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Unsteady Two-Dimensional Drift of a Viscoelastic Fluid with Magnetic and Thermal Effects: A Perturbation Approach

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Abstract:
This study explores the unsteady, two-dimensional flow of an electrically conductive, viscoelastic, and incompressible fluid past a uniformly moving, infinite, non-conducting vertical plate, in the presence of a uniform first-order chemical process involving mass and heat transfer. A uniform magnetic field is applied perpendicular to the flow, with the magnetic Reynolds number assumed small to neglect the induced magnetic field. The governing equations are solved using a regular perturbation method, yielding approximate solutions for concentration, velocity, temperature, and shear stress at the plate. Additionally, mass and heat transfer rates are determined, with the influence of viscoelasticity and other physical parameters emphasized. Where applicable, results are graphically represented.

Keywords: Chemical reaction, free convection, MHD, porous plate, Visco-elastic.

1. Introduction:

Viscoelastic fluids exhibit both viscous and elastic properties when subjected to deformation. For decades, the study and modeling of viscoelastic fluid flow have significantly contributed to research involving various industrially important fluids such as lubricants, paints, polymers, and colloidal oils. These studies have gained particular attention due to their applications in fields such as geothermal systems, oil extraction, geophysics, and magnetohydrodynamic (MHD) bearings. As a result, researchers have focused on MHD fluid flows with heat and mass transfer through porous media.

Several studies have explored these phenomena. Anghel et al. [1] examined the effects of Dufour and Soret on free convective boundary layers over vertical surfaces in porous media. Kafousias and Williams [2] investigated the impacts of thermal diffusion on convective mass transfer, while Dursunkaya and Worek [3] explored the diffusion-thermo effects in natural convection. Additional research has been conducted on MHD free convection flow and mass transfer through porous media by Raptis and Kafousias [4], and on combined heat and mass transfer by Chaudhary and Jain [5]. Postelincus [6] studied the influence of magnetic fields on heat and mass transfer, while Kolar and Sastri [7], Kim and Vafai [8], Alam and Rahman [9], and Nazmul and Mahmud [10] all contributed to this area of research, particularly focusing on free convection and the Dufour and Soret effects.

The current paper discusses the unsteady MHD flow of an incompressible viscoelastic fluid past an infinitely large vertical plate, considering heat and mass transfer in the presence of a first-order chemical reaction.

2. Mathematical formulation:

The study focuses on the unsteady, two-dimensional free convective flow of an electrically conducting visco-elastic fluid past an infinitely large, non-conducting vertical plate that moves uniformly. The flow involves both heat and mass transfer, and the plate experiences a normal periodic suction velocity. A uniform magnetic field, denoted by strength R_0, is applied perpendicular to the main flow direction. Apart from density, all fluid properties are constant in the buoyancy force, and the magnetic Reynolds number is small enough to ignore the induced magnetic field compared to the applied one. The flow occurs along the x-axis (parallel to the vertical plate), with the y-axis normal to it. Using these conditions and Boussinesq's approximation, the governing equations for flow and transport are derived as follow:

$$\frac{\partial \overline{u_y}}{\partial \bar{y}} = 0 \tag{2.1}$$

$$\frac{\partial \overline{u_x}}{\partial \bar{t}} + \overline{u_y}\frac{\partial \overline{u_x}}{\partial \bar{y}} + \frac{\sigma {R_0}^2}{\rho}\overline{u_x} + \frac{\vartheta_1}{\bar{K}}\overline{u_x} = \vartheta_1 \frac{\partial^2 \overline{u_x}}{\partial \bar{y}^2} + g\beta(\bar{\theta} - \bar{\theta}_\infty) + \vartheta_2 \frac{\partial^3 \overline{u_x}}{\partial \bar{y}^2 \partial \bar{t}} + \vartheta_2 \overline{u_y}\frac{\partial^3 \overline{u_x}}{\partial \bar{y}^3} + g\bar{\beta}(\bar{\phi} - \bar{\phi}_\infty) \tag{2.2}$$

$$\frac{\partial \bar{\theta}}{\partial \bar{t}} + \overline{u_y}\frac{\partial \bar{\theta}}{\partial \bar{y}} - \frac{k}{\rho C_p}\frac{\partial^2 \bar{\theta}}{\partial \bar{y}^2} - Q(\bar{\theta}_\infty - \bar{\theta}) = \frac{\vartheta_1}{C_p}\left(\frac{\partial \overline{u_x}}{\partial \bar{y}}\right)^2 + \frac{\vartheta_2}{C_p}\left(\frac{\partial \overline{u_x}}{\partial \bar{y}}\right)\left(\frac{\partial^2 \overline{u_x}}{\partial \bar{y}\,\partial \bar{t}}\right) + \frac{\vartheta_2}{C_p}\overline{u_y}\left(\frac{\partial \overline{u_x}}{\partial \bar{y}}\right)\left(\frac{\partial^2 \overline{u_x}}{\partial \bar{y}^2}\right) \tag{2.3}$$

$$\frac{\partial \bar{\phi}}{\partial \bar{t}} + \overline{u_y}\frac{\partial \bar{\phi}}{\partial \bar{y}} = D_T \frac{\partial^2 \bar{\phi}}{\partial \bar{y}^2} + (\bar{\phi}_\infty - \bar{\phi})\xi + D\frac{\partial^2 \bar{\phi}}{\partial \bar{y}^2} \tag{2.4}$$

The relevant boundary conditions are:

$$\bar{y} = 0: \overline{u_x} = \bar{u}_w, \bar{\theta} = \bar{\theta}_w + \varepsilon(\bar{\theta}_w - \bar{\theta}_\infty)e^{i\bar{\omega}\bar{t}},\ \bar{\phi} = \bar{\phi}_w + \varepsilon(\bar{\phi}_w - \bar{\phi}_\infty)e^{i\bar{\omega}\bar{t}}$$

$$\bar{y} \to \infty: \overline{u_x} \to 0, \bar{\theta} \to \bar{\theta}_\infty, \bar{\phi} \to \bar{\phi}_\infty$$

Equation (2.1) is trivially satisfied by $\overline{u_y} = -u_0\left(1 + \varepsilon A e^{i\bar{\omega}\bar{t}}\right)$, where A is a constant $(A > 0)$ such that $\varepsilon A < 1$.

For convenience we introduce the following dimensionless variables :

$$y = \frac{\bar{y}u_0}{\vartheta_1}, t = \frac{\bar{t}{u_0}^2}{\vartheta_1}, \omega = \frac{\vartheta_1\bar{\omega}}{{u_0}^2},\ u_x = \frac{\overline{u_x}}{u_0},\ u_w = \frac{\bar{u}_w}{u_0}, \theta = \frac{\bar{\theta} - \bar{\theta}_\infty}{\bar{\theta}_w - \bar{\theta}_\infty}, \phi = \frac{\bar{\phi} - \bar{\phi}_\infty}{\bar{\phi}_w - \bar{\phi}_\infty}, Gr = \frac{g\beta\vartheta_1(\bar{\theta}_w - \bar{\theta}_\infty)}{{u_0}^3},\ Gm = \frac{g\bar{\beta}\vartheta_1(\bar{\phi}_w - \bar{\phi}_\infty)}{{u_0}^3}, M = \frac{\sigma {R_0}^2\vartheta_1}{\rho {u_0}^2},\ S = \frac{Q\vartheta_1}{{u_0}^2}, E = \frac{{u_0}^2}{C_p(\bar{\theta}_w - \bar{\theta}_\infty)}, \Pr = \frac{\mu_1 C_p}{k},\ K = \frac{\bar{K}{u_0}^2}{{\vartheta_1}^2}, Sc = \frac{\vartheta_1}{D},\ \ Sr = \frac{D_T(\bar{\theta}_w - \bar{\theta}_\infty)}{\vartheta_1(\bar{\theta}_w - \bar{\theta}_\infty)}, \vartheta_1 = \frac{\mu_1}{\rho}, \vartheta_2 = \frac{\mu_2}{\rho},\ \alpha_1 = \frac{\vartheta_2{u_0}^2}{{\vartheta_1}^2}, Kc = \frac{S\xi D}{{u_0}^2} \tag{2.5}$$

Substitution of (2.5) into the equations (2.2) to (2.4) yields the following dimensionless equations:

$$\frac{\partial u_x}{\partial t} + \left(M + \frac{1}{K}\right)u_x = \frac{\partial^2 u_x}{\partial y^2} + \alpha_1\left\{\frac{\partial^3 u_x}{\partial y^2 \partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial^3 u_x}{\partial y^3}\right\} + \theta Gr + \phi Gm + \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial u_x}{\partial y} \tag{2.6}$$

$$Pr\frac{\partial \theta}{\partial t} + Pr\left(1 + \varepsilon A e^{i\omega t}\right)\alpha_1 E \frac{\partial u_x}{\partial y}\frac{\partial^2 u_x}{\partial y^2} + PrS\theta = \frac{\partial^2 \theta}{\partial y^2} + PrE\left(\frac{\partial u_x}{\partial y}\right)^2 + Pr\alpha_1 E\left(\frac{\partial u_x}{\partial y}\right)\left(\frac{\partial^2 u_x}{\partial y \partial t}\right) + Pr\left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial \theta}{\partial y} \tag{2.7}$$

$$Sc\frac{\partial \phi}{\partial t} + \phi ScKc = SrSc\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \phi}{\partial y^2} + Sc\left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial \phi}{\partial y} \tag{2.8}$$

The corresponding boundary conditions are:

$$y = 0: u_x = u_w, \theta = 1 + \varepsilon e^{i\omega t}, \phi = 1 + \varepsilon A e^{i\omega t}$$

$$y \to \infty: u_x \to 0, \theta \to 0, \phi \to 0$$

3. Method of solution:

Using perturbation techniques for $\varepsilon \ll 1$, $E \ll 1$, and $\alpha_1 \ll 1$ (based on Nowinski and Ismail [11]), we derived expressions for the velocity, temperature, and concentration profiles. With these profiles, key flow characteristics such as mass flux, wall shear stress, and local heat flux are analyzed. The non-dimensional shearing stress at the plate (y = 0) is expressed as:

$$\tau = \left(\frac{\sigma_{xy}}{\rho {u_0}^2}\right)_{y=0}$$

Similarly, the non-dimensional heat flux at the plate (y = 0), represented by the Nusselt number (Nu), is:

$$Nu = \left(\frac{\partial\theta}{\partial y}\right)_{y=0}$$

And the non-dimensional mass flux at the plate (y = 0), represented by the Sherwood number (Sh), is:

$$Sh = \left(\frac{\partial\phi}{\partial y}\right)_{y=0}$$

4. Results and Discussion:

Analytical calculations were performed for velocity, temperature, concentration, shearing stress, Nusselt number, and Sherwood number to gain a physical understanding of the problem. We used various values for the visco-elastic parameter α_1, thermal Grashof number *Gr*, solutal Grashof number *Gm*, magnetic parameter *M*, Soret number *Sr*, and chemical reaction parameter *Kc*, while keeping constants such as E=0.001, ε=0.01, K=0.2, A=0.3, S=1, Sc =0.8, $u_w = 1$, Pr =4, ω=1, and ωt =π/2 throughout the calculations. When $\alpha_1 \neq 0$, the fluid is visco-elastic, and when α_1=0, it represents Newtonian fluid flow.

Figures 1 through 6 depict the variation in fluid velocity u_x against *y* with different flow parameters. The graphs show that fluid velocity decreases as the distance from the plate increases for both Newtonian and visco-elastic fluids. Additionally, fluid velocity slows down as the visco-elastic parameter α_1 increases (α_1=0, -0.02, -0.04), compared to Newtonian fluid flow. Higher values of the thermal Grashof number Gr (Figs: 1 and 2) and solutal Grashof number Gc (Figs: 1 and 3) lead to an increase in fluid velocity, indicating that both concentration and thermal buoyancy forces tend to accelerate the fluid. Increased magnetic parameter M reduces fluid velocity in both types of fluids (Figs: 1 and 4), as the transverse magnetic field generates a Lorentz force that slows down the fluid. Higher values of the Soret number Sr (Figs: 1 and 5) and chemical reaction parameter Kc (Figs: 1 and 6) show a decreasing trend in fluid velocity. Likewise, variations in shearing stress, Nusselt number, and Sherwood number are observed with changes in *Gr, Gm, M, Sr,* and *Kc* as well as other flow parameters.

5. Conclusions:

The study concludes that the velocity field is strongly influenced by the viscoelastic parameter at every point in the fluid flow region. As the absolute value of the viscoelastic parameter increases, it indicates a deceleration in fluid velocity compared to the behavior seen in Newtonian fluids. Enhancements in various key flow parameters significantly impact the shear stress, Nusselt number, and Sherwood number in both Newtonian and non-Newtonian fluid scenarios.

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Figures:

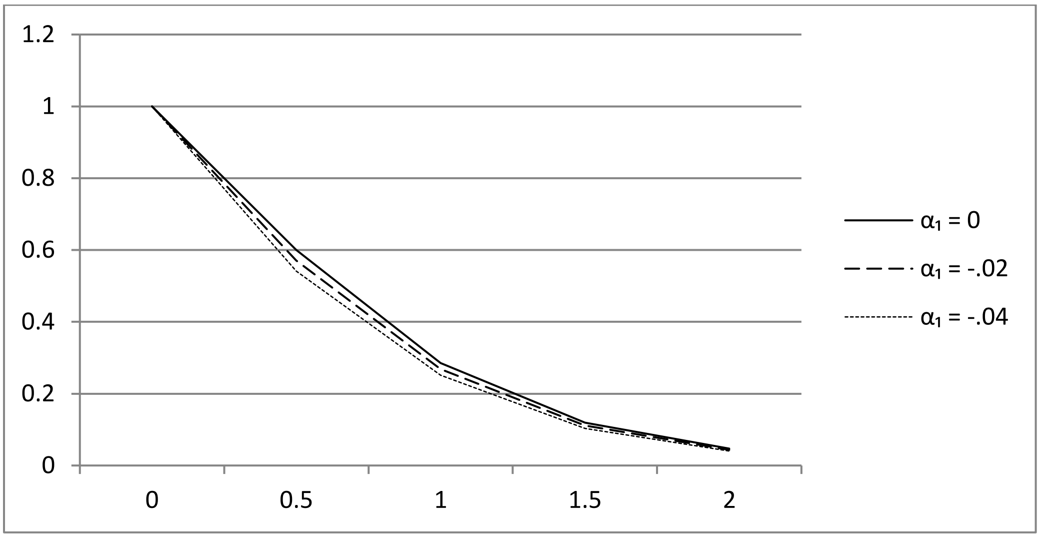

Fig 1. Variation of u_x against y for $Gr = 5,\ Gm = 4,\ M = 2,\ Pr = 4,\ Sr = 3$ and $Kc = 3$

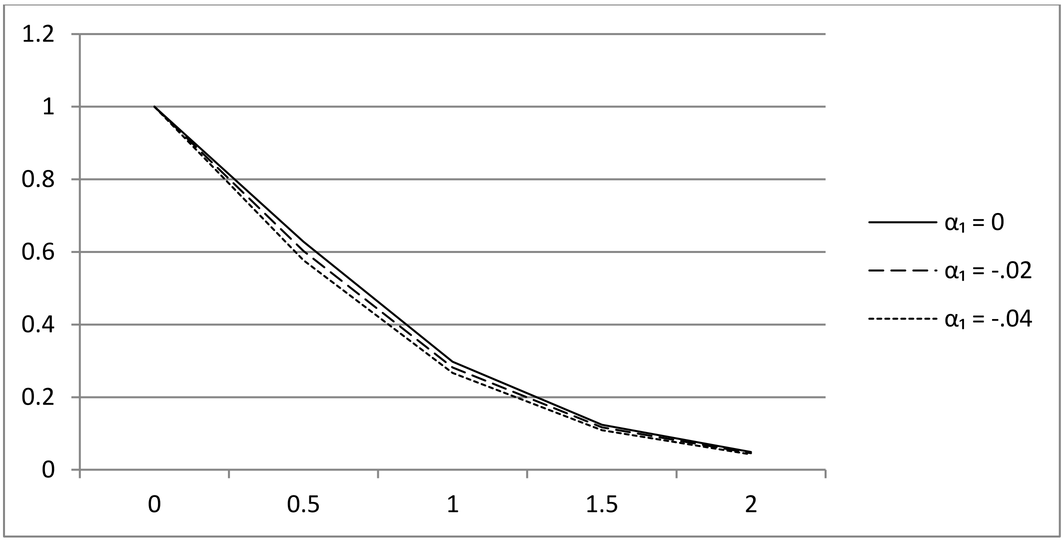

Fig 2. Variation of u_x against y for $Gr = 7,\ Gm = 4,\ M = 2,\ Pr = 4,\ Sr = 3$ and $Kc = 3$

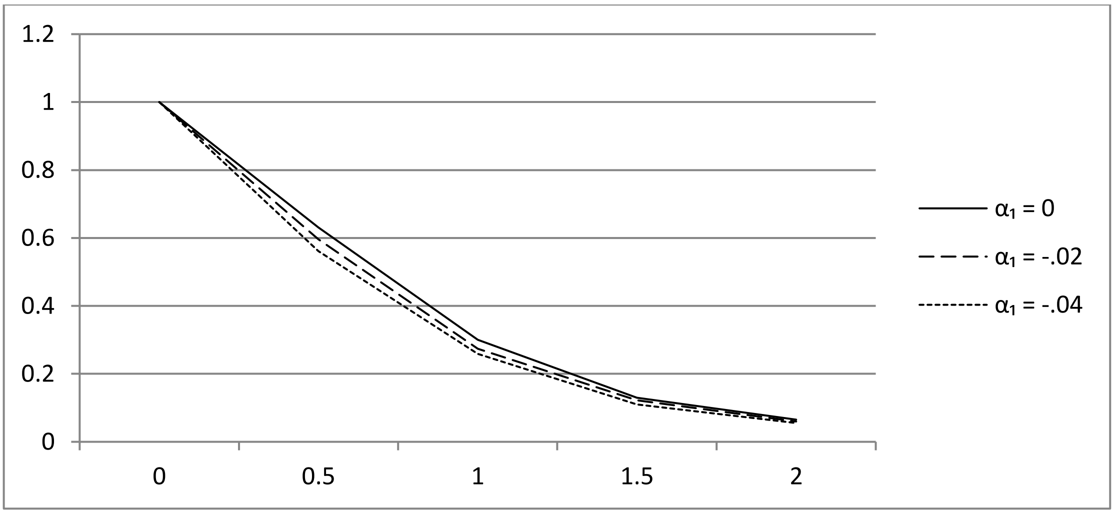

Fig 3. Variation of u_x against y for $Gr = 5,\ Gm = 5,\ M = 2,\ Pr = 4,\ Sr = 3\ and\ Kc = 3.$

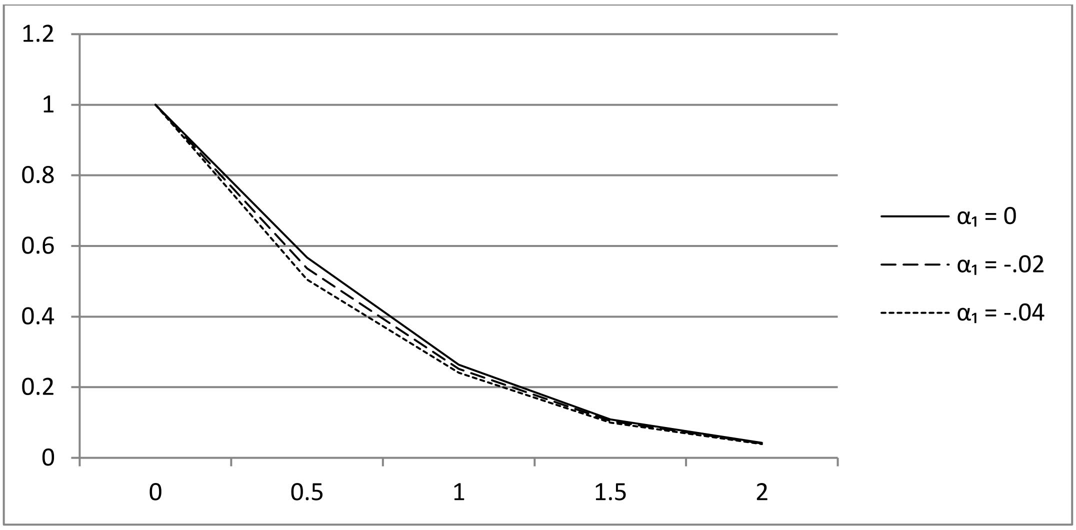

Fig 4. Variation of u_x against y for $Gr = 5.\ Gm = 4,\ M = 3,\ Pr = 4,\ Sr = 3\ and\ Kc = 3.$

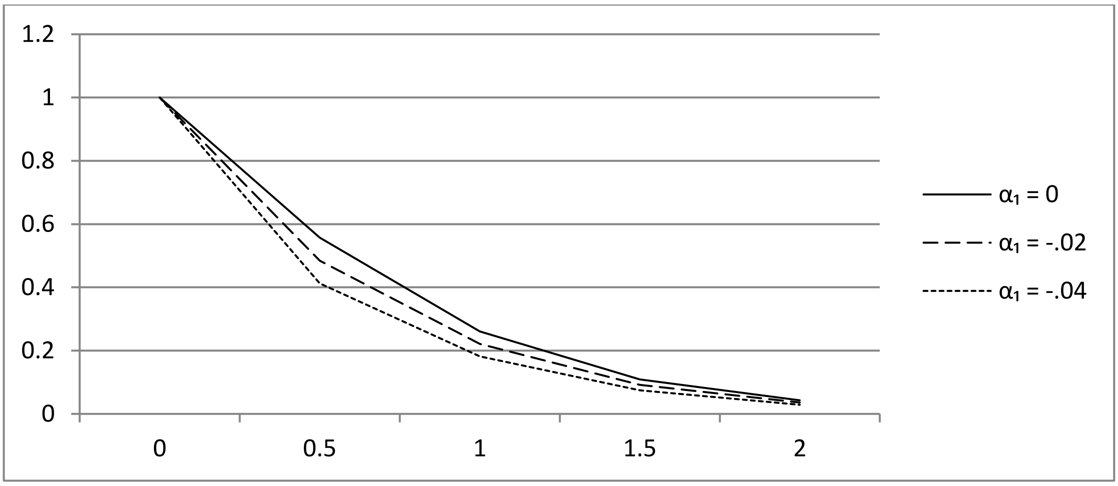

Fig 5. Variation of u_x against y for $Gr = 5,\ Gm = 4,\ M = 2,\ Pr = 4,\ Sr = 4$ and $Kc = 3$

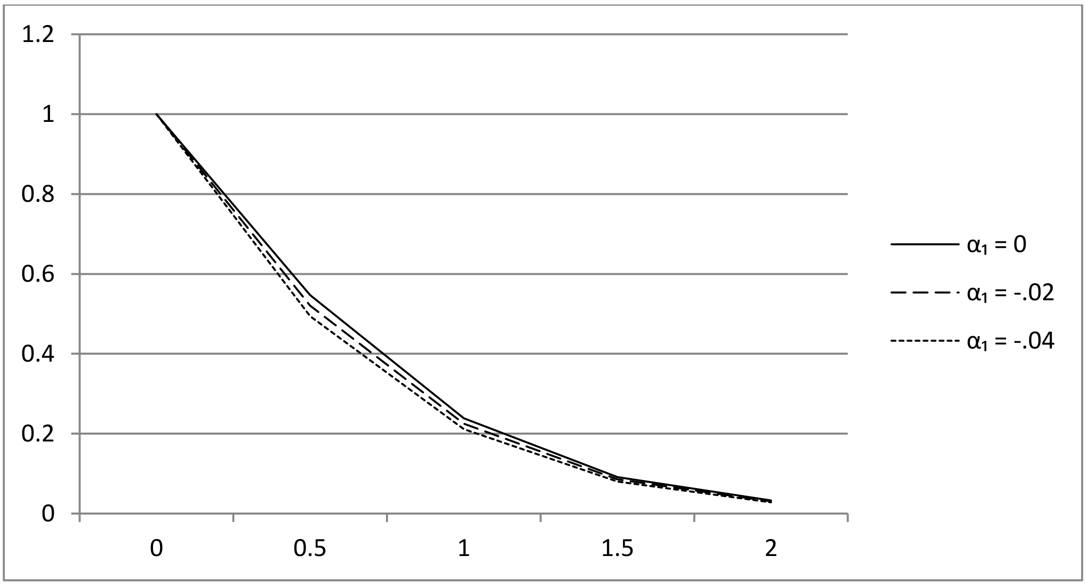

Fig 6. Variation of u_x against y for $Gr = 5,\ Gm = 4,\ M = 2,\ Pr = 4,\ Sr = 3$ and $Kc = 4$

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