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Fuzzy Graphs: An Overview

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Abstract

An extension of traditional graph theory, fuzzy graphs are made to deal with ambiguous or imprecise data. These graphs have many uses in the social sciences, network research, and decision-making because they incorporate the idea of fuzziness. The basic definitions, characteristics, and kinds of fuzzy graphs are examined in this article. Additionally, it talks about how they are used in a variety of fields, such as artificial intelligence, communication networks, and transportation.

Keywords: Fuzzy graph

1. Introduction

There are a number of intriguing uses for graph theory in economics, operations research, and system analysis. It is convenient to handle the characteristics of graph problems using fuzzy logic techniques because these aspects are frequently unknown. In his seminal work "Fuzzy sets" from 1965, Zadeh invented the idea of fuzzy relations, which are widely used in pattern recognition. Rosenfeld initially presented fuzzy graphs and a number of fuzzy analogues of graph theoretic ideas in 1975. Fuzzy graph theory is therefore finding more and more uses in the modelling of real-time systems where the amount of information present in the system varies with varying degrees of accuracy.

Binary relationships are the basis of classical graph theory; an edge between two vertices either exists or does not. However, uncertainty, ambiguity, or imprecise relationships are present in many real-world issues. By giving vertices and edges degrees of membership, fuzzy graphs solve these problems and adhere to the fuzzy set theory outlined by Lotfi A. Zadeh in 1965.

The work of Zadeh [2], which addressed the idea of partial truth between absolute true and absolute false, is credited with giving rise to a fuzzy set as a superset of a crisp set. Zadeh's brilliant concept has numerous uses in a variety of domains, such as computer science, discrete mathematics, networking, decision-making, communications, and the chemical industry. The idea of a fuzzy subgroup of a group was first introduced by Rosenfeld using the idea of a fuzzy subset of a set. Fuzzy abstract algebra was first developed as a result of Rosenfeld's paper [3].

A fuzzy equivalent of certain graph theoretical concepts was described by Rosenfeld [4]. Afterwards, Bhattacharya [5] offered some observations regarding fuzzy graphs. This class was studied by T. AL-Hawary et.al in [6,7]. Certain properties of conjunction of fuzzy graphs, regular fuzzy graphs, certain sequences in fuzzy graphs, and the degree of vertex in various fuzzy graphs were described by Nagoor Gani and Radha [8,9]. Some fuzzy graphs and operations on fuzzy graphs were defined by Mordeson and Chang-Shyh [10].

2. Definition and Properties

FuzzySet: A fuzzy set A is defined as $A=\{(x,\mu_A(x))|x\in X,\mu_A(x)\in[0,1]\}$, where $\mu_A(x)$ represents the membership degree of x in A.

Fuzzy Graph: A fuzzy graph G is a pair G=(V,σ,μ) where V is the set of vertices.
σ:V→[0,1] assigns a membership degree to each vertex.
μ:V×V→[0,1] assigns a membership degree to each edge.

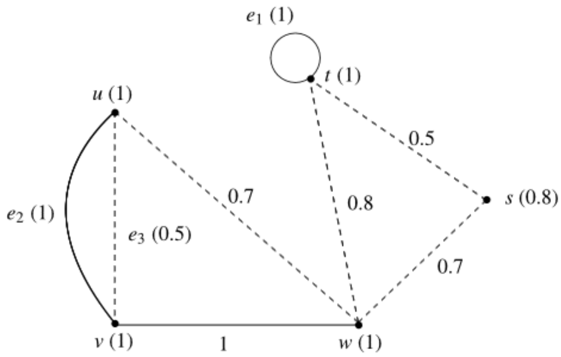

Fig:1 A Fuzzy graph

Edge irregular Fuzzy graph : Let ˜G = (σ, μ) be a connected fuzzy graph on G = (V, E). Then ˜G is said to be a strongly edge irregular fuzzy graph if every pair of edges having distinct degrees (or) no two edges have same degree

Fuzzy Subgraphs: A fuzzy subgraph is defined by reducing the membership degrees of vertices and edges of the parent fuzzy graph.

Complement: The complement of a fuzzy graph is derived by reversing the membership degrees, highlighting what is absent in the original graph.

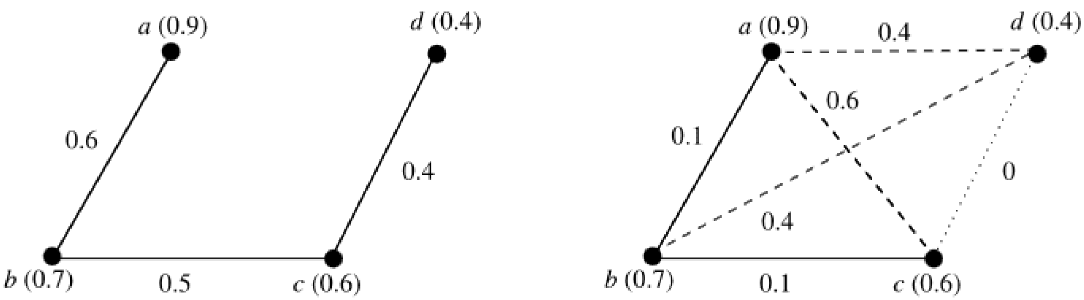

Fig: 2 Fuzzy graph and its complement

Operations: Union, intersection, and Cartesian products are extended from classical graph theory, with membership functions combined using fuzzy set operations.

Fuzzy graphs are very flexible when modelling complicated systems because of these characteristics. Many of the characteristics of classical graphs are retained by fuzzy graphs, which expand them to account for fuzziness. linkages in a fuzzy graph. Sandeep Narayanan et al. demonstrated that if a fuzzy graph without m-strong arcs is linked, then so is its complement. Additionally, it was demonstrated that if a given graph contains at least one connected spanning fuzzy subgraph devoid of any m-strong arcs, then the graph and its complement are linked.

Mcallister [11] demonstrated that the intersection of two fuzzy graphs is itself a fuzzy graph. Radha et

al. provided pictures to describe the degree of edges in fuzzy graph unions and joins. Nagoor Gani et al. explored the degree of a vertex in composition and Cartesian product. Bhattacharya et al. developed a fuzzy graph approach to determine a map's maximum and minimum powers. Moderson et al. established the necessary and sufficient conditions for a graph to be a Cartesian product of two fuzzy subgraphs or a union of two fuzzy subgraphs.
Moderson et al.[12] established a necessary and sufficient condition for a graph to be a Cartesian product of two fuzzy subgraphs. Additionally, the union of two fuzzy subgraphs of a graph results in another fuzzy subgraph. Nair mentioned a few characteristics of complete fuzzy graphs and fuzzy trees. He demonstrated triangle and parallelogram rules, as well as the lack of equivalence between bridges in fuzzy graphs. Sunitha et al. investigated fuzzy bridge and cut node features and used them to characterise fuzzy trees and cut nodes. Nagoor et al. demonstrated the order-size inequality in fuzzy graphs. Mordeson defined the complement of fuzzy graphs.

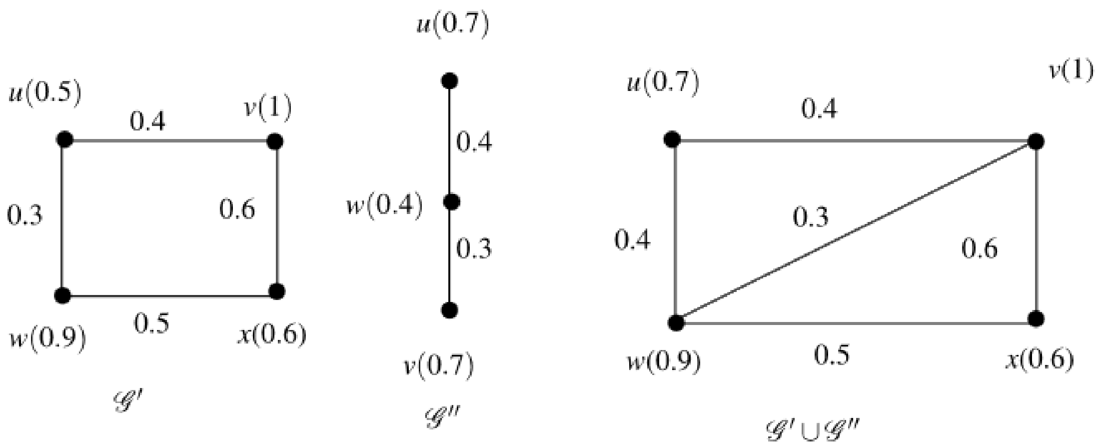

Fig: 3 Union of two Fuzzy graphs

Arindam Dey et al. presented a program that calculates the fuzzy complement of a fuzzy graph.Nagoor Gani and Chandrasekaran defined the μ-complement of fuzzy graphs. Nagoor Gani et al. explored the properties of μ-complements in fuzzy graphs. A strong fuzzy graph is required for the μ-complement to have isolated nodes, according to their findings.
Mini Tom et al. demonstrated that if there are at least two internally disjoint strongest u v pathways, the fuzzy graph meeting the requirements that either (u, v) is a δ-arc or μ(u, v) = 0 is a block. Additionally, they demonstrated that the fuzzy graph G without δ-arc is a block if the underlying graph G is a complete graph. A fuzzy graph that is a cycle is a fuzzy cycle if it is not a fuzzy tree, as demonstrated by Mordeson et al. Assuming that the dimension of the cycle space of the underlying graph (σ, μ) is unity, they also demonstrated that the fuzzy graph (σ, μ) lacks a fuzzy bridge if it is a cycle and μ is a constant function.
Moderson proposed a required and sufficient condition for a fuzzy graph to be a fuzzy line graph. Craine examined the properties of fuzzy interval graphs. Naga Maruthi Kumari et al. demonstrated that the combination of two strong interval valued fuzzy graphs results in a strong interval valued fuzzy graph. Hossein Rashmanlou et al. demonstrated that the semi-strong and strong products of two interval-valued fuzzy graphs are complete. Akram proposed that interval-valued fuzzy graphs are equivalent to interval-valued fuzzy intersection graphs. Sen et al. demonstrated that a fuzzy intersection graph is chordal when a, b, c, and d in the semigroup have a right common multiple.

We can associate a group with a fuzzy graph as an automorphism group, according to Bhattacharya's fuzzy analogy of fuzzy graph theory, which he derived from graph theory. Bhutani presented the idea of fuzzy graph isomorphism. He demonstrated that each fuzzy group has an embedding in the fuzzy group of a fuzzy graph's group of automorphisms. Let (σ1, μ1) and (σ2, μ2) be fuzzy subgraphs of the corresponding graphs (σ1, μ1) and (σ2, μ2). Any weak isomorphism of (σ1, μ1) onto (σ2, μ2) is, in turn, an isomorphism of (σ1, μ1) onto (σ2, μ2), as demonstrated by Mordeson[12].

Eslahchi et al. introduced fuzzy coloring of a fuzzy graph. Arindam Dey et.al introduced an algorithm with an illustration to color the complement of a fuzzy graph through α-cuts by considering three cases. Nivethana et al. used the executive committee problem as an example to determine the chromatic number of a fuzzy network. Ananthanarayanan et al. illustrated how to find the chromatic number of a fuzzy graph using α-cuts for crisp vertices and fuzzy edges. Sameena proposed a technique for creating ϵ-clusters with strong arcs and provided images for $0 \leq \epsilon < 1$.

Akram et al.[13] explored certain metric properties of intuitionistic fuzzy graphs. In an intuitionistic fuzzy graph, Nagoor Gani et al. discovered that the sum of the degree of membership value of all vertices is twice the sum of the membership value of all edges. Similarly, the sum of the degree of non-membership value of all vertices is two times the sum of the non-membership value of all edges. Karunambigai et al.[14] identified three instances where a strong path is the strongest path in intuitionistic fuzzy graphs. Akram et al. demonstrated that combining two strong intuitionistic fuzzy graphs results in another strong intuitionistic fuzzy graph. Karunambigai et al.[15] demonstrated that two isomorphic intuitionistic fuzzy graphs have the same order and size. He demonstrated that all complete intuitionistic fuzzy graphs are balanced.

3. Applications

Transportation Networks: Fuzzy graphs are used to simulate uncertainty in traffic, connectivity, and trip durations. Travel times, connectivity, and traffic flow are all naturally unpredictable in transportation networks. These uncertainties are modelled using fuzzy graphs, which help with congestion control and route optimisation.

Social Networks: Represent relationships where connections have varying strengths or probabilities.The strength and dependability of relationships in social networks vary. By quantifying these variances, fuzzy graphs give analysts information on community structure, influence, and trust.

Decision-Making: Used to assess options under uncertainty in situations involving multi-criteria decision-making. Evaluating options in the face of uncertainty is a common component of multi-criteria decision-making. A computational and visual framework for option selection and ranking is offered by fuzzy graphs.

Artificial Intelligence: Applied in machine learning for clustering, pattern recognition, and knowledge representation.In fields where imprecision is crucial, such as clustering, knowledge representation, and natural language processing, fuzzy graphs are used.

4. Challenges

Despite their promise, fuzzy graphs have a number of drawbacks.

Complexity: Adding membership functions makes computations more difficult, particularly in large-scale networks.

Standardization: Inconsistent outcomes may arise from the absence of standardized techniques for esta-

blishing membership functions.
Integration: More research is needed to combine fuzzy graphs with alternative uncertainty models, such as probabilistic or rough set theories.
Unlocking fuzzy graphs' full potential requires addressing these obstacles.

5. Scope for further research

Fuzzy Graph Algorithms: One direction for further research is to develop algorithms for shortest paths, spanning trees, and network flows in fuzzy contexts. A fuzzy graph algorithm works with fuzzy graphs, which are mathematical constructs used to express ambiguous or imprecise relationships. Fuzzy graphs are an extension of classical graphs in which each edge and/or vertex is assigned a membership value in the [0,1] range, indicating the level of association or certainty.
Fuzzy Hypergraphs: Another area of further scope is generalization of fuzzy graphs where edges can connect more than two vertices.A fuzzy hypergraph is an extension of a fuzzy graph that generalizes the concept of edges to include subsets of vertices, called hyperedges, where both vertices and hyperedges are associated with membership values. Fuzzy hypergraphs are particularly useful for modeling complex relationships and interactions in systems where uncertainty or vagueness is present.
Consider a fuzzy hypergraph H=(V,E,μ), where: V is the set of vertices, E is the set of hyperedges, each hyperedge connecting multiple vertices, μ is the membership function for vertices and hyperedges, assigning a value in [0,1] to indicate the degree of membership.
Example Scenario: Research Collaboration Vertices (V): Researchers involved in a project: V={A,B,C,D,E}.Each researcher has a membership value indicating their expertise relevance to the project. Hyperedges (EE): Groups of researchers collaborating on specific tasks: E={e1,e2,e3}
e1={A,B,C}: Literature Review, e2={B,C,D}: Data Collection, e3={A,D,E}: Data Analysis
Membership Function (μ):Assigns a membership degree to each vertex and hyperedge:
Vertices:
μ(A)=0.9: Researcher AA is highly relevant.
μ(B)=0.8: Researcher BB is moderately relevant.
μ(C)=0.7: Researcher CC is moderately relevant.
μ(D)=0.6: Researcher DD has partial relevance.
μ(E)=0.5: Researcher EE has lower relevance.
Hyperedges:
μ(e1)=0.85: Literature review team is highly cohesive.
μ(e2)=0.75: Data collection team is moderately cohesive.
μ(e3)=0.65: Data analysis team is somewhat cohesive.
Fuzzy Hypergraph Representation
Vertices and Membership Values:
V={A(0.9),B(0.8),C(0.7),D(0.6),E(0.5)}
Hyperedges and Membership Values:
E={e1({A(0.9),B(0.8),C(0.7)},0.85),e2({B(0.8),C(0.7),D(0.6)},0.75),e3({A(0.9),D(0.6),E(0.5)},0.65)}
Intuitionistic Fuzzy Graphs: Yet another idea is extension incorporating both membership and non-membership functions.An intuitionistic fuzzy graph (IFG) is an extension of fuzzy graphs that incorporates two membership functions: a membership degree and a non-membership degree, for both vertices and edges. These values collectively provide a richer representation of uncertainty by not only

describing how strongly an element belongs to a set but also how strongly it does not belong. The difference between the membership and non-membership degrees allows for additional flexibility in modeling hesitation or indeterminacy.

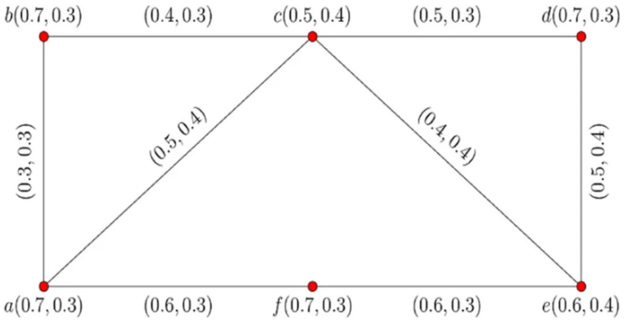

Fig: 4 Example of an Intutionistic Fuzzy Graph

Fuzzy graph research's future depends on how well it integrates with other fields and technologies:
Quantum Computing: Since uncertainty is essential in quantum networks, fuzzy graphs may be useful for modelling them.
Bioinformatics: Fuzzy graphs are a perfect fit for modelling biological systems, such as gene networks and protein interactions, which frequently involve managing ambiguous input.
Big Data: One interesting area for advancement is the capacity of fuzzy network algorithms to scale to handle large datasets.
As processing power and mathematical skills advance, fuzzy graphs are projected to become even more crucial in tackling complicated, uncertain systems.

6. Conclusion

A strong foundation for examining systems with ambiguous relationships is offered by fuzzy graphs. Computational methods are becoming more and more applicable as they develop, providing potential answers to challenging real-world issues.A major development in graph theory, fuzzy graphs allow practitioners and researchers to efficiently model uncertainty. They close the gap between theoretical accuracy and practical ambiguity by expanding the traditional framework to include degrees of membership. With their many uses and increasing significance in contemporary science and technology, fuzzy graphs remain an essential tool for comprehending and resolving challenging issues. They are positioned to open up new possibilities and insights in a variety of fields as research advances.

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