_1, then U_2 is minimal regular open in X_1.

Suppose U_2 is not minimal regular open set in X_1,then there exist a regular open set U_1 in X_1 such that $U_1 \subset U_2, U_1 \cap B_1 \subset U_2 \cap B_1 = U_2 \Longrightarrow U_1 \cap B_1 \subset U_2$.
The proofs follow from the proofs of case-1 and case-2.

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