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Comparative Performance of ARIMA and ARCH Models on Time Series of Monthly Libyan Brent Oil Price

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Abstract
Many economic and financial time series exhibit a phenomenon known as heteroscedastic, where the variance of the series changes over time. This research study focuses on financial time series modelling, with special application to modelling the price of Libyan Brent Oil. In particular, the theory of univariate nonlinear time series analysis is explored and applied to the price Libyan of Brent Oil, spanning from January 2000 to December 2010. The data was obtained from Bullent of the Platts Market Wire of Statistics. This study aims to evaluate the performance of ARIMA as a linear model and ARCH as a nonlinear modelling data. Multiple time series models were considered for fitting the data, and the best ARIMA models were selected based on the Akaike Information Criteria (AIC). ARIMA (0, 1, 1), and ARIMA (1, 1, 0) were identified as the best models. After estimating the parameters of the selected models, model checking revealed that these models were not suitable for modeling the data, as they lacked validity according to the test of squared residuals. The goodness of fit was assessed using the AIC, and based on minimum AIC values, the best fit ARCH models were found to be ARIMA (0, 1, 1) - ARCH (1). After estimating the parameters of the selected model, a series of diagnostic and forecast accuracy tests were performed. Based on this model, a twelve-month forecast of the price of Libyan Brent crude was made.

Keywords: ARIMA model, ARCH model, AIC, Hetroscedacity, TIME SERIES models

Introduction

Time series analysis plays a significant role in modeling various economic phenomena and forecasting their future values. A key requirement for effective modeling is that the series should be stationary. However, most financial and economic data do not meet this condition, often exhibiting varying levels of volatility over different periods, with variance changing over time. For example, when analysing the time series of a stocks in financial markets, we might observe periods of both low and high volatility across different periods of the series.

This variability necessitates the use of specialised models to handle the changing volatility in time series data. Robert Engel (1982) was the first to introduce models capable of addressing this issue through his research on estimating inflation variation in the United Kingdom. These models, known as

Autoregressive Conditional Heteroscdasticity (ARCH) models, are formula for nonlinear models and based on the use of unconditional variance to examine the time dependent fluctuations in the data, discarding the assumption of constant error variances.

The aim of this study is to evaluate the performance of Autoregressive Integrated Moving Average ARIMA as linear model compared to ARCH, as nonlinear model, in modeling the monthly price of Libyan Brent Oil. The general objective of the study is to address the problem of variance instability in time series models. Furthermore, the study aims to highlight the importance of ARCH models in volatility modeling, and demonstrate that the Box-Jenkins methodology is less efficient than ARCH models in dealing with financial time series.

Literature Review

Engle (1982) introduced ARCH models, revealing that, these models were designed to address the assumption of non-stationary often found in real life financial data. He further not that these models have become widely used tools for dealing with time series heteroscedastic. The ARCH and GARCH models treat heteroscedasticity as a variance that needs to be modelled. The goal of such models is to provide a measure of volatility, such as the standard deviation, which can be used in financial decisions concerning risk analysis.

Hamilton (1994) emphasized the importance of forecasting conditional variance, nothing that sometimes we may be interested not only in forecasting the level of the series but also its changing variance. He further described those changes in variance are crucial for understanding financial markets, since investors require higher expected returns as compensation for holding riskier assets.

Chatfield (2000) discussed in his book that the idea behind a GARCH model is similar to that behind ARMA model. In the sense that a higher-order AR or MA model can often be approximated by a mixed ARMA model with fewer parameters using a rational polynomial approximation. Thus, a GARCH model can be seen as an approximation to a higher-order ARCH model, as similarly suggested by Ngailo Edward (2011).

Hansen and Lunde (2005) compared 330 ARCH-type models based on their ability to describe the conditional variance. They conduced out-of-sample comparisons using DM- $ exchange rate data and IBM return data, with the latter based on a new dataset of realized variance.

Igogo (2010) studied the effect of real exchange rate volatility on trade owes in Tanzania from 1968 to 2007. The studies employed recent ARCH family models to measure volatility. Initially, the GARCH (1,1) model was employed but was found to violate the non-negativity condition. The study then used the EGARCH (1,1) model proposed by Nelson (1991) to resolve this issue. The adequacy of the EGARCH (1,1) model to measure real exchange rate volatility was confirmed by testing for ARCH effect after running the model.

1.1 Methodology

1.1. 1 Time Series Models

The Box-Jenkins method is founded on statistical concepts and principles, providing a range of models that adequately represent many time series encountered in practice. This methodology, often referred to as ARIMA (Autoregressive Integrated Moving Average) models, is based on the assumption that the processes being modeled are dynamic and subject to statistical fluctuations (Box & Jenkins, 1976).

1.1.2 Determine the Order of The Model

Identifying the order of AR and MA models is often best done using the PACF (Partial Autocorrelation Function) and ACF (Autocorrelation Function), respectively. For ARIMA models, our aim was to find an appropriate model based on the ACF and PACF plots. Initial analysis suggested that ARIMA (1, 1, and 0) and ARIMA (0, 1, and 1) might be the best fit for the data.

Table (1.1): Summarizes the characteristics of theoretical ACF and PACF for stationary process.

Process	ACF	PACF
AR(p)	Tails off as exponential decay or damped sine wave	Cut off after lag p
MA(q)	Cut off after lag p	Tails off as exponential decay or damped sine wave
ARMA(p,q)	Tails off after (q-p)	Tails off after (p-q)

1.1.3 Asset Return

Most financial studies involve returns rather than prices of assets. Campbell, Lo, and MacKinlay (1997) provide two main reasons for using returns: first, the return of an asset is a complete and scale-free summary of the investment opportunity, and second, return series are easier to handle than price series due to their more attractive statistical properties.

There are several definitions of an asset return. Where p_t denote the price of a financial series at time t; the return at time t can be defined as:

$$a_t = \frac{p_t - p_{t-1}}{p_{t-1}} \quad \text{or} \quad a_t = \frac{p_t}{p_{t-1}} - 1$$

For modeling the changing volatility frequently observed in such series, Engle (1982) introduced the Autoregressive Conditional Heteroscedastic process of order q, ARCH (q) (see chapter three for more details)

1.1.4 Heteroskedasticity

In statistics, heteroscedasticity refers to the phenomenon where the standard deviations of a variable, monitored over a specific amount of time, are non-constant. This condition can arise in two forms: conditional and unconditional. Conditional heteroskedasticity occurs when the volatility of a series id non-constant, and future periods of high and low volatility cannot be predicted. In contrast, unconditional heteroskedasticity is observed when such periods of varying volatility can be identified in advance.

In finance, conditional heteroskedasticity is often seen in the prices of stocks and bonds, where the level of volatility cannot be predicted over any period of time (Lopez, 1999). Unconditional heteroscedasticity, on the other hand, is typically seen in variables with identifiable seasonal variability, such as electricity usage.

1.1.5 ARCH(q) Model

The first model that provides a systematic framework for volatility modeling is the **ARCH** model introduced by Engle (1982). The basic idea of **ARCH** models is as follows:

a) The shock at of an asset return is serially uncorrelated but dependent.

b) The dependence of a_t can be described by a simple quadratic function of its lagged values.
In **ARCH** models, the conditional variance has a structure very similar to that of the conditional expectation in an AR model (Bera & Higgins,1993). An ARCH (q) model assumes that:

$$a_t = \sigma_t \varepsilon_t,$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2$$

Let $G_t = \{a_1, a_2, \ldots, a_{t-1}\}$. Then:

$$E(a_t \backslash G_t) = 0$$

$$Var(a_t \backslash G_t) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i a_{t-1}^2$$

And the error term ε_t is such that:

$$E(\varepsilon_t \backslash G_t) = 0 \qquad Var(\varepsilon_t \backslash G_t) = 1$$

Here, $\{\varepsilon_t\}$ is a sequence of independent and identically distributed (iid) random variables with mean zero and variance 1. Additionally, $\alpha_0 > 0$, and $\alpha_i \geq 0$ for $i > 0$. The coefficients αi must satisfy some regularity conditions to ensure that the unconditional variance of a_t is finite.

1.1.6 Case Study

The data employed in this study comprises 132 monthly observations of the Libyan crude price for the Brent field, as reported in the bulletin of the Platts Market Wire, spanning from January 1st, 2000, to December 31st, 2010. Table 1.2 presents a summary of the descriptive statistics for the oil price series.

Table (1.2): Descriptive Statistics for the data series.

Statistics	Mean	Median	Std. Dev.	Kurtosis	Skewness	Maximum	Minimum
Price	52.19	49.64	25.4778	-0.2039	0.676	132.44	18.68

To check if the data follows a normal distribution, a Jarque-Bera test was performed. The p-value obtained was 0.829, which is greater than 0.05. Therefore, we accept the null hypothesis at the 5% significance level, indicating that the distribution is normal.
As stated earlier, most economic time series are non-stationary, with variance changing over time. By examining Figure 1.1, plot (a) shows the oil price series, and it is evident that the level of prices does not appear to be stationary. The plot indicates a clutter of activity, reflecting the non-stationary nature of the series. Plot (b), the Autocorrelation Function (ACF), shows that the data decays slowly, which further supports the non-stationarity of the time series. Plot (c), the Partial Autocorrelation Function (PACF), shows that the data has a cut-off after the first lag.
To address the non-stationarity of the data, various methodologies can be employed to achieve stationarity. While visual inspection of the plots provides initial insights, it is not sufficient to conclusively prove the non-stationarity of the series. Therefore, a more robust statistical tool, the unit root test, is used for further clarification.

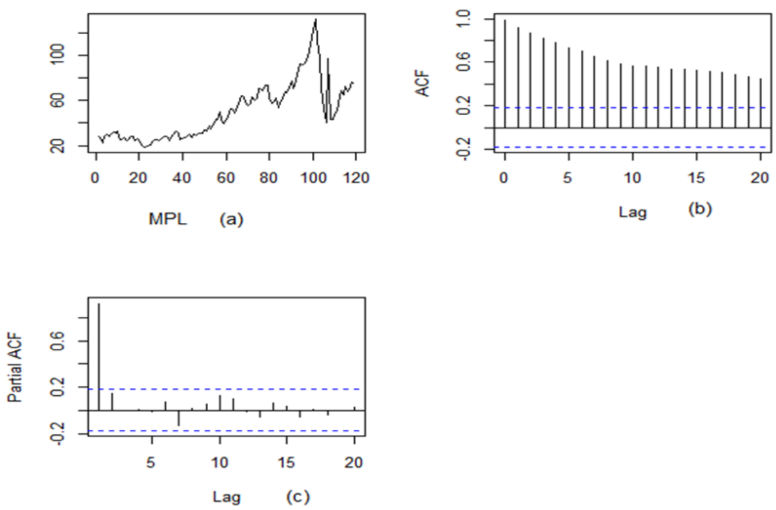

Figure (1.1) :(a) The series of monthly price Libyan of Brent oil (MPL). **(b)** The Autocorrelation Function (ACF) of the MPL time series, indicating the correlation between values over successive lags. **(c)** The Partial Autocorrelation Function (PACF) of the MPL time series, showing the correlation of the series with its own lagged values, after removing the effects of earlier lags.

4.3 The Unit Root Tests of Series (data)

Table 1.3 presents the results of the unit root tests applied to the time series data of monthly Libyan crude oil prices for the Brent field. The p-values for the Phillips-Perron (PP) and Augmented Dickey-Fuller (ADF) tests are 0.2248 and 0.1948, respectively, both of which are greater than 0.05. This suggests that we cannot reject the null hypothesis of a unit root, indicating that the series is not stationary. Conversely, the p-value for the KPSS test is 0.01, which is less than 0.05, indicating rejection of the null hypothesis of stationarity. Together, these results confirm that the time series is not stationary.

Table (1.3): Result of the unit roots test of Libyan Crude Oil Price Series.

Test	Value of statistic	P-value	Decision
PP	-24.405	0.2248	**Accepted H_0 (non-stationary)**
ADF	-2.918	0.1948	**Accepted H_0 (non-stationary)**
KPSS	3.4421	0.01	**Accepted H_1 (non-stationary)**

2.2 The First Difference

In Figure (1.2) we observe that plot (b), the Autocorrelation Function (ACF) and plot (c), the Partial Autocorrelation Function (PACF), of the first differenced time series for Libyan crude price for the Brent field indicate that the series has become stationary. This means that there is no evident trend, and both the ACF and PACF are within confidence interval supporting the stationarity of series.

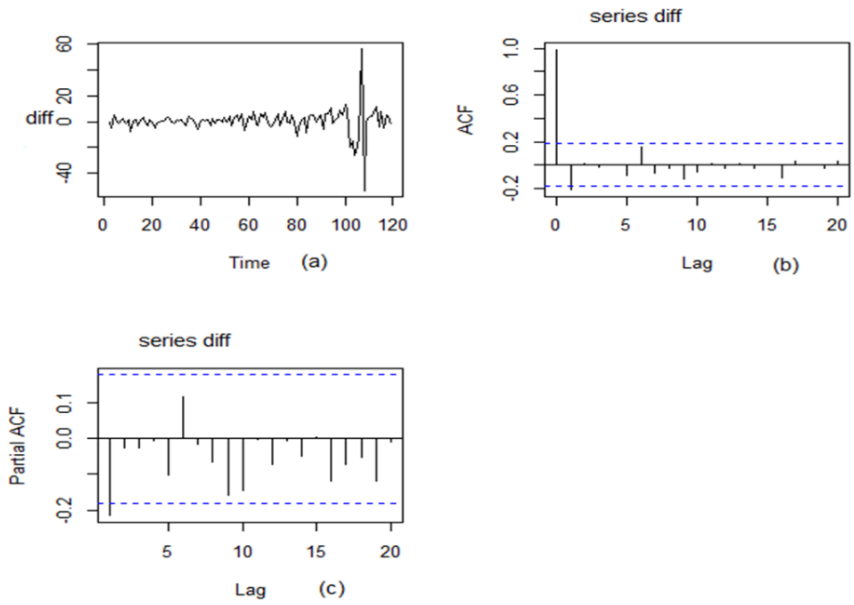

Figure (1.2) First difference of the time series, with corresponding ACF and PACF plots.

Table (1.4): Result of the Unit Roots test for First Differenced Libyan Crude Oil Price Series for the Brent field

Test	Value of statistic	P-value	Decision
PP	-150.8932	0.01	**Accepted H_1(stationary)**
ADF	-4.5532	0.01	**Accepted H_1(stationary)**
KPSS	0.0848	0.1	**(stationary)Accepted H_0**

From Table (1.4) the p-value of ADF and PP tests are both 0.01, which are less than 0.05, indicating that we reject the null hypothesis of a unit root, thus confirming stationarity. The p-value for KPSS test is 0.1, which is greater than 0.05, supporting the acceptance of the null hypothesis of stationarity. These results indicates that the time series for the monthly of Libyan crude Oil price for the Brent field is stationary after first differencing, and therefore, no further differencing is required.

2.3 Fitting of ARIMA Model

2.3.1 Model Identification

Identification of an AR model is often best done with the PACF, and while the identification of an MA model is typically done using the ACF. Our aim is to find an appropriate ARIMA model based on the ACF and PACF shown in Figure (4.2). It seems possible that an AR (1) might be best fit for our data, based on the significant spike at the first lag of PACF plot. On the other hand, an MA (1) might work well, as indicated by the significant spike in the first lag of ACF plot. This primary analysis suggests that the models to consider for this data are ARIMA (1,1,0), ARIMA (0,1,1), ARIMA (1,1,1).

2.3.2 Model Selection

The strategy used for selecting the appropriate model from the competing models is based on the Akaike Information Criteria (AIC) and Root Mean Square Error (RMSE). R programming software was used to perform trials and determine the best-fitting model. Table 1.5 provides the suggested models along with their respective fit statistics.

Table (1.5): The values of (AIC) and (RMSE) for suggested ARIMA models

ARIMA	AIC	RMSE
ARIMA (1,0,0)	946.77	9.102
ARIMA (1,1,0)	932.87	8.696
ARIMA (0,1,1)	**932.77**	8.690
ARIMA (1,1,1)	934.75	8.724
ARIMA (2,1,0)	934.78	8.725
ARIMA (1,2,0)	990.58	11.201
ARIMA (2,1,1)	934.91	**8.517**
ARIMA (2,1,2)	938.08	8.562

The results from the above Table 1.5 show that the ARIMA (0,1,1) and ARIMA (1,1,0) models had the smallest AIC values, while the ARIMA(1,1,1) model had the second smallest AIC value (934.75). The ARIMA (2,1,1) model has lowest RMSE (8.517). Therefore, we will proceed to estimate the parameters of our suggested models.

2.3.3 Model Estimation

By using R software packages to estimate the parameters of the models ARIMA (0,1,1), ARIMA (1,1,1), ARIMA (2,1,1), and ARIMA (1,1,0), we obtained the following results:

1- ARIMA (0,1,1) model:

Table (1.6): Parameter estimate for ARIMA (0,1,1)

Parameter	Ma1(θ_1)
Estimate	-0.218
P-value	0.01

From Table (1.6) the coefficient of the ARIMA (0,1,1) model is significantly different from zero. Fitted model in this case is $(\Delta\hat{z}_t) = (1 + o.218B)(\varepsilon_t)$ **(4.1)**
with estimated variance, $\hat{\sigma}_t^2$= 75.3, and log likelihood = -465.38

2- ARIMA (1,1,1) model:

This model is chosen because it seemed to fit the data well, the model is chosen based on the ACF and PACF behavior see Figure (1.2).

Table (1.7): Parameters estimate for ARIMA (1,1,1)

Parameter	AR1(ϕ_1)	MA1(θ_1)
Estimate	-0.030	0.195
P-value	0.940	0.617

From Table (1.7), the p-value of all parameters are greater than 0.05, so we accept the null hypothesis that there are not different from zero and conclude that the coefficients of the ARIMA (1,1,1) model are not significant.

3- ARIMA (2,1,1) model:

Table (1.8): Parameters estimate for ARIMA (2,1,1)

Parameter	AR1(ϕ_1)	AR2(ϕ_2)	MA1(θ_1)
Estimate	0.695	0.115	1.0
P-value	0.00	0.00	0.610

From Table (1.8), the p-value of ϕ_1 and ϕ_2 are less than 0.05, indicating that these parameters are significant. However, the p-value of θ is greater than 0.05, indicating that this coefficient. Therefore, we conclude that the model is not adequate.

4- ARIMA (1,1,0) model

From Table (1.9) we can see that the p-value is 0.013, which is less than 0.05, indicating that the coefficient is significant.

Table (1.9): Parameters estimate for ARIMA (1,1,0)

Parameter	AR1(ϕ_1)
Estimate	-0.216
P-value	0.013

fitted model in this case is $(\hat{\boldsymbol{z}}_t) = (\mathbf{1} + \mathbf{0.216})\boldsymbol{B} + \boldsymbol{\varepsilon}_t$

with estimated variance, $\hat{\sigma}_t^2$= 75.36, and log likelihood = -465.44

At this point, it can be established that, among all the identified models, the ARIMA (0,1,1), and ARIMA (1,1,0) have proven to be the best fitting models.

4.5.4 Model Checking

In time series modelling, the selection of the best-fitting model is directly related to how well the residual analysis is performed. Moreover, we can assess the properties of the residuals using the following tests:

1. We can check the randomness of the residuals by examining the ACF and PACF plots of the residuals.
2. We can assess the normality of the residuals by considering the p-value from the Jarque-Bera test and the Q-Q plot of the residuals.
3. We can check for autocorrelation in the residuals by examining the p-value from the Ljung-Box test.

First, in Figures 1.3 and 1.4, we show that (a) the residual plots of the ARIMA (0,1,1) and ARIMA (1,1,0) models, and (b) the ACF and PACF plots, do not have any significant lags, indicating that these models are good fits for representing our data.

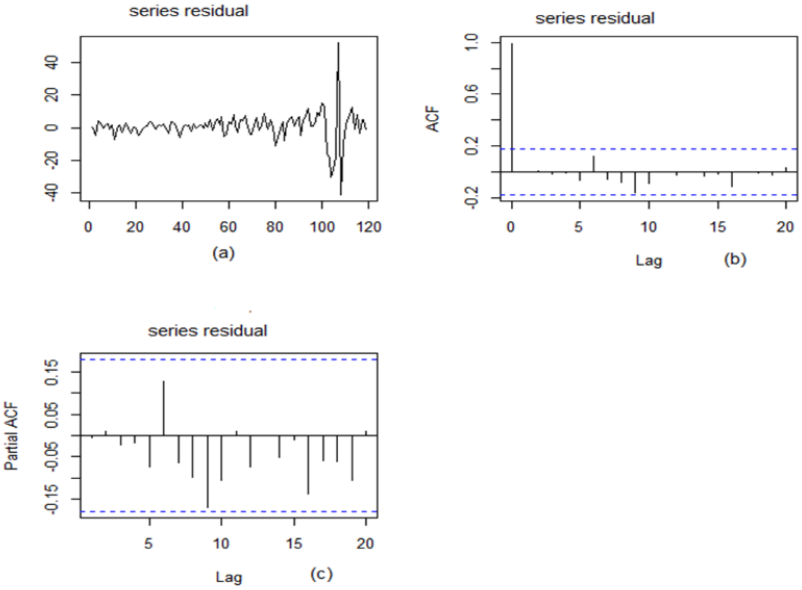

Figure (1.3): (a) The residual plot of ARIMA (0,1,1). (b) The Autocorrelation Function (ACF) of the residuals. (c) The Partial Autocorrelation Function (PACF) of the residuals.

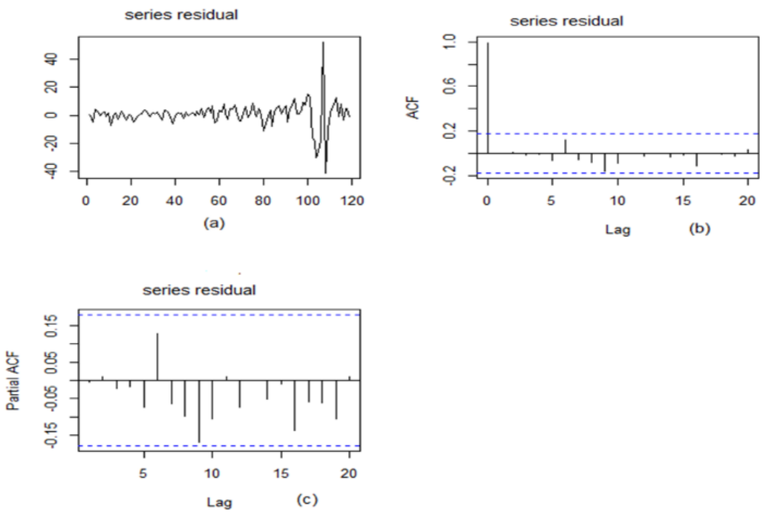

Figure (1.4): (a) The residual plot of ARIMA (1,1,0), (b) ACF, and (c) PACF

Second, from Table 1.10, the Jarque-Bera test statistic for the residuals of ARIMA (0,1,1) is 1134.173, and for ARIMA (1,1,0), it is 1223.642, both with p-values equal to 0, which are significant. The test rejects the null hypothesis at the 5% level, leading us to conclude that these distributions are not normal. Furthermore, the kurtosis values for ARIMA (0,1,1) and ARIMA (1,1,0) are 14.1 and 14.96, respectively, indicating that the distributions of these models have high peaks.

Table (1.10): Jarque-Bera test and kurtosis for residual ARIMA (0,1,1) and ARIMA (1,1,0)

Model	JB	Kurtosis
ARIMA (0,1,1)	1134.173	14.1
ARIMA (1,1,0)	1223.642	14.96

Figures 1.5 show the Q-Q normal plots of residuals, which explore the distributional shapes. These figures suggest that the distributions exhibit some non-normality in the tails, while the nearly straight lines indicate that the residuals follow approximately normal distributions.

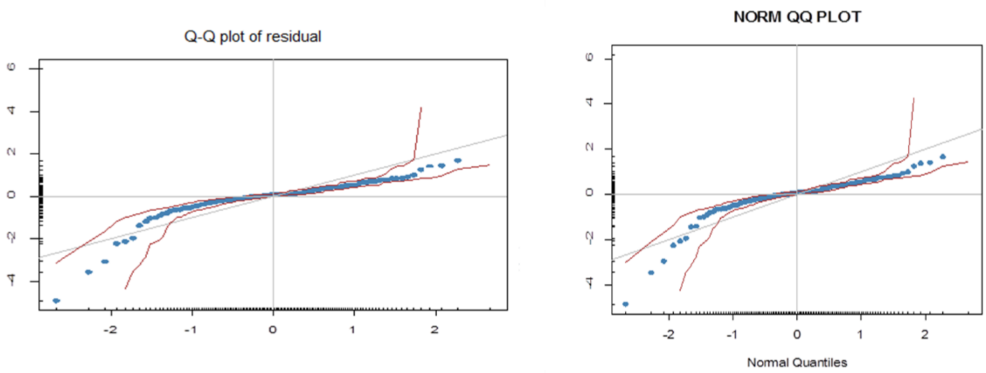

Figure (1.5): The Q-Q plot of residuals for the ARIMA (0,1,1) fit. The Q-Q plot of residuals for the ARIMA (1,1,0) fit

Finally, for further diagnostic checking, we present the results of the Ljung-Box test in Tables 1.10 and 1.11.

Table (1.10): The Ljung-Box test for ARIMA (0,1,1)		Table (1.11): The Ljung-Box test for ARIMA (1,1,0)	
Lag	**p-value**	**Lag**	**p-value**
10	0.5429	10	0.498
15	0.8262	15	0.7967
20	0.9525	20	0.9386

From Tables 1.11 and 1.12, the output from the R program shows that the p-values of all results are greater than 0.05, so we cannot reject the hypothesis that the autocorrelation is different from zero. Therefore, the selected model is appropriate for modeling the price of oil for the Brent field.

4.6 ARCH Model

By looking of the Figures 1.3 and 1.4, we see that ACF and PACF of residuals have no significant lags, but the time series plots of residuals show some cluster of volatility at the end of series. In order to model volatility, ARCH method comes into play. For double check, we will plot the squared of residual to ensure that there are indeed clusters of volatility.

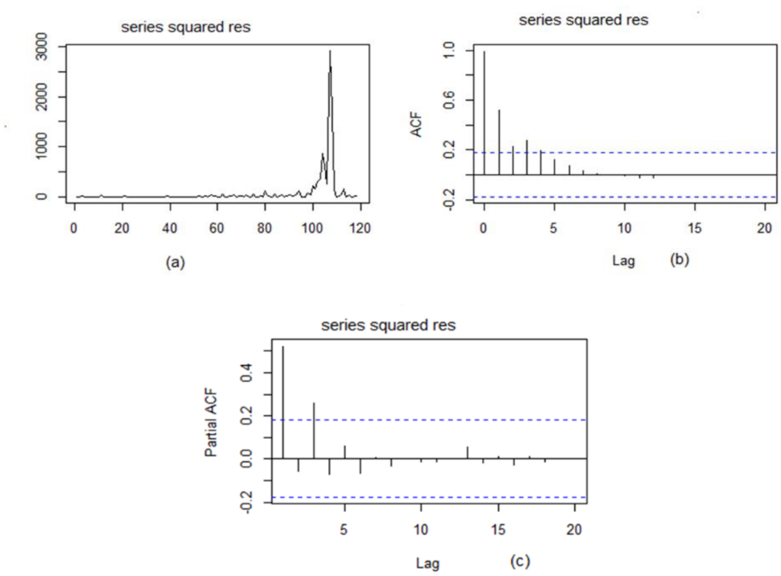

Figure (1.6): (a) The squared residual plot for the ARIMA (0,1,1) fit, (b) ACF, and (c) PACF.

From the Figure 1.6, we can see the squared residuals. Plot (a) shows cluster of volatility at certain points in time. Plot (b) shows the ACF, which seems to be die down gradually, and plot (c) shows the PACF, which cuts off after lag 2, even though some remaining lags are significant. However, the ARCH model is necessary to properly capture the volatility of the series. As the name suggests, this method focuses on modeling the conditional variance of the series.

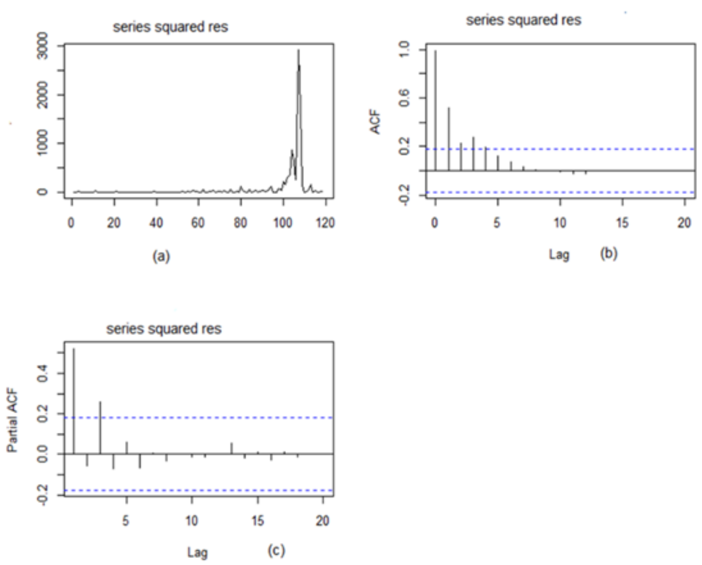

Figure (1.7): (a) The squared residual plot for the ARIMA (1,1,0) fit, (b) ACF, and (c) PACF

Figure 1.7 is similar to Figure 1.6, and the same results and comments apply.

4.7 Testing for ARCH Effects

The ARCH-LM test statistic at lags 10, 15, and 20 was computed for the ARIMA (0,1,1) and ARIMA (1,1,0) models. From Tables 1.12 and 1.13, all the p-values are less than 0.05, which means that the null hypothesis is rejected, indicating that there is an ARCH effect in both models.

Table (1.12): LM ARCH Test for residuals of ARIMA (0,1,1)			Table (1.13): LM ARCH Test for residuals of ARIMA (1,1,0)		
Lag	**Chi-squared**	**p-value**	**Lag**	**Chi-squared**	**p-value**
10	47.96	6.304e-07	10	41.59	8.85e-08
15	46.10	5.118e-05	15	46.10	0.00046
20	44.17	0.001428	20	44.17	0.0082

4.8 Model Selection and Analysis

The strategy used to select the appropriate model from competing models is based on the minimum value of Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). A quick comparison of the results obtained from R software was conducted to fit ARCH models, using the residuals tests of the ARIMA (0,1,1) and ARIMA (1,1,0) series to determine the best fitting model.
Table 1.14 presents the suggested models along with their respective fit statistics. The goal is to have a parsimonious model that captures as much variation in the data as possible. Typically, the mixed ARCH model captures most of the variability in stabilized series. In the table below, a smaller AIC value indicates a better fit.

Table (1.14): Comparison of suggested ARIMA-ARCH models

Model	AIC	BIC
Model (1) =ARIMA (0,1,1)-ARCH (1)	6.15	**6.24**
Model (2) =ARIMA (0,1,1)-ARCH (2)	6.15	6.25
M0del (3) =ARIMA (0,1,1)-ARCH (3)	6.17	6.30
Model (4) =ARIMA (1,1,0)-ARCH (1)	6.17	6.26
Model (5) =ARIMA (1,1,0)-ARCH (2)	6.15	6.26
Model (6) =ARIMA (1,1,0)-ARCH (3)	6.17	6.30

Table 1.14 shows the competing models along with their AIC and BIC values. Notice that models 1, 2, and 5 have the same lowest AIC value (6.15), while model 1 has the smallest BIC value (6.24). Therefore, we can conclude that model 1 is the best fit for the data.

4.9 Model fit for model (1)

Using the method of maximum likelihood, we derived our models and used the 'garchFit' function from the R package 'fGarch' to estimate the coefficients of model 1.

Table (1.15): Parameters estimates for model (1)

Parameter	**mu(μ)**	**Ma1(θ_1)**	**Alpha1(α_1)**
Estimates	0.86	-0.24	0.76
p-value	0.004	0.001	0.00017

From Table 1.15, the coefficients of model 1 are significantly different from zero, and the estimated values satisfy the stationarity condition, where $\alpha_0 > 0$, $\alpha_i \geq 0$. The full model 1 with the estimated coefficients is represented as:

$$a_t = \sigma_t \epsilon_t \quad z_t = -0.24\varepsilon_{t-1} + \varepsilon_t$$

$$\sigma_t^2 = 0.86 + 0.76 a_{t-1}^2$$

4.10 Diagnostic Checking of Model (1)

Before we accept a fitted model and interpret its findings, it is essential to verify whether the model is correctly specified. In time series modeling, the selection of the best-fitting model is directly related to whether residual and squared residual analyses are performed correctly. One of the assumptions of the ARCH model is that, for a good model, the residuals must follow a white noise process. Therefore, we conducted the same analysis for model 1 that we did for ARIMA(0,1,1) and ARIMA(1,1,0) in Section 4.5.4.

For the model (1) fitted to the price for the Brent Oil residual data, we obtained the following results:

First, the p-value of the Jarque-Bera test is 0.15, so the test accepts the null hypothesis at the 5% significance level, indicating that the distribution is normal.

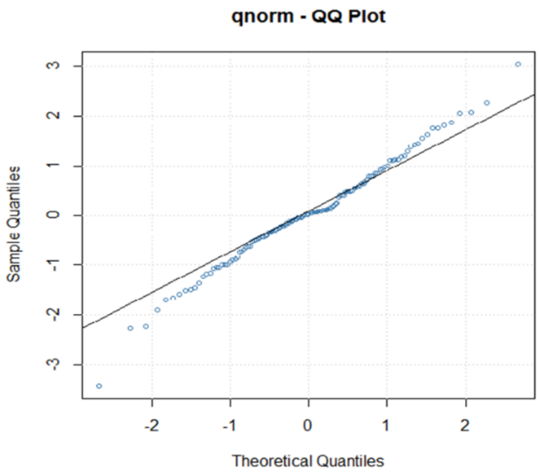

Figure (1.8): Normal Q-Q plot of residuals

Figure (1.8) shows that the Q-Q plot of the residuals is nearly a straight line, suggesting that the residuals follow an approximately normal distribution.

Second, if the model successfully captures the serial correlation structure in the conditional mean and conditional variance, then there should be no autocorrelation left in the residuals and squared residuals. Figure 1.9 provides the ACF plot of the residuals, which shows that no correlation remains.

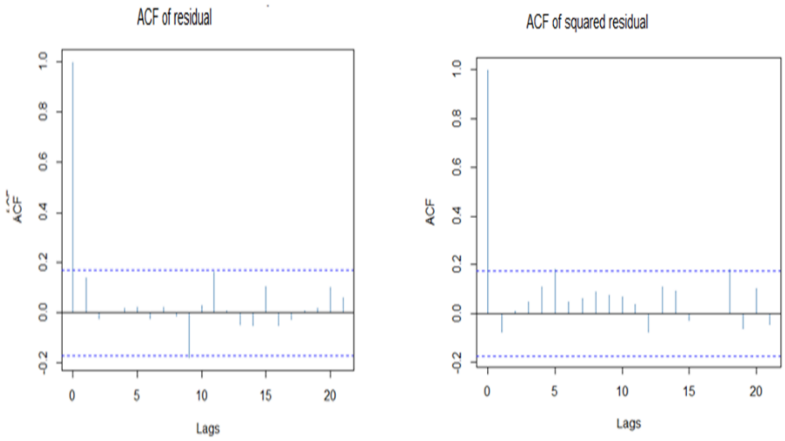

Figure (1.9): ACF plot of residuals of model (1). Figure (1.10): ACF plot of squared residuals of model (1)

Figure 1.10 provides the ACF plot of the squared residuals. The diagnostic checking reveals no new patterns, so we can assume that our model is adequate. We generated forecasts for a few periods (one year ahead), which are shown in the next section.

Finally, Table 1.16 shows that the p-values of the Box-Ljung Q statistics for the residuals of model 1 are greater than the 0.05 significance level. Therefore, we fail to reject the null hypothesis, indicating that there is no autocorrelation left in the residuals.

Table (1.16): Q - Test of residuals using model (1)		**Table (1.17): Q - Test of squared residuals using model (1)**	
Lag	**p-value**	**Lag**	**p-value**
10	0.64	10	0.33
15	0.49	15	0.439
20	0.67	20	0.12

Table 1.17 shows that the p-values of the Box-Ljung Q statistics for the squared residuals of model 1 are greater than the 0.05 significance level. Therefore, we accept the null hypothesis, indicating that there is no autocorrelation left in the squared residuals.

ARCH Effect Test:

Furthermore, we can check if there is an ARCH effect in the residuals using the ARCH test. The p-value is 0.31, which is greater than 0.05, so we accept the null hypothesis that there is no ARCH effect left (no heteroscedasticity). Based on all the results, we conclude that model 1 is the appropriate model for our data, and we proceed to use the model to forecast future values of the oil price series.

4.11 Forecasting With the Model (1)

In Figure 1.11, we observe that there are no significant changes in the forecasted values, which means that most of them remain the same after the first month. Our explanation for this is that in the IMA (1,1) model, the forecast will be flat after the first point. If there is an ARCH component, the forecast will technically never be flat, but the oscillations will "die out" and become smaller and smaller, potentially appearing flat to the eye, as seen in Figure 1.11.

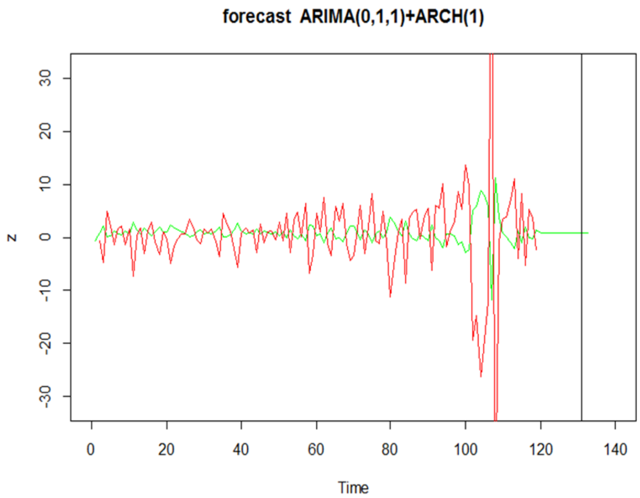

Figure (1.11): Forecasting model (1)

Next, we compare the ARIMA (0,1,1) model with model 1 using three benchmarks: Mean Error (ME), Root Mean Square Error (RMSE), and Mean Absolute Error (MAE). The results of these tests are presented in Table 1.20.

Table (1.20): Results of the benchmark evaluation for the two types of models

Model forecast	MSE	RMSE	MAE
ARIMA (0,1,1)	73.96	8.6	4.9
Model (1)	73.96	8.6	4.7

The results show that the **model (1**) has smaller benchmarks values, with an MAE of 2.7, while the MSE and RMSE of both models are 73.96 and 8.6, respectively. Hence, we can conclude that the **model (1)** is better than the **ARIMA (0,1,1) model**.

5.1 Conclusions

This study aimed to identify the appropriate model to predict the monthly price of Libyan Brent Oil. The data, characterized by heteroscedasticity and variability across different periods, is typical of economic and financial time series. The research focused on comparing two different models: the Box-Jenkins ARIMA model and the ARCH model.

The study was conducted in two parts. In the first part, a detailed examination of the ARIMA model was performed. Two models, ARIMA (0,1,1) and ARIMA (1,1,0), were selected from eight suggested models based on the minimum AIC and RMSE values. The residuals of both models were then diagnostically checked using the Jarque-Bera test and Q-Q plot for normality, and ACF and PACF diagrams for randomness. The Ljung-Box test was used to check for autocorrelation, and both models were found to be suitable. However, the time series plot of residuals revealed clusters of volatility. This prompted a re-examination of the squared residuals, where clusters were indeed detected. Consequently, the ARCH effect was tested using the ARCH-LM test, which confirmed the presence of the ARCH effect, making these models less ideal for this data.

In the second part of the study, ARCH models were explored. The selected model was the mixed ARIMA (0,1,1)-ARCH (1), chosen from six suggested models based on the smallest AIC and BIC values. This model underwent a detailed examination similar to the ARIMA models, with the residuals and squared residuals analysed. It was concluded that the ARIMA (0,1,1)-ARCH (1) model is the most accurate for data classification. A 12-month in-sample prediction and a 12-month out-of-sample forecast were conducted, and the results were compared with those from the ARIMA (0,1,1) model using the ME, RMSE, and MAE evaluation criteria. The mixed model ARIMA (0,1,1)-ARCH (1) outperformed the ARIMA (0,1,1) model.

This leads to the conclusion that the ARCH model is more suitable than the ARIMA model for this specific type of data.

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