International Journal for Multidisciplinary Research (IJFMR)

E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

The Extended Infinite Geometric Series Theorem for 10-adic Number Conversion

Shllok Rawat

Student, IBDP, Hill Spring International School

Abstract

This paper looks at an extension of the infinite geometric series, exploring its applications in the 10-adic number system. This paper aims to prove that the conventional sum-to-infinity formula,

$$S_\infty = \frac{a}{1-r}$$

can be used to quantify 10-adic numbers. This study acts as a mathematical bridge between the decimal and p-adic worlds, aiming to prove that the infinite geometric series formula can be used while converting from 10-adic to decimal numbers, when $r = 10^n$ and where $n \in Z^+$.

1. The Extended Infinite Geometric Series Theorem for 10-adic Number Conversion

The quantifiable decimal value of a 10-adic number can be denoted by the formula:

$$S_\infty = \frac{a}{1-10^n}$$

Where $n \in Z^+$.

2. Introduction

This mathematical theorem aims to bring a new perspective to the infinite geometric series formula, proving that it is not only valid for converging numbers but can also hold validity when it comes to numbers exhibiting a cascading effect. This paper aims to prove the theorem's validity as well as shining light on the unique characteristics of the 10-adic number system.

3. Definitions

- Infinite Geometric Series[4]: The sum of an infinite number of terms where the terms form a geometric series, i.e. each term after the first is found by multiplying the previous term with a fixed non zero number, referred to as the common ratio(r)
- Common ratio of an infinite geometric series (r)[3]: A fixed non-zero number acting as a multiplier between consecutive terms of a sequence.
- Decimal number system[1]: A base-10 number system using digits 0 through 9, with each digit's position represented by a power of 10.
- P-adic numbers:[5]A family of number systems indexed by a prime number 'p'.
- 10-adic numbers: A specific class of p-adic numbers where the index number 'p' is 10. 10-adic numbers exhibit properties not found in the conventional number system, such as digit recurrence to the right of the decimal place.

- Quantifiable values (in 10-adic numbers): The ability to assign a finite numeric value to an infinite repeating decimal representation.

4. Formulae

1. The infinite geometric series formula:[2]

$$S_\infty = \frac{a}{1-r} \; for \; |r| < 1$$

2. The extension to the infinite geometric series formula

$$S_\infty = \frac{a}{1-10^{\mathrm{n}}} \; where \; \mathrm{n} \in \mathrm{Z}^{+}$$

5. Mathematical Notation

- Infinite geometric series: S_∞
- First term of a geometric series: a
- Common ratio of a geometric series: r

6. Characteristics of 10-adic Numbers

10-adic numbers have unique characteristics that allow them to exist outside the axioms of mathematics. One such characteristic is the recurrence of digits to the left of the decimal point rather than the right. This property allows the formation of numbers such as ...9999, referring to the number 9 recurring infinitely.

7. Quantifiable Value of a 10-adic Number

Due to the interesting nature of 10-adic numbers, they allow for an infinite number to have a quantifiable integer value. To illustrate, let a 10-adic number ...9999 = x:

$$x = \ldots 9999$$

Therefore, 10x = ...9990. Subtracting x − 10x:

$$x - 10x = \cdots 9999 - \ldots 9990$$

Since all the recurring nines cancel out:

$$-9x = 9$$

Therefore, $x = -1$. This example explains the quantifiable nature of 10-adic numbers, letting an infinite number have a finite value.

8. Alternative Calculation Using the Infinite Geometric Series Formula

An alternative method suggested by this theorem is using the infinite geometric series formula to determine the quantifiable decimal value of a 10-adic number. Taking the same example, if we rewrite the 10-adic number ...9999 as an infinite geometric series:

$$a(u_1) = 9$$

$$r = 10$$

$$S_\infty = \frac{a}{1-r}$$

Therefore, $S_\infty = \frac{9}{1-10} = -1.9 = -1$. This example illustrates that the infinite geometric series can be used to give quantifiable values to 10-adic numbers.

9. Proof

To establish the validity of this theorem, a mathematical proof is needed that emphasizes the versatility and usability of this formula.

9.1 Analysis of the Cascading Effect

To prove that this theorem holds validity, the analysis of the cascading effect that these infinite numbers possess is needed. For every term in the sequence, when ‘a’ is multiplied by 10^n, each digit of ‘a’ undergoes a left shift by ‘n’ places. This results in a cascading effect wherein every digit of ‘a’ has an influence over two different places: its original position and the position shifted by ‘n’ places.

For example, the first digit of a (a_0) will influence the first and the nth digit of the number, and every nth digit after that will also be influenced by the a_0 term.

However, this only holds true if n is a positive integer, as the numbers will not have a cascading effect unless they are multiplied by a multiple of 10.

Using this logic of recursion, let’s create a mathematical proof for the theorem:

$$S_\infty = a + a \cdot 10^n + a \cdot 10^{2n} + a \cdot 10^{3n} + \ldots$$

Therefore, $10^n \cdot S_\infty = a + a \cdot 10^n + a \cdot 10^{2n} + a \cdot 10^{3n} + \ldots$

If you subtract $10^n \cdot S_\infty$ from S_∞, your answer should be a, as all the cascading digits will cancel out:

$$(1 - 10^n) \cdot S_\infty = (a + a \cdot 10^n + a \cdot 10^{2n} + \ldots) - (a \cdot 10^n + a \cdot 10^{2n} + \cdots)$$

Therefore, $(1 - 10^n) \cdot S_\infty = a$, and $S_\infty = \frac{a}{1-10^n}$

Compare with the formula for the infinite geometric series:

$$S_\infty = \frac{a}{1-r}$$

Therefore, $r = 10^n$

10. Extended Example 1

To further illustrate this theorem’s validity, let’s consider the following equation:

$$a = 1234, \qquad n = 2$$

$$S_\infty = \frac{1234}{1 - 10^2}$$

Therefore, $S_\infty = -\frac{1234}{99}$,

To compare it with the actual 10-adic representation of the geometric series:

$$S_\infty = 1234 + 123400 + 12340000 + \ldots$$

$$S_\infty = ...464634$$

Therefore, $100 \cdot S_\infty = ...46463400$

$$(1-100)S_\infty = ...46463400 - ...464634$$

$$\therefore (1-100)S_\infty = 1234$$

$$S_\infty = -\frac{1234}{99}$$

11. Extended Example 2

To further illustrate this theorem's validity, let's consider the following equation:

$$a = 2, \qquad n = 2$$

$$S_\infty = \frac{2}{1 - 10^2}$$

Therefore, $S_\infty = -\frac{2}{99}$

To compare it with the actual 10-adic representation of the geometric series:

$$S_\infty = 2 + 200 + 20000 + \ ...$$

$$S_\infty = \ ...20202$$

Therefore, $100 \cdot S_\infty = \ ...20200$

$$\therefore (1-100)S_\infty = \ ...20200 - \ ...20202$$

$$\therefore (1-100)S_\infty = 2$$

Therefore, $S_\infty = -\frac{2}{99}$

12. Conclusion

This extension of the infinite geometric series helps act as a bridge between the p-adic and decimal number systems. It helps compute these large seemingly infinite 10-adic numbers into finite decimal solutions. This theorem's proof resembles the well-known mathematical paradox where 0.999... = 1. This connection lies in the principles of the cascading effect. In both these cases, the infinitely recurring digits create a pattern of convergence.

This theorem acts as a unique outlook on the interplay of number systems, proposing a unified perspective on the idea of convergence among number systems. This theorem can help enhance calculations with 10-

adic numbers, with the potential to increase the efficiency of 10-adic calculations in cryptography, algorithm development, and computation.

Not only does this theorem have implications in the world of mathematics, but it can also expand the computations in the physical realm, especially in systems where non-standard number systems are widely used. This theorem's ability to redefine the way we interpret patterns of convergence could lead to the creation of new mathematical frameworks, spearheading advancements in both science and engineering.

13. References

1. BYJUS. "Decimal Number System (Definition, Conversions & Examples)," n.d. https://byjus.com/maths/decimal-number-system/.
2. Cuemath. "Infinite Geometric Series Formula – Learn the Formula of Infinite Geometric Series," https://www.cuemath.com/infinite-geometric-series-formula/. n.d.
3. Study.com. "Common Ratio: Definition & Concept - Video & Lesson Transcript Study.com," 2019. https://study.com/academy/lesson/common-ratio-definition-lesson quiz.html.
4. Varsitytutors.com. "Infinite Geometric Series," 2016. https://www.varsitytutors.com/hotmath/hotmath help/topics/infinite-geometric-series.
5. Weisstein, Eric W. "P-Adic Number." mathworld.wolfram.com, n.d. https://mathworld.wolfram.com/p-adicNumber.html.