Study of Heat Transfer in An Incompressible Electrically Conducting Non-Newtonian Fluid Over: A Linear Horizontal Stretching Sheet

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Abstract

This paper concerns with a steady two-dimensional flow of an electrically conducting non newtoinan fluid over a heated non linear stretching sheet. The flow is permeated by uniform transverse magnetic field the fluid viscosity is assumed to vary as a linear function of temperature. The effects of free convection and internal heat generation or absorption are also considered. Variable fluid properties flow and temperature dependent heat source/sink render the problem intractable. The shooting method is used to solve boundary value problems.

Keywords: stream function, stretching sheet, partial differential equations, shooting method.

Introduction

The study of laminar boundary layer flow and heat transfer in a non-Newtonian fluid over a stretching sheet, issuing from a slit, has gained tremendous interest in the past three decades. A great number of investigations concern the boundary layer behavior on a stretching surface and this is important in many engineering and industrial applications. Flow due to stretching sheet is often encountered in extrusion processes where a melt is stretched into a cooling liquid. Apart from this, many metallurgical processes including chemical engineering processes involve cooling of continuous stripes or filaments by drawing them into a cooling system. The fluid mechanical properties desired for the outcome of such a process would mainly depend on the rate of cooling and stretching rate. So, one has to pay considerable attention in knowing the heat transfer characteristics of the stretching sheet as well.

Mathematical Formulation of the Problem

The steady two-dimensional flow of an incompressible, electrically conducting, non-Newtonian liquid over a non-linear stretching sheet. The flow is generated by the action of two equal and opposite forces along the x-axis and the sheet is stretched with a velocity that is proportional to the distance from the origin. Two different types of sheet velocities $u_w(x)$ are considered, namely linear and quadratic velocities, Further, the sheet is assumed to warmer than the ambient liquid, i.e., $T_w(x) > T_\infty$.

The boundary layer equations governing the flow and heat transfer in a non Newtonian liquid over a horizontal stretching sheet are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{k}{\rho}\lambda^*\left(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}\right), \qquad (2)$$

$$u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}=\frac{\partial}{\partial y}\left(\frac{k}{\rho C_p}\frac{\partial t}{\partial y}\right)+\frac{q'''}{\rho C_p}, \qquad (3)$$

The following boundary conditions are made use :

$$u=u_w=cx+dx^2,\ \ v=v_c+ex,\ \ t=t_w=t_\infty+A\left(\frac{X}{L}\right)^{\lambda}\quad at\quad y=0,$$
$$u\to 0,\qquad t\to t_\infty\qquad as\qquad y\to\infty. \qquad (4)$$

where u and v are the velocity components along x and y directions respectively, t is the temperature of the fluid, t_w is the temperature of the sheet , t_∞ is the temperature of the liquid far away from the sheet, A is the constant, λ is the temperature parameter , L is the characteristic length, ρ is the density of the liquid, σ is the electrical conductivity of the fluid, C_p is the specific heat at constant pressure, k is the thermal conductivity of the liquid which is assumed to vary linearly with temperature and it is of the form, $k=k_\infty\left[1+\varepsilon\left(\frac{t-t_\infty}{t_w-t_\infty}\right)\right]$ with ε being a small parameter. The non-uniform heat source/sink q''' is modeled as

$$q'''=\frac{\rho k u_w(x)}{xK}\left[A^*(t_w-t_\infty)f'+(t-t_\infty)B^*\right], \qquad (5)$$

where A^* and B^* are the coefficients of space and temperature dependent heat source/sink, respectively. Here we make a note that the case :

(i) $A^*>0$, $B^*>0$ corresponds to internal heat generation

(ii) $A^*<0$, $B^*<0$ corresponds to internal heat absorption.

We have adopted the following boundary conditions:

$$X=\frac{x}{L},\quad Y=\left(\frac{\rho U_0 L}{K}\right)^{\frac{1}{2}}\frac{y}{L},\quad U=\frac{u}{U_0},$$

$$V=\left(\frac{\rho U_0 L}{K}\right)^{\frac{1}{2}}\frac{v}{U_0},\quad V_C=\left(\frac{\rho U_0 L}{K}\right)^{\frac{1}{2}}\frac{v_c}{U_0},$$

$$E=\left(\frac{\rho U_0 L}{K}\right)^{\frac{1}{2}}\frac{Le}{2U_0},\qquad D=\frac{L^2 d}{U_0},$$

$$\mathrm{Re}_L=\frac{\rho U_0 L}{K},\qquad T=\frac{t-t_\infty}{t_w-t_\infty}. \qquad (6)$$

The boundary layer equation (1) – (3) on using (6) takes the following form :

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \qquad (7)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+\lambda_1\left(U\frac{\partial^3 U}{\partial X\partial Y^2}+V\frac{\partial^3 U}{\partial Y^3}\right), \qquad (8)$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}+\frac{UT\lambda}{X}=\frac{1}{\Pr_L}\left\{\varepsilon\left(\frac{\partial T}{\partial Y}\right)^2+(1+\varepsilon T)\frac{\partial^2 T}{\partial Y^2}\right\}+(1+\varepsilon T)(\alpha f'+\beta T), \qquad (9)$$

where $\Pr_L=\dfrac{\rho C_p U_0 L}{k_\infty \operatorname{Re}_L}$ is the uniform Prandtl number where $\operatorname{Re}_L=\dfrac{\rho U_0 L}{K}$ is the Reynolds number, $\alpha=\dfrac{k_\infty A^*}{KC_p}$ is the space-dependent heat source/sink parameter, $\beta=\dfrac{k_\infty B^*}{KC_p}$ is the temperature-dependent heat source/sink parameter and $\lambda_1=c\lambda^*$ is the viscoelastic parameter.

The boundary conditions given in (4) takes the form :

$$U=X+DX^2,\quad V=V_c+2EX,\quad T=1\quad at\quad Y=0,$$
$$U\to 0,\quad T\to 0\quad as\quad Y\to\infty, \qquad (10)$$

where D is the quadratic stretching parameter. Here, we note that quadratic stretching sheet problem is generalization of linear one. If D=0, E=0 the linear velocity will be recovered from the quadratically varying velocity.

Introducing the stream function $\psi(X,Y)$ that satisfies the continuity equation in the dimensionless form (7), we obtain :

$$U=\frac{\partial\psi}{\partial Y},\quad V=-\frac{\partial\psi}{\partial X}, \qquad (11)$$

using (11) equations in the boundary layer equations (8) and (9) can be written as

$$\frac{\partial\psi}{\partial Y}\frac{\partial^2\psi}{\partial X\partial Y}-\frac{\partial\psi}{\partial X}\frac{\partial^2\psi}{\partial Y^2}=\frac{\partial^3\psi}{\partial Y^3}+\lambda_1\left(\frac{\partial\psi}{\partial Y}\frac{\partial^4\psi}{\partial X\partial Y^3}-\frac{\partial\psi}{\partial X}\frac{\partial^4\psi}{\partial Y^4}\right), \qquad (12)$$

$$\frac{\partial\psi}{\partial Y}\frac{\partial T}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial T}{\partial Y}+\frac{\lambda T}{X}\frac{\partial\psi}{\partial Y}=\frac{1}{\Pr_L}\left\{\varepsilon\left(\frac{\partial T}{\partial Y}\right)^2+(1+\varepsilon T)\frac{\partial^2 T}{\partial Y^2}\right\}+(1+\varepsilon T)(\alpha f'+\beta T), \qquad (13)$$

The boundary conditions in (10) can be written in terms of stream function in the following form :

$$\frac{\partial\psi}{\partial Y}=X+DX^2,\quad -\frac{\partial\psi}{\partial X}=V_C+2EX,\quad T=1\quad at\quad Y=0,$$
$$\frac{\partial\psi}{\partial Y}\to 0,\quad T\to 0\quad as\quad Y\to\infty. \qquad (14)$$

In order to convert the partial differential equations (12) and (13) in to ordinary differential equations the following similarity transformation are adopted :

$$\psi=Xf_0(Y)-DX^2 f_1(Y),$$
$$T=\theta_0(Y)+2DX\theta_1(Y). \qquad (15)$$

Using the transformation (15) in the boundary layer equations (12) and (13) we have the following boundary value problems :

$$f_0'''+f_0 f''-(f_0')^2+\lambda_1\left(f_0' f_0'''-f_0 f_0^{IV}\right)=0, \qquad (16)$$

$$f_1^{\prime\prime\prime} - 3 f_0^{\prime} f_1^{\prime} + f_0 f_1^{\prime\prime} + 2 f_1 f_0^{\prime\prime} + \lambda_1 \left(2 f_0^{\prime} f_1^{\prime\prime\prime} + f_1^{\prime} f_0^{\prime\prime\prime} - f_0 f_1^{IV} - 2 f_1 f_0^{1V} \right) = 0 \tag{17}$$

$$\theta_0^{\prime\prime}(1+\varepsilon\theta_0) + \Pr_L \left(f_0 \theta_0^{\prime} - \lambda \theta_0 f_0^{\prime} \right) + \Pr_L \{ (\alpha f^{\prime} + \beta \theta_0)(1+\varepsilon\theta_0) \} + \varepsilon (\theta_0^{\prime})^2 = 0, \tag{18}$$

$$\theta_1^{\prime\prime}(1+\varepsilon\theta_0) + \Pr_L \left(\beta \theta_1 + 2\beta\, \theta_1 \theta_0 \varepsilon + \varepsilon \theta_1\, \alpha f^{\prime} \right) + \Pr_L \left\{ \left(\theta_1^{\prime} f_0 - \theta_1 f_0^{\prime} - f_1 \theta_0^{\prime} \right) + \lambda \left(\frac{\theta_0 f_1^{\prime}}{2} + \theta_1 f_0^{\prime} \right) \right\} + \varepsilon \theta_1 \theta_0^{\prime\prime} + 2\varepsilon \theta_0^{\prime} \theta_1^{\prime} = 0. \tag{19}$$

$$f_0(0) = -V_C, \quad f_1(0) = \frac{E}{D}, \quad f_0^{\prime}(0) = 1, \quad f_1^{\prime}(0) = -1, \quad \theta_0(0) = 1,$$
$$\theta_1(0) = 0, \quad f_0^{\prime}(\infty) \to 0, \quad f_1^{\prime}(\infty) \to 0, \quad \theta_0(\infty) \to 0, \quad \theta_1(\infty) \to 0, \tag{20}$$

here, the prime denote differentions with respect to Y in case of linear and quadratic stretching sheet problems. We now move forward to explain the method of solution of the boundary value problem obtained in case of linear and quadratic stretching sheet problems.

Method of Solution

1. Decision on ∞
2. Converting BVP to IVP b choosing suitable initial condition for f_1, f_0, θ_1 & θ_0.
3. The choice of $f_0''(0)$, $f_1''(0)$, $\theta_1'(0)$ & $\theta_0'(0)$ required for the solution of initial value problem by the classical, explicit Runge-Kutta method of four slopes.

The decision on an appropriate '∞' for the problem depends on the parameter values chosen. In view of this, for each parameter combination, the appropriate value of '∞' has to be decide.

We chosen guess values as $f_1''(0) = \alpha_1$, $f_0''(0) = \alpha_2$, $\theta_1'(0) = \beta_1$ & $\theta_0'(0) = \beta_2$, We solve the equations (16) - (20) with initial conditions :

$$f_0(0) = -V_C, \quad f_1(0) = \frac{E}{D}, \quad f_0^{\prime}(0) = 1,$$
$$f_1^{\prime}(0) = -1, \quad f_1''(0) = \alpha_1, \quad f_0''(0) = \alpha_2, \tag{21}$$
$$\theta_0(0) = 1, \quad \theta_1(0) = 0, \quad \theta_1'(0) = \beta_1, \quad \theta_0'(0) = \beta_2,$$
$$f_0^{\prime}(\infty) \to 0, \quad f_1^{\prime}(\infty) \to 0, \quad \theta_0(\infty) \to 0, \quad \theta_1(\infty) \to 0.$$

The boundary value problem is converted into an initial value problem by guessing the missed initial conditions as follows:

$$\frac{dF}{dY} = F_1, \quad \frac{dF_1}{dY} = F_2, \quad \frac{dF_2}{dY} = F_3, \quad \frac{dF_4}{dY} = \frac{1}{\lambda_1 F} \left\{ F_3 + \lambda_1 F_1 F_3 - (F_1)^2 + F F_2 \right\},$$

$$\frac{dH}{dY} = H_1, \quad \frac{dH_1}{dY} = H_2, \quad \frac{dH_2}{dY} = H_3,$$

$$\frac{dH_3}{dY} = \frac{1}{\lambda_1 F} \left\{ \begin{array}{l} H_3 + 2\lambda_1 F_1 H_3 + \lambda_1 H_1 F_3 - 2\lambda_1 H F_4 \\ -3 F_1 H_1 + F H_2 + 2 H F_2 \end{array} \right\},$$

$$\frac{dI}{dY}=I_1,\quad \frac{dI_1}{dY}=\frac{1}{1+I}\begin{Bmatrix}-\Pr_L\left(FI_1-\lambda IF_1\right)-\Pr_L\left(\alpha F_1+\beta I\right)(1+\varepsilon I)\\ -\varepsilon\left(I_1\right)^2\end{Bmatrix},$$

$$\frac{dJ}{dY}=J_1,\quad \frac{dJ_1}{dY}=\frac{1}{1+\varepsilon I}\begin{Bmatrix}-\Pr_L\left(\beta J+2\beta\ JI\varepsilon+\alpha\,\varepsilon JF_1\right)-\\ \Pr_L\left(J_1F-JF_1-HI_1+\lambda\left(\frac{IH_1}{2}+JF_1\right)\right)\\ -\varepsilon JI_2-2\varepsilon I_1J_1\end{Bmatrix}. \tag{22}$$

Subsequently the boundary conditions (21) takes the form

$$\begin{gathered}F(0)=-V_C,\quad H(0)=\frac{E}{D},\quad F_1(0)=1,\\ H_1(0)=-1,\quad F_2(0)=\alpha_2,\quad H_2(0)=\alpha_1,\\ I(0)=1,\qquad J(0)=0,\qquad I_1(0)=\beta_2,\quad J_1(0)=\beta_1,\\ F_1(\infty)=0,\quad H_1(\infty)=0,\qquad I(\infty)=0,\quad J(\infty)=0,\end{gathered} \tag{23}$$

here, $F=f_0(Y)$, $H=f_1(Y)$, $I=\theta_0(Y)$ *and* $J=\theta_1(Y)$.

The initial value problem (22) is integrated using the fourth order Runge-Kutta method. Newton-Raphson method is implemented to correct the uses values $\alpha_1,\alpha_2,\beta_1$ & β_2. Appropriate '∞' value of the solution is determined through the actual computation.

Results and Discussion

The boundary layer flow and heat transfer in an electrically conducting fluid over a stretching sheet with variable thermal conductivity is investigated in the presence of non-uniform heat source/sink. The effect of $\lambda_1,\Pr,\alpha,\beta,\varepsilon$ *and* λ on flow and heat transfer is shown graphically in Figs. 1 – 13.

Figs. 1 - 2 show the effect of viscoelastic parameter λ_1 on axial velocity profiles. The effect of increasing λ_1 is to increase $f_0'(Y)$ and decrease $f_1'(Y)$.

Figs. 3 and 9 shows the effect of Prandtl number on $\theta_0(Y)$ and $\theta_1(Y)$. Prandtl number has significant impact on the dynamics. The effect of Pr is to decrease $\theta_0(Y)$ and the value of Pr increases the peak in the $\theta_1(Y)$ profiles that shift towards the wall.

The effect of temperature-dependent heat source/sink parameter α on heat transfer $\theta_0(Y)$ and $\theta_1(Y)$ is demonstrated in Figs. 4 and 10. These graphs show that the thermal boundary layer generates energy which causes the temperature to increase in magnitude with increasing values of $\alpha>0$ where as in the case $\alpha<0$ boundary layer absorbs energy resulting a substantial fall in temperature with decreasing values of $\left|\overline{\alpha}\right|$.

Figs. 5 and 11, illustrates the effect of temperature-dependent heat source/sink parameter β. These graphs show that energy is released for increasing values of $\beta\,(>0)$ and this causes the magnitude of temperature to increase, where as energy is absorbed for decreasing values of $\beta\,(<0)$ resulting in temperature dropping significantly near the boundary layer.

The effect of variable thermal conductivity parameter ε on temperature profiles is shown in Figs.6 and 12. It is observed from these plots that the increasing values of ε results in increasing the magnitude of temperature causing thermal boundary layer thickening.
The effect of λ on the heat transfer is shown in Figs.7 and 13. The effect of λ on the heat transfer is typical in above some critical negative value λ_c, the increasing effect of λ is to decrease the magnitude of the temperature. There will be transfer of heat form sheet to the liquid for $\lambda > \lambda_c$. Below this critical value the effect of λ is opposite, i.e., if $\lambda < \lambda_c$ the heat flow from the liquid to sheet itself. This case is of least interest because the present investigation concerns most about cooling the sheet hence the values of λ are chosen above the critical value λ_c. When $\lambda = \lambda_c$, there is no heat transfer between the stretching surface and the ambient liquid.

Conclusion

- The viscoelastic normal stress results in thickening of thermal boundary layer.
- The individual effects of increasing λ_1, α, β are to be increase the magnitude of heat transfer. The opposite effect is observed for increasing values of Prandtl number Pr and temperature parameter λ.
- The variable thermal conductivity parameter ε increases the magnitude of temperature.
- The magnitude of λ dictates the direction of heat transfer.
- The effect of flow and temperature dependent heat source/sink parameters is to generate temperature for increasing positive values and absorb temperature for decreasing negative values. Hence flow and temperature-dependent heat sinks are better suited for cooling purposes.

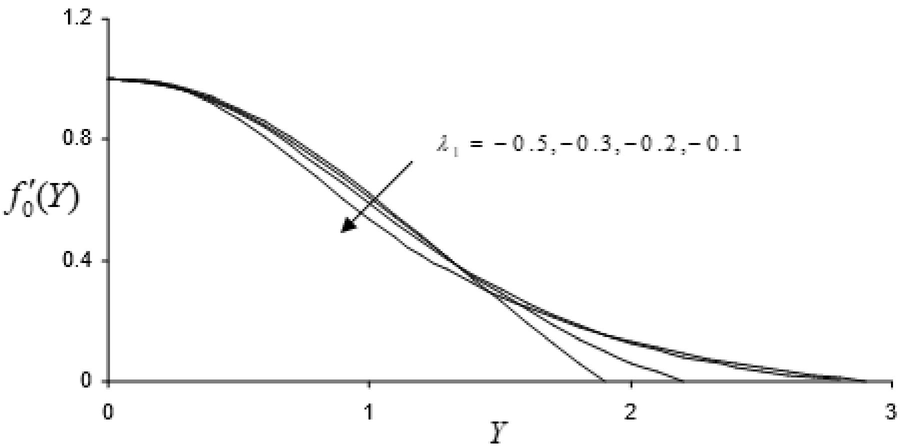

Fig. 1 : Effect of λ_1 on velocity profile $f_0'(Y)$ in case of non-linear stretching sheet problem.

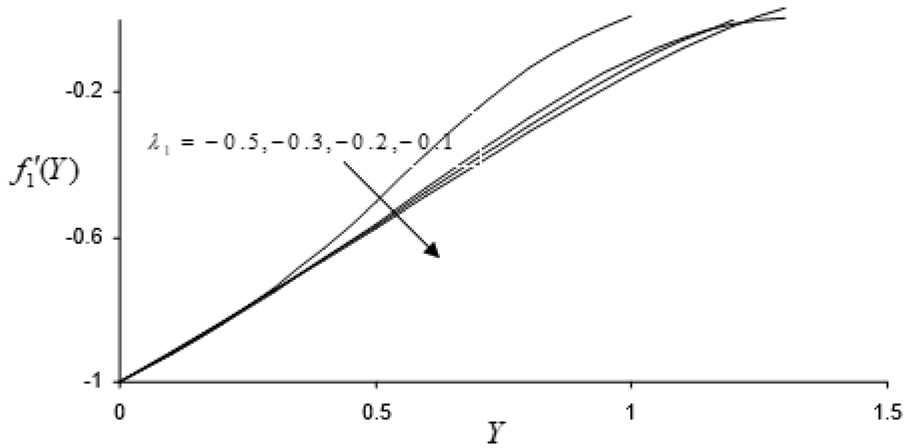

Fig. 2: Effect of λ_1 on velocity profile $f_1'(Y)$ in case of non-linear stretching sheet problem.

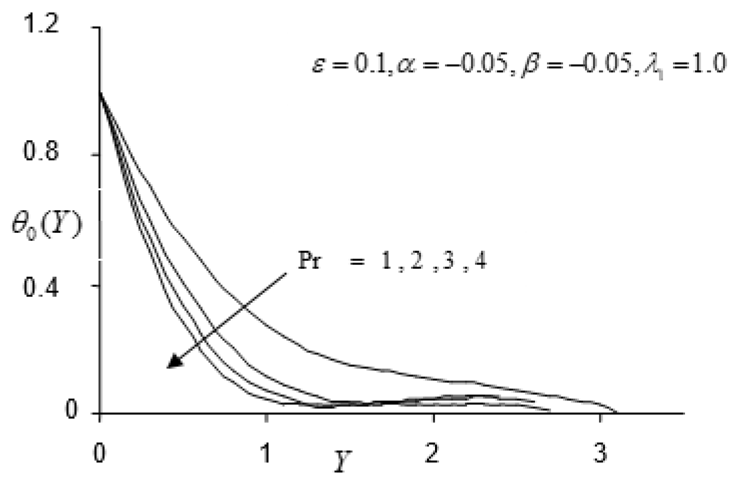

Fig. 3 :Effect of Pr on temperature profile $\theta_0(Y)$ in case of non-linear stretching sheet problem.

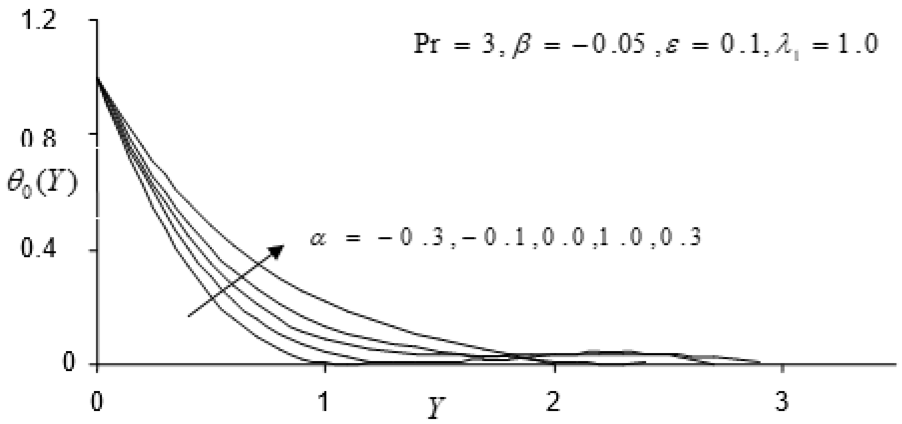

Fig. 4 : Effect of α on temperature profile $\theta_0(Y)$ in case of non-linear stretching sheet problem.

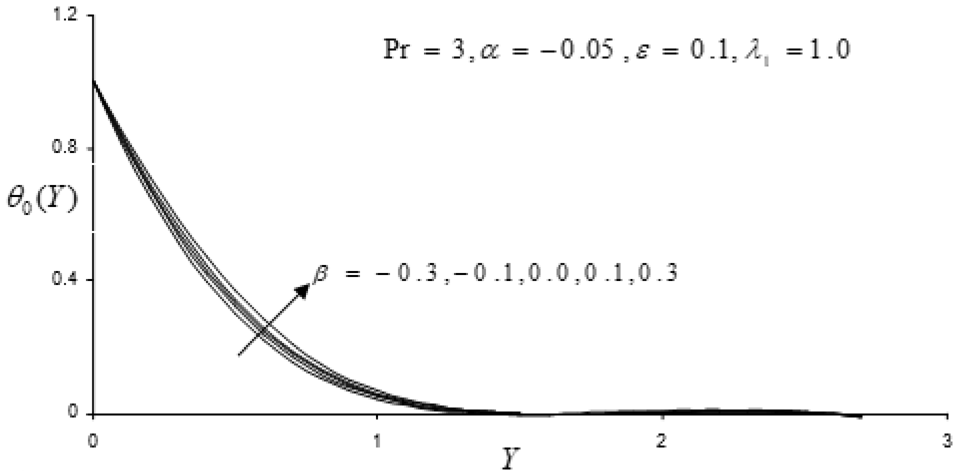

Fig. 5: Effect of β on temperature profile $\theta_0(Y)$ in case of non-linear stretching sheet problem.

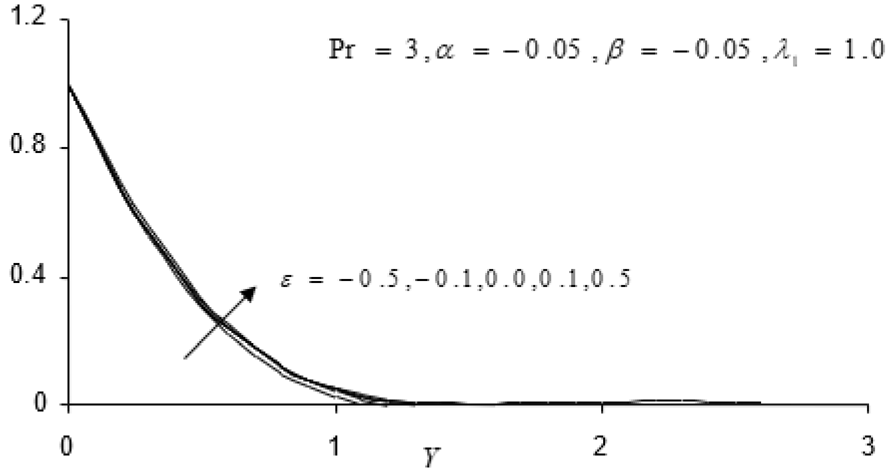

Fig. 6 : Effect of ε on temperature profile $\theta_0(Y)$ in case of non-linear stretching sheet problem.

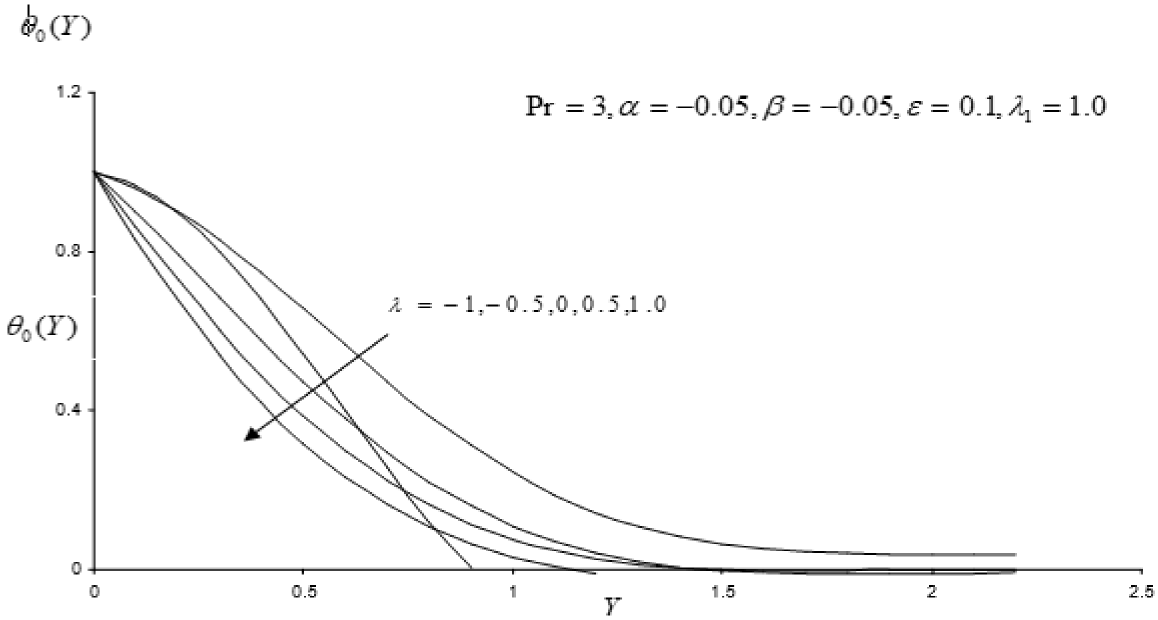

Fig. 7 : Effect of λ on temperature profile $\theta_0(Y)$ in case of non-linear stretching sheet problem.

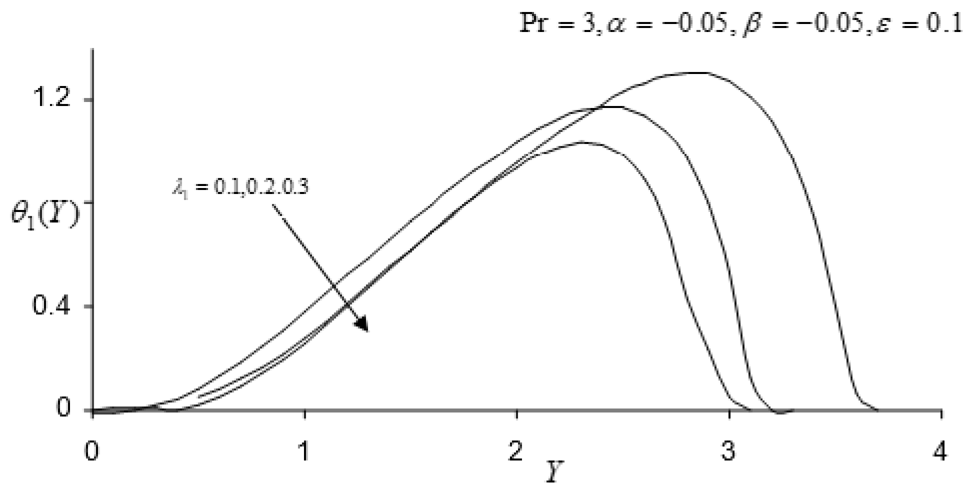

Fig. 8 : Effect of λ_1 on temperature profile $\theta_1(Y)$ in case of non-linear stretching sheet problem.

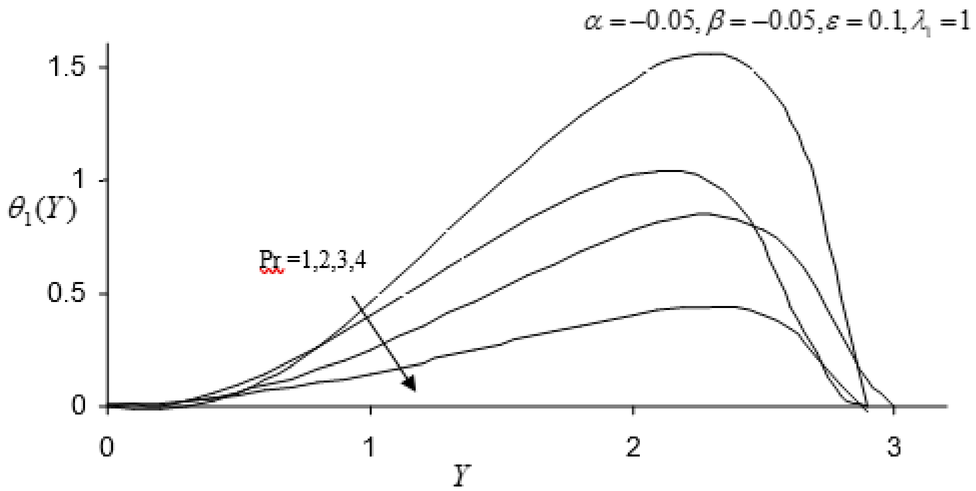

Fig. 9 : Effect of Pr on temperature profile $\theta_1(Y)$ in case of non-linear stretching sheet problem.

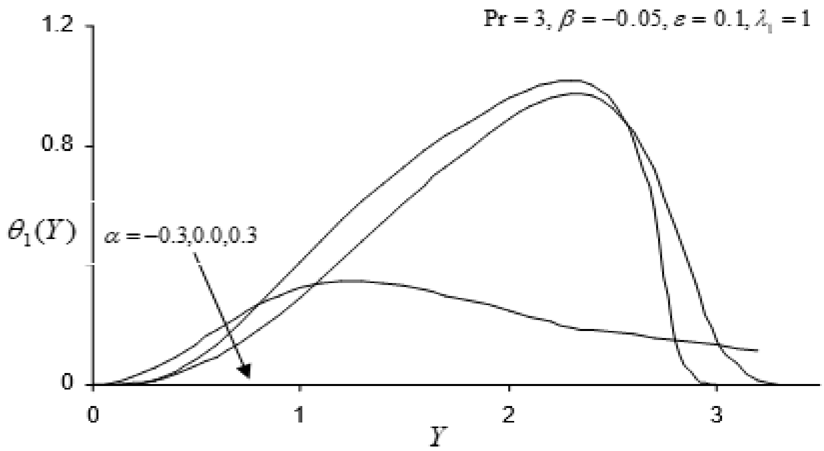

Fig. 10: Effect of α on temperature profile $\theta_1(Y)$ in case of non-linear stretching sheet problem.

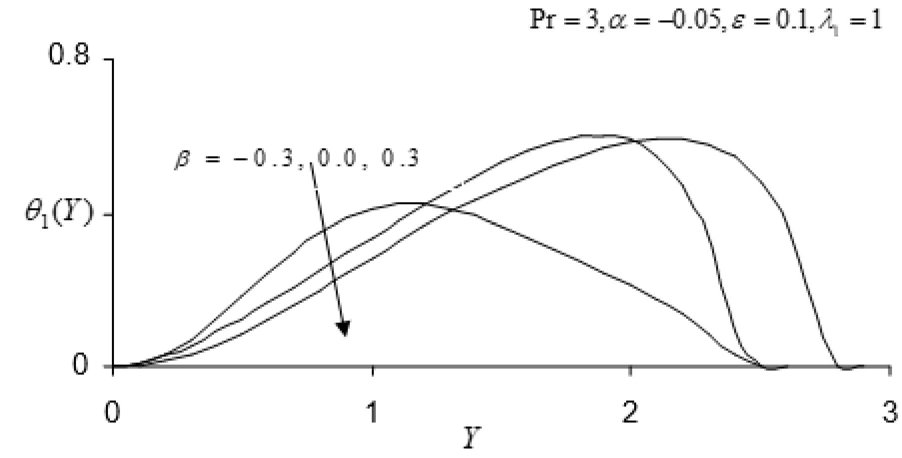

Fig. 11 : Effect of temperature profile β on $\theta_1(Y)$ in case non-linear stretching sheet problem.

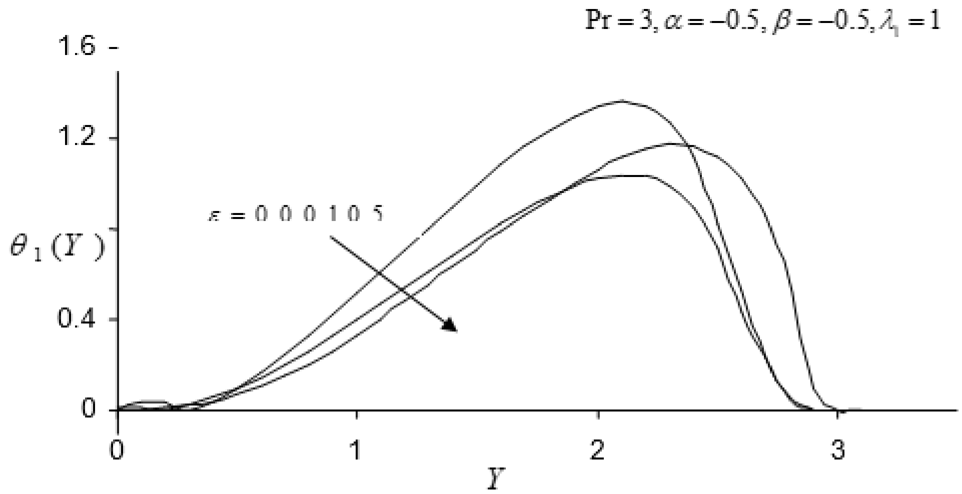

Fig. 12 : Effect of ε on temperature profile $\theta_1(Y)$ in case of non-linear stretching sheet problem.

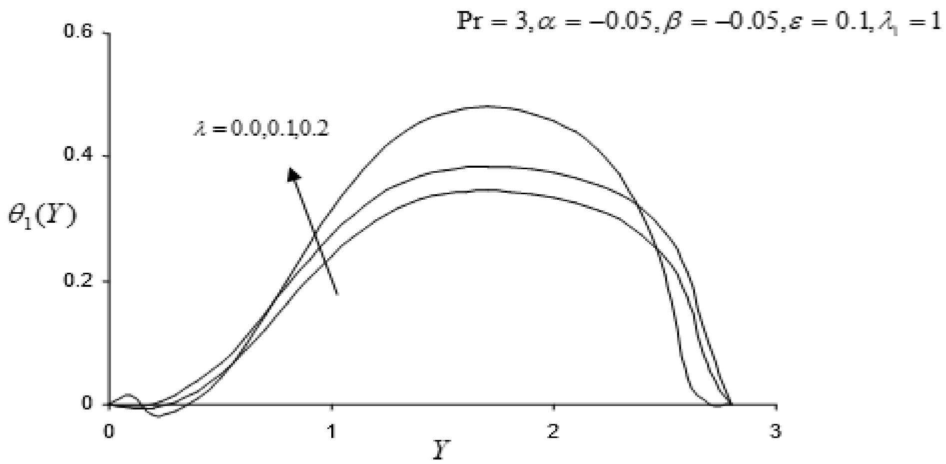

Fig. 13 : Effect of λ on temperature profile $\theta_1(Y)$ in case non-linear stretching sheet problem.

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