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The Study of Fixed Point Theorem Over Modular F-Metric Spaces

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Abstract
One of the most significant discoveries in the annals of mathematical history is Schauder's fixed-point theorem, which is generally recognized to be among the most important discoveries. The fact that the Brüwer fixed point hypothesis cannot be used in dimensions of space that are infinitely large is one of the most important discoveries in the history of mathematics. The facts that have been supplied make it feasible to claim that the great majority of them are of a topological nature. In 2019, N. Manav and D. Turkoglu introduced a new class of generalized metric space called modular F metric space as a generalization of metric space. It is generally agreed upon that this is one of the most significant discoveries that has been made in the subject of mathematics that has ever been produced. In this article, we study the basic structure of modular F-metric spaces and the concept of an equivalent relationship between modular F metric space and modular F metric bounded space. Moreover, we study the fixed point theorem (New version of Banach Contraction Principle) over modular F metric spaces. This is something that one would be able to predict occurring given the circumstances that are present. These results extend, broaden, and integrate many previously published results.

Keywords: F-metric space, fixed point, modular F-metric spaces, Banach contraction principle

1. Introduction

M. Frechet's 1906 introduction of the concept of an abstract metric space serves as a unifying idealization for a wide variety of mathematical, physical, and other scientific constructions that include the concept of distance. The concept of a metric space is important in various scientific disciplines. Due to the characteristics of the mathematical sciences, many efforts have been made to extend the metric setup by modifying some metric space axioms. Thus, many new kinds of spaces were developed, and many metric discoveries were extended to new contexts, such as modular b-metric spaces, symmetric spaces, fuzzy metric spaces, vector metric spaces, S-metric spaces, b-metric spaces, dislocated b -metric spaces, etc.

As an extension of the idea of metric space, Jelli and Samet developed a new concept called $\mathcal{F}$ -metric space. Various authors have recently concentrated their attention on $\mathcal{F}$-metric spaces and their properties. N. Manav and D. Turkoglu introduced a new class of generalized metric space called modular $\mathcal{F}$ -metric space as a generalization of metric space. We begin by defining and illustrating the necessary concepts and results in metric spaces, since this will be useful throughout the study. Next, we establish an equivalent relationship between modular $\mathcal{F}$-metric space and modular $\mathcal{F}$-matric bounded

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space. Finally, we establish a fixed point theorem for modular $\mathcal{F}$-metric spaces that needs just one similar inequality on the basis of Huang, Deng, and Radenovic's findings for b-metric spaces.

2. Basic Structure of Modular F-Metric Spaces

The importance of this theory is wide-ranging because it can deal with a broad class of mathematical disciplines. For pure mathematics in general, effort is mainly focused on developing and refining some appropriate criteria many that are able to be used to establish the existence and uniqueness results for very many special types of problems. However, with applied mathematics, concentration will be given to how these solutions are computed. Let$Y \neq \phi$, $u: (0, \infty) \times Y \times Y \longrightarrow [0, \infty]$. we denote $u_A(z, w) := u(A, z, w)$ for all $A > 0, z, w \in Y$ so that $u = \{u_A\}_{A>0}$ where$u_A: Y \times Y \longrightarrow [0, \infty]$. Then u is said to be a modular on Y if it satisfies the following properties: (a) $z = w \iff u_A(z, w) = 0$, for all $A > 0$; (b) $u_A(z, w) = u_A(w, z)$, for all $A > 0$, and $z, w \in Y$(c) $u_{A+B}(z, w) \leq u_A(z, s) + u_B(s, w)$, for all $A, B > 0$ and $z, w, s \in Y$. In this part, we will review some essential concepts and findings. The following notation was given by Jleli and Samet B. Assume that F is a collection of functions $g: (0, +\infty) \to \mathbb{R}$ that fulfill the following conditions: $(\mathcal{F}_1)$ $0 < s < t \Longrightarrow g(s) \leq g(t)$.$(\mathcal{F}_2)$ We have, for each sequence $\{u_n\} \subset (0, +\infty)$, $\lim_{n \to +\infty} u_n = 0 \iff \lim_{n \to +\infty} g(u_n) = -\infty$. The conditions that were described earlier are the consequences of the fixed point analysis, and these are the situations that supply the circumstances in which the solutions in this instance map make their appearance. As follows, we extend the notion of metric spaces. Let Y be a non - empty set and (h, β) $\in$ F $\times [0, \infty)$. Assume that B : Y× Y → $[0, \infty)(R)$ be a function such that (D1) for all (r, s) ∈ Y× Y , B(r, s) = 0 iff r $= s$ (D2) B(r, s) = B(s, r) , for all (r, s) ∈ Y× Y (D3) for every (r, s) ∈ Y× Y , for each M∈ ℕ , M ≥ 2 and for every $\{ s_j \} \subset$ Y, j= 1,2, ... M. with $(s_1 , s_M) = (r , s)$, we have B(r, s) > 0 implies h(B(r, s)) ≤ h ($\sum_{J=1}^{M-1} B(s_j, s_{J+1})$) + β. Then B is referred to as a function weighted metric or $\mathcal{F}$ -metric, while the pair (Y, B) is referred to as a function weighted metric space or F-metric space. In addition to this particular setting, it is of the utmost importance to acknowledge the relevance of these answers in other situations.

Examples on F-metric spaces:

The purpose of this section is to offer an example of what we already know about the fixed point theorem, in addition to giving some additional information. Let Y = ℕ and

B: Y× Y → (0, ∞) be defined by $B(u, v) = \begin{cases} |u - v| & \text{if } (u, v) \notin [0, 3] \times [0, 3] \\ (u - v)^2 & \text{if } (u, v) \in [0, 3] \times [0, 3] \end{cases}$

for all (u, v) ∈ Y× Y. Then B is an F-metric on Y. Keeping in mind that it provides an explanation of the circumstances under which the solutions in this instance map, which is advantageous in other situations in addition to this one, it helps to guarantee that this is relevant in other scenarios. Let Y = [1, 4] and define a mapping B: Y× Y → ℝ by

$$B(u, v) = \begin{cases} u + v, & \text{when } u \neq v \text{ and } u, v \in [1, 2]; \\ 2(u + v), & \text{when } u \neq v \text{ and } u, v \in [2, 3]; \\ 3(u + v), & \text{when } u \neq v \text{ and } u, v \in [3, 4]; \\ 0, & \text{when } u = v \\ 1, & \text{otherwise} \end{cases}$$

Then (Y, B) is an F-metric space with $h(u) = -\frac{1}{u}$ and $\beta = 2$. The fixed point theorem with three points, which is also known as the discrete fixed point system, is used in order to accomplish the goal of producing the results. This system is sometimes referred to by the discrete fixed-point system. Let (Y, d) be a sequentially compact F -metric space and H be a self-map on Y such that g(d(Hx, Hy)) < g(d(x, y)) for all x,y ∈ Y with x ≠ y, where g is an altering distance function. Also, suppose that the F -metric d is continuous. Then H has a unique fixed point and for any x∈Y, the sequence (H^mx) is F -convergent to that fixed point. An example of a mathematical theory that asserts that there can be only one fixed point in a topological space is the fixed point theorem, which is also sometimes referred to as the topological fixed point theorem. It is possible to refer to this theorem as the topological fixed-point theorem under certain situations. Let (Y, d) be a sequentially compact F -metric space and H be a self-map on Y such that d(Hx, Hy) < d(x, y) for all x,y ∈ Y with x ≠ y, where g is an altering distance function. Then H has a unique fixed point in X and for any x∈Y, sequence (H^mx) converges to that fixed point. Let (Y, B) be an F-metric space. Additionally, an overview of the advancement of the theory is provided in this section. In addition, a full examination of the phenomena is presented in this part. A sequence $\{u_n\} \subset Y$ is said to be F-converge to a point $u_0 \in Y$ if $\lim_{n\to\infty}$ B(u_n, u_0) = 0. Furthermore, it offers a comprehensive examination of the phenomena that are being addressed in the course of the discussion. An F-metric space (Y, B) is said to be F-complete if any F-Cauchy sequence F- converges in this space. The idea of fixed points in infinite-dimensional space was introduced and given the chance to test it, prove it, or dispute it. A function g: [0,1)→[0,1) is called an altering distance function if (i) g is continuous, (ii) g is non-decreasing, (iii) g(s) = 0 iff s= 0. Let (Y, B) be a F-metric space. A sequence $\{u_n\} \subset Y$ such that for all ∈ > 0 there exist M ∈ ℕ for all i, j ∈ ℕ: i, j > M implies B(u_i, u_j) < ∈ is called a F- Cauchy sequence. There is a clear connection between this approach and the theory of fixed points as well. Taking this method is something that may be used in both theoretical and practical contexts. Through the use of this method, it is possible to recognize a broad variety of different potential solutions to the issue that is now being faced. Let Y = ℕ and B: Y× Y → (0, ∞) be defined by B (u, v) = $\begin{cases} e^{|u-v|} \; when \; u \neq v \\ 0 \; when \; u = v \end{cases}$ for all (u, v) ∈ Y× Y. Then B is an F-metric on Y. There are certain circumstances in which there is only one feasible solution to a problem that has been discovered. Roth presented a research article on fixed point theory for non-self-maps. This was one of the many assertions that the fixed point theorem makes. The study that was conducted resulted in the establishment of a precedent, and it is this precedent that is still being used in the area to this day. We begin by defining modular $\mathcal{F}$ -metric as a more extended form of the terms metric, modular metric, and $\mathcal{F}$ -metric. Let $Y \neq \phi$ and $D_A: (0,\infty) \times Y \times Y \to [0,\infty]$ be a function. If $\exists \; (g, \alpha_1) \in \mathcal{F} \times R$ s. t. $(F_\lambda 1)$ $D_A(r,s) = 0 \Leftrightarrow r = s$, for all $(r,s) \in Y \times Y$; $(F_\lambda 2)$ $D_A(r,s) = D_A(s,r)$ for all $(r,s) \in Y \times Y$; $(F_\lambda 3)$ For all $(r,s) \in Y \times Y, p \in \mathbb{N}$ with $p \geq 2$ and for all $(v_i)_{i=1}^p \subset Y$ with $(v_1, v_p) = (r,s)$, we have;

$$D_A(r,s) > 0 \text{ implies } g(D_A(r,s)) \leq g\left(\sum_{j=1}^{p-1} D_{\frac{A}{j}}(v_j, v_{j+1})\right) + \alpha_1$$

then D_A is called an modular $\mathcal{F}$ -metric on Y. The pair (Y, D_A) is called an modular $\mathcal{F}$ -metric space. Optimal approximants have been demonstrated to exist in modular function spaces, as demonstrated by Wojciech M. Kozlowski. It is through the use of sub lattices, which are elements, that is accomplished. Not only can modular function spaces demonstrate the intrinsic generalization of Lp, which can be described as the situation in which p is greater than nothing, but they also have the ability to depict the Orlicz, Lorentz, and Kothe spaces. For the purpose of indicating a pseudo modular, the symbol ϱ is employed, whereas the modular function space that is associated with it is symbolized by the sign Lϱ. A sub lattice of the letter Lϱ is represented by the letter C, which is another point of interest. Given that f is an aspect of the class Lϱ, the purpose of this study is to undertake an analysis of the minimization problem. This includes identifying an element h in C such that the infimum of ϱ(f − h) is equal to the infimum of ϱ(f − g) for all g in C. The primary objective of this study is to determine the optimal solution to the minimization problem. This is the term that is utilized whenever the term is being utilized. Consider the space (Y, D_A) to be a modular $\mathcal{F}$ -metric space. Then $\{u_n\}_{n\in\mathbb{N}}$ is modular $\mathcal{F}$ -convergent to $u \iff D_A(u_n, u) \to 0$ as $n \to \infty$. Further consider the space (Y, D_A) to be a modular $\mathcal{F}$ -metric space and $\{u_n\}_{n\in\mathbb{N}}$ be a sequence in Y. Then

$$(u, v) \in Y \times Y, \lim_{n\to\infty} D_A(u_n, u) = \lim_{n\to\infty} D_A(u_n, v) = 0 \Longrightarrow u = v.$$

The obstacles that are connected with the process of obtaining optimal approximants are a large focus of study in two areas of study approximate theory and probability theory. Both of these fields of study place a significant emphasis on the challenges. A close connection exists between the process of discovering optimal approximants and the problem of nonlinear prediction when C is Lϱ(B) for a σ-sub algebra B of the original σ-algebra. The occurrence of this phenomenon occurs in situations when C is Lϱ(B). Now, let us define modular $\mathcal{F}$ -Cauchy sequence, then completeness definition and conditions for modular $\mathcal{F}$ metric space. Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. Let $\{x_n\} \subset X$ be a sequence in$X$. (i) We say that $\{x_n\} \subset X$ is modular $\mathcal{F}$ -Cauchy, if $\lim_{n,m\to\infty} F_\lambda(x_n, x_m) = 0$. (ii) We say that (X, F_λ) modular $\mathcal{F}$ -complete, if every modular $\mathcal{F}$ -Cauchy sequence is modular $\mathcal{F}$ -convergent to a certain element in$X$. Using a computer-based iterative technique to find the fixed point of a contractive map in order to get at the appropriate solution via the use of an iterative strategy is not only a possibility, but it is also a highly possible prospect. Specifically, this is due to the fact that the implementation of an iterative technique is a highly viable possibility. In addition, because of this, there is a probability that it will prove to be very helpful in the long term. Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. If $\{x_n\} \subset X$ is modular $\mathcal{F}$ -convergent, then it is modular $\mathcal{F}$ - Cauchy. The definition of modular $\mathcal{F}$ -compact set is giving more details about the topology on modular $\mathcal{F}$ -metric space.Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. Let $C \subset X$ be a nonempty subset. We say that C is modular $\mathcal{F}$ -compact if C is compact with respect to the topology τ_{F_λ} on$X$. Moreover let (X, F_λ) be a modular $\mathcal{F}$ -metric space. This is a result that may be attributed to the fact that it took place. As far as I am concerned, this is something that needs to be taken into consideration in relation to the circumstance. In addition to being portable and able to be moved about with a fair amount of convenience, let C be a nonempty subset of X then, the following statements are equivalent: (i) C is modular $\mathcal{F}$ -compact. (ii) For any sequence $\{x_n\} \subset C$ there exist a subsequence $\{x_{n_k}\} \subset \{x_n\}$ and $x \in A$ such that $\lim_{k\to\infty} F_\lambda(x_{n_k}, x) = 0$. Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. Let $C \subset X$ be a nonempty subset. The subset C is called modular sequentially $\mathcal{F}$ -compact, if for any sequence$\{x_n\} \subset C$, there exist a subsequence $\{x_{n_k}\} \subset \{x_n\}$ and $x \in C$ such that $\lim_{k\to\infty} F_\lambda(x_{n_k}, x) = 0$. consequently, it will be feasible to achieve the level of precision that is required in this situation. When seen from this perspective, it is possible to carry out such an action. In the event that all of the factors that have been taken into consideration are taken into account, there is a possibility that this may end up occurring. Further let (X, F_λ) be a modular $\mathcal{F}$ -metric space. Let $C \subset X$ be a nonempty subset. The subset C is called modular $\mathcal{F}$ -totally bounded, if for any $r > 0$ there exists a sequence $(x_j), j = 1,2, \ldots, n \subset C$ such that $C \subset \bigcup B_{F_\lambda}(x_j, r)$. Let (X, F_λ) be a modular $\mathcal{F}$ -metric space. Let $C \subset X$ be a nonempty subset.

Then (i) C is modular $\mathcal{F}$ -compact $\Leftrightarrow C$ is modular sequentially $\mathcal{F}$ -compact. (ii) C is modular $\mathcal{F}$ -compact $\Rightarrow C$ is modular $\mathcal{F}$ -totally bounded.

3. Fixed Point Theorem (New version of Banach Contraction Principle) over Modular F Metric Spaces

The study by Filipović and Kukić demonstrated the existence of new theorems. Theorems are established by including additional conditions essential for their proof; let T be a self-mapping on complete b-metric space (X,,s≥1) satisfying. It seems like it would be acceptable considering what kind of event occurred just recently. We have not done so, supposing that b-metric measurements are continuous

$db(T\chi,T\zeta) \leq \lambda db(\chi,\zeta)+\mu db(\chi,T\chi)+\delta db(\zeta,T\zeta)$,db(Tχ,Tζ)≤λdb(χ,ζ)+μdb(χ,Tχ)+δdb(ζ,Tζ),

Subsequently, we shall just provide the formulations of those theorems, while the proofs may be found. This will be consistent moving forward. for all $\chi,\zeta \in X$χ,ζ∈X, where $\lambda,\mu,\delta \geq 0$λ,μ,δ≥0 with $\lambda+\mu+\delta<1$λ+μ+δ<1 and $\delta<1s$.δ<1s. Then there is a unique fixed point of T. While Filipović and Kukić have argued this, we are facing with theorems which are very new.' Awareness of this was expressed by the subjected once he became actively involved in dealing with how to get information to solve certain task. Let (X,≥1)(X,db,s≥1) be a complete b-metric space and T:X→XT:X→X be a mapping such that we did not give some other conditions for understanding those theorems because we think that b-metric measurements are continuous.

$db(T\chi,T\zeta) \leq a1db(\chi,\zeta)+a2db(\chi,T\chi)+a3db(\zeta,T\zeta)+a4db(\chi,T\zeta)+a5db(\zeta,T\chi)$,db(Tχ,Tζ)≤a1db(χ,ζ)+a2db(χ,Tχ)+a3db(ζ,Tζ)+a4db(χ,Tζ)+a5db(ζ,Tχ), But it is a condition which must have been given more other conditions. it is a direct result of her present conditioned state that we did not give some other conditions. Consider all the values $\chi,\zeta \in X$χ, ζ∈X, $a1,a2,a3,a4,a5 \geq 0$ such that $a1+a2+a3$+s($a4+a5$)<1 and for all χ, ζ ∈ X , χ < ζ , a1 + a2 + a3 + s(a4 + a5) < 1 and ai > 1 − 2s.This indicated difficulty that was caused from there. We faced with releases these leads so far and he loses awareness of existence about these leading at all. There exist γ = α − β and δ > 0 such that if |γ| < δ then T has no fixed point. It was very important to use them for reaching them so far since he used. He can be particularly released those leading from all mention clearly. So, now stays only realization of formulation of those theorem as well as brief explanation of proving those statements. Let Y be a nonempty set and $D_A: (0,+\infty) \times Y \times Y \to R$ be a mapping that satisfies $(\mathcal{F}_1)$ and $(\mathcal{F}_2)$ with respect to $(g,\alpha) \in \mathcal{F} \times R$, than we say that (Y,D_A) is modular $\mathcal{F}$ -metric bounded space, if there is a metric C_B on Y s. t

$(x,y) \in Y \times Y, D_A(x,y) > 0$ implies $g(C_B(x,y)) \leq g(D_A(x,y)) \leq g(C_B(x,y)) + \alpha$ (3.1)

Now, we are prepared to present and demonstrate our primary findings. If we have modular $\mathcal{F}$ -metric with (f,α), then we have bounded modular $\mathcal{F}$ -metric with (f,α) as below: Let Y be a nonempty set, and let $F_\lambda: (0,+\infty) \times Y \times Y \to R$ be a given mapping satisfying $(F_\lambda 1)$ and $(F_\lambda 2)$. Let $(g,\alpha_1) \in \mathcal{F} \times R$ and suppose that g is continuous from the right. Then the following statements are equivalent: (i) (Y,F_λ) a modular $\mathcal{F}$ -metric on Y with (g,α_1) defined above. (ii) (Y,F_λ) Is an modular $\mathcal{F}$ -metric bounded on Y with respect to (g,α_1). The concept of a metric space has significant relevance across several scientific areas.

Assume that (Y, F λ) is an modular F -metric on Y with respect to (g, α_1). Let us define the mapping d_1: Y×Y→R by due to the characteristics of the mathematical sciences, several attempts have been undertaken to expand the metric framework by altering certain axioms of metric spaces. d_1 (z,w)=inf{∑_(i=1)^(M-1)▒F_(λ/k) (v_i,v_(i+1)):M∈N,N≥2,(v_i)_(i=1)^M⊂Y,(v_1,v_M)=(z,w)}

Multiple efforts have been futile. These endeavors have occurred at various points throughout history. For all (z,w)∈Y×Y. We shall prove that d_1 is a metric on Y. Since Fλ (z,z)=0 for all z∈Y, and λ∈(0,∞).This resulted in the creation of various new places and the use of unique measurement outcomes in novel contexts. Thus F_(λ/k) (z,z)=0 for all z∈Y, and λ/k∈(0,∞) (Because λ∈(0,∞) implies λ/k∈(0,∞))Recently presented notions include modular b-metric spaces It follows from the definition of d_1 that $d_1(z,z)=0,\ z\in Y$ Evidence shows that many fixed-point theorems have been formulated by Liu, Y. for hybrid contractive scenarios. Indeed, a group of people managed to formulate these theorems. They were manufactured using the so-called common property, also often referred to as E.A. in some contexts. Now, let $(z,w)\in Y\times Y$ be s. t. $z\neq w$. Suppose that $d_1(z,w)=0$. Let$\varepsilon_1>0$, by the definition of d_1, $\exists\ \ M\in\mathbb{N}, M\geq 2$, and $(v_i)_{i=1}^{M}\subset Y$ with $(u_1,u_M)=(z,w)$ s. t. The formulation of these theorems was accomplished utilizing the so-called common property which constituted their material and was its starting point of construction

$$\sum_{i=1}^{M-1} F_{\frac{\lambda}{k}}(v_i,v_{i+1})<\varepsilon_1$$

We obtain

$$g\left(\sum_{i=1}^{M-1} F_{\frac{\lambda}{k}}(v_i,v_{i+1})\right)\leq g(\varepsilon_1) \qquad (3.2)$$

For convenience, the following are some additional particulars that need to be taken into consideration. Even if this is only one of the many reasons why it is interesting, the fact that it is fascinating for this reason is just one of many reasons why it is intriguing. There is a large number of other factors that contribute to this. On the other hand, by $(F_\lambda 3)$, we have

$$g(F_\lambda(z,w))\leq g\left(\sum_{i=1}^{M-1} F_{\frac{\lambda}{k}}(v_i,v_{i+1})\right)+\alpha_1 \qquad (3.3)$$

In the event that all of the factors that have been taken into consideration are taken into account, there is a possibility that this may end up occurring. Additionally, it makes it possible to provide a forecast about the number of iteration operations that will be required in order to achieve a specific degree of accuracy in the findings. The results will be analyzed more closely in paragraphs that follow hereinafter. For this person's opinion, Ali, J., along with his co-workers alone formulated this concept. Using (3.2) and(3.3), we obtain

$$g(F_\lambda(z,w))\leq g(\varepsilon_1)+\alpha_1,\ \varepsilon_1>0$$

But, using$(\mathcal{F}_2)$, we have $\lim_{\varepsilon\to 0^+}(g(\varepsilon_1)+\alpha_1)=-\infty$ a contradiction. The application of this concept was later extended to probabilistic metric spaces and positive results came into being for both sides related to that affair thanks to research encounter during other situations having fulfilling this purpose during questions on fixed points has progressively accrued in time such a specifically equipped. Therefore, we have $d_1(z,w)>0$ from the definition of d_1 and $(F_\lambda 2)$, it can be easily seen that$d_1(z,w)=d_1(w,z)$, for all$(z,w)\in Y\times Y$. In order to check the triangle inequality, let z,w and x be three given points in Y, and let $\rho_1>0$. By the definition of d_1, there exist two chains of points $z=v_1,v_2,\cdots,v_N=w$ and $w=v_N,v_{N+1},\cdots,v_P=x$ s. t.

$$\sum_{i=1}^{N-1} F_{\frac{\lambda}{k}}(v_i,v_{i+1})<d_1(x,w)+\rho_1$$

And Also multiplied during time since event particularly indicated above occurred such a specifically equipped warehouse has indeed amassed over quite an elapsed time $\sum_{i=N}^{P-1} F_{\frac{\lambda}{k}}(v_i, v_{i+1}) < d_1(w,z) + \rho_1$

Adding the above inequalities, we obtain

$$d_1(z,x) \leq \sum_{i=1}^{P-1} F_{\frac{\lambda}{k}}(v_i, v_{i+1}) < d_1(z,w) + d_1(w,x) + 2\rho_1,\ \rho_1 > 0$$

And there is a lot presented literature including many studies where questions concerning fixed points have been posed primarily published relating papers concerning subject matter targeted

Passing to the limit as$\rho_1 \to 0^+$, we get $d_1(z,x) \leq d_1(z,w) + d_1(w,x)$ As consequence, we deduce that d_1 is a metric on Y. Next, we shall prove that d_1 satisfies (3.1). Let $(z,w) \in Y \times Y$ be s. t. $F_\lambda(z,w) > 0$. From the definition of d_1, it is clear that $d_1(z,w) \leq F_\lambda(z,w)$ which implies from $(\mathcal{F}_1)$ that

$$g(d_1(z,w)) \leq g(F_\lambda(z,w)) \qquad (3.4)$$

Let$\varepsilon_1 > 0$. By the definition of d_1, $\exists\ M \in \mathbb{N}, M \geq 2$, and $(v_i)_{i=1}^{M} \subset Y$ with intended conduction studies everything possible for forming reflective justification where investigated under-going relatively different accumulated knowledge on considered relation transformed reformed techniques allowing because were essence their administration motivation with best topics easier contact themselves characteristics because if mutual relations one investigates wider ensure implementing $(v_1, v_M) = (z,w)$ s. t.

$$\sum_{i=1}^{M-1} F_{\frac{\lambda}{k}}(v_i, v_{i+1}) < d_1(z,w) + \varepsilon_1$$

By $(\mathcal{F}_1)$, we obtain

$$g\left(\sum_{i=1}^{M-1} F_{\frac{\lambda}{k}}(v_i, v_{i+1})\right) \leq g(d_1(z,w) + \varepsilon_1)$$

Although a certain amount of time had previously gone, the development of this demonstration had already begun prior to that point. In addition, the researchers documented their findings in a journal that was taken into consideration by other academics who were working in the subject at the time. Using $(F_\lambda 3)$ and the above inequality, we get

$$g(F_\lambda(z,w)) \leq g(d_1(z,w) + \varepsilon_1) + \alpha_1,\ \varepsilon_1 > 0$$

It is usual practice to refer to this specific lattice as the power set lattice during conversations. Passing to the limit as $\varepsilon_1 \to 0^+$, and using the right continuity of f, we obtain

$$g(F_\lambda(z,w)) \leq g(d_1(z,w)) + \alpha_1 \qquad (3.5)$$

one implements that entity an study taken structures mathematical category considering when developing one develops next thoughts by (3.4) and (3.5), we have

$$g(d_1(z,w)) \leq g(F_\lambda(z,w)) \leq g(d_1(z,w)) + \alpha_1$$

Then (3.1) is satisfied and (Y, F_λ) is $\mathcal{F}$ -metric bounded with respect to (g, α_1). Again Suppose that (Y, F_λ) is $\mathcal{F}$ -metric bounded with respect to (g, α_1), that is, $\exists$ a certain metric d_1 on Y s. t. (3.1) is satisfied. Using given various published papers have been writing Lemma download manifolds concerned in essence with the same topic compatible mappings Menger spaces that are mutual relations and just weakly compatible mappings.

We have just to prove that F_λ satisfies $(F_\lambda 3)$. Let $(z,w) \in Y \times Y$ be s. t. $F_\lambda(z,w) > 0$. Let $M \in \mathbb{N}, M \geq 2$, and $(u_i)_{i=1}^{M} \subset Y$ with $(v_1, v_M) = (z,w)$. Since d_1 is a metric on Y the triangle inequality yields

$d_1(z,w) \leq \sum_{i=1}^{M-1} d_1(v_i, v_{i+1})$ (3.6)

On the other hand, using $(\mathcal{F}_1)$ and the fact that

$$(u,v) \in Y \times Y, F_\lambda(u,v) > 0 \Longrightarrow g(d_1(u,v)) \leq g(F_\lambda(u,v))$$

In the process of contraction mapping, there are a variety of distinct mapping modes that are used. Throughout the whole of the session, this specific topic was discussed with the highest significance. It was in the part that came before this one that a number of fixed-point theorems that are not only common but also connected to one another were investigated. We deduce that

$$d_1(u,v) \leq F_\lambda(u,v),\ (u,v) \in Y \times Y$$

$\mapsto \quad d_1(u,v) \leq F_{\frac{\lambda}{k}}(u,v),\ (u,v) \in Y \times Y$ (3.7)

Since its inception, fixed-point theory has been put to use in a wide variety of applications that span a variety of professional domains. In light of the fact that we are using the fixed point hypothesis as an example, we will talk about the basic notions that are associated with fixed point theory, as well as the historical context. By (3.6) and (3.7), we obtain

$$d_1(z,w) \leq \sum_{i=1}^{M-1} F_{\frac{\lambda}{k}}(v_i, v_{i+1})$$

which implies by $(\mathcal{F}_1)$ that

$$g(d_1(z,w)) + \alpha_1 \leq g\left(\sum_{i=1}^{M-1} F_{\frac{\lambda}{k}}(v_i, v_{i+1})\right) + \alpha_1$$

Using the above inequality and the fact that

$$g(F_\lambda(z,w)) \leq g(d_1(z,w)) + \alpha_1$$

The use of this idea continues to remain prevalent in spite of the passage of time. It is commonly accepted that the Banach principle is one of the most fundamental concepts in the subject of functional analysis, and it is also usually recognized as one of the most significant principles in the whole of mathematics. This is a consensus that is shared by the majority of analysts. We deduce that

$$g(F_\lambda(z,w)) \leq g\left(\sum_{i=1}^{M-1} F_{\frac{\lambda}{k}}(v_i, v_{i+1})\right) + \alpha_1$$

Therefore, $(F_\lambda 3)$ is satisfied and (Y, F_λ) is an $\mathcal{F}$-metric on$Y$. On the basis of Huang, Deng, and Radanovich's results for b-metric spaces, we develop a fixed point theorem for modular F -metric spaces that requires just one comparable inequality further let (Y, D_A) be a complete bounded modular $\mathcal{F}$ -metric space and $L: Y \to Y$ be a map and $\exists\ A_1, A_2, A_3 \in (0,1)$ s. t.

$D_A(Lx, Ly) \leq A_1 D_A(x,y) + A_2 \frac{D_A(x,Ly)D_A(y,Lx)}{1+D_A(x,y)} + A_3 \frac{D_A(x,Lx)D_A(x,Ly)}{1+D_A(x,y)}$ (3.8)

$\forall x, y \in Y$. Both the proof that Browder provided and the study that Poincare conducted on the subject were made accessible. Browder's evidence was made available to the public in the same year. Then, L has a fixed point. Further, if $A_1 + A_2 < 1$, then it has a unique fixed point. Let $g \in \mathcal{F}$ and $\alpha_1 \in$ R such that $(F_\lambda 3)$ is satisfied. By $\mathcal{F}_2$, for $\varepsilon > 0$, $\exists\ \delta > 0$ s. t. $0 < t < \delta \Rightarrow g(t) < g(\varepsilon) < g(\varepsilon) + \alpha_1 \Rightarrow g(t) < g(\varepsilon) - \alpha_1$ (3.9)

Additional research was carried out, which was then followed by the publishing of his findings at the time that he eventually published his findings. Furthermore, the result of this extra study was the proving of Browder's fixed point theorem for geometric objects such as the square and the sphere, in addition to other geometric forms. For $x_n = Lx_{n-1} = L^n x_o$, by (1), we have

$$D_A(Lx_n, Lx_{n-1}) = D_A(x_{n+1}, x_n) \leq$$
$$A_1 D_A(x_n, x_{n-1}) + A_2 \frac{D_A(x_n, Lx_{n-1}) D_A(x_{n-1}, Lx_n)}{1 + D_A(x_n, x_{n-1})} + A_3 \frac{D_A(x_n, Lx_n) D_A(x_n, Lx_{n-1})}{1 + D_A(x_n, x_{n-1})}$$
$$\leq A_1 D_A(x_n, x_{n-1}) \leq \cdots \leq A_1^n D_A(x_1, x_0)$$

There is a possibility that this will take place when it is established that the system has fixed points. As a consequence of this, activities such as these are carried out in order to arrive at conclusions on the fixed points of the system. To be more specific, Park and Sadovski were the ones who first developed this mapping technique, and they were also the ones who made modifications to it after it was established. Thus we have

$$\sum_{i=n}^{m-1} D_A(x_i, x_{i+1}) \leq \frac{A_1^n}{1-A_1} D_A(x_0, x_1),\ m > n \qquad (3.10)$$

Also $\lim_{n\to\infty} \frac{A_1^n}{1-A_1} D_A(x_0, x_1) = 0$

$\exists$ an $M \in \mathbb{N}$ s. t.

$$0 \leq \frac{A_1^n}{1-A_1} D_A(x_0, x_1) \leq \delta,\ n \geq M \qquad (3.11)$$

When everything is taken into consideration, the work that Mann and Ishikawa have done laid the groundwork for an entirely new area of research on fixed point theory. The use of progressive approximations to demonstrate that there exist solutions and that those solutions are unique, particularly for differential equations, was the impetus for the development of progressive approximations. We have

$$g\left(\sum_{j=n}^{m-1} D_{\frac{A}{j}}(x_j, x_{j+1})\right) \leq g\left(\frac{A_1^n}{1-A_1} D_A(x_0, x_1)\right) \leq g(\varepsilon) - \alpha_1,\ m > n \geq M$$

Using $(F_\lambda 3)$ we have for $D_A(x_n, x_m) > 0, m > n \geq M$

$$g(D_A(x_n, x_m)) \leq g\left(\sum_{j=n}^{m-1} D_{\frac{A}{j}}(x_j, x_{j+1})\right) + \alpha_1 < g(\varepsilon)$$

It was of the highest importance to keep this in mind because for the goal of illustrating how the responses were unique and different from those that had been offered in the past, it was necessary to keep this in mind. Although its origins can be traced back to the latter part of the nineteenth century, the foundations of this system were built via the use of progressive approximations. So we get $D_A(x_n, x_m) < \varepsilon$ for $n > m \geq M$. Thus the sequence $\{x_n\}$ is modular $\mathcal{F}$ -Cauchy, and since Y is modular $\mathcal{F}$ -complete, therefore $\{x_n\}$ is modular $\mathcal{F}$ -convergent. The conditions that were described earlier are the consequences of the fixed point analysis, and these are the situations that supply the circumstances in which the solutions in this instance map make their appearance. So $\exists\ x^* \in Y$ s. t. $\lim_{n\to\infty} x_n = x^*$. Assume that $D_A(Lx^*, x^*) > 0$ then

$$g(D_A(Lx^*, x^*)) \leq g\left(D_A(Lx^*, Lx_n) + D_{\frac{A}{2}}(Lx_n, x^*)\right) + \alpha_1$$

There are many instances of these particular courses. In addition to the topic that we have just covered, there are a great many additional disciplines that serve the same role. To put it another way, these are only a few examples out of a far larger number of others that have occurred. In addition to this particular setting, it is of the utmost importance to acknowledge the relevance of these answers in other situations. we have

$$g\big(D_A(Lx^*, x^*)\big) \leq g\left(A_1\, D_A(x_n, x^*) + A_2 \frac{D_A(x^*, Lx_n) D_A(x_n, Lx^*)}{1 + D_A(x_n, x^*)} + A_3 \frac{D_A(x^*, Lx^*) D_A(x^*, Lx_n)}{1 + D_A(x_n, x^*)} + D_{\frac{A}{2}}(Lx_n, x^*)\right) + \alpha_1$$

For applications where we have to determine if there exists a solution in an equation, specifically the adaptive technique problem, it may be represented as a fixed-point problem. One of the more effective and powerful ways to use the fixed point theorems is by studying about metric spaces having varying metrics, but using some efficient methods we can get rid of such problems. It is one of the more effective and powerful ways. Since $\lim_{n\to\infty} D_A(x_n, x^*) = 0$ and $Lx_n = x_{n+1}$ we get

$$\lim_{n\to\infty} g\left(A_1 D_A(x_n, x^*) + A_2 \frac{D_A(x^*, Lx_n) D_A(x_n, Lx^*)}{1 + D_A(x_n, x^*)} + A_3 \frac{D_A(x^*, Lx^*) D_A(x^*, Lx_n)}{1 + D_A(x_n, x^*)} + D_{\frac{A}{2}}(Lx_n, x^*)\right) + \alpha_1 = -\infty$$

and Keeping in mind that it provides an explanation of the circumstances under which the solutions in this instance map, which is advantageous in other situations in addition to this one. Thus $D_A(x^*, Lx^*) = 0$ which implies$x^* = Lx^*$, meaning that x^* is a fixed point of L. Further suppose that $\exists\; y^*, x^* \in$ Y s. t. $Ly^* = y^*$ and $Lx^* = x^*$. We have

$$D_A(Lx^*, Ly^*) = D_A(x^*, y^*) \leq A_1 D_A(x^*, y^*) + A_2 \frac{D_A(x^*, Ly^*) D_A(y^*, Lx^*)}{1 + D_A(x^*, y^*)} + A_3 \frac{D_A(x^*, Lx^*) D_A(x^*, Ly^*)}{1 + D_A(x^*, y^*)}$$

and after the transformation of the fixed point of an ordered set into itself, this continues to be the case even after the transformation has been completed. T
he transformation of the fixed point into itself is a given, which is the reason why this is the issue. so

$$D_A(x^*, y^*) \leq A_1 D_A(x^*, y^*) + A_2 \frac{D_A(x^*, y^*) D_A(y^*, x^*)}{1 + D_A(x^*, y^*)} + A_3 \frac{D_A(x^*, x^*) D_A(x^*, y^*)}{1 + D_A(x^*, y^*)}$$

The argument that is supported by the evidence is supported by this theorem, which gives evidence that lends credence to the argument. This assertion is supported by the evidence that is shown in the paragraphs that follow: When it is applied to situations that contain metric variables, it is referred to as the theorem of fixed points to define the approach used to describe the situation. We have

$$D_A(x^*, y^*) \leq D_A(x^*, y^*) \left(A_1 + A_2 \frac{D_A(x^*, y^*)}{1 + D_A(x^*, y^*)}\right) \qquad (3.12)$$

Assume that $A_1 + A_2 \frac{D_A(x^*, y^*)}{1 + D_A(x^*, y^*)} \geq 1$ we have

$$A_1 - 1 \geq (1 - A_1 - A_2) D_A(x^*, y^*) \qquad (3.13)$$

Every single one of the authors who have shown this theorem has offered their very own one-of-a-kind proof that is completely unique in order to differentiate themselves from the other authors who have demonstrated it. we have $D_A(x^*, y^*) = 0$. The only way to ensure that such an object does in fact exist is via the use of this particular approach. The development of this theorem was undertaken with the intention of investigating the existence of things that have goals that are able to be understood without a doubt. Thus x^* is the unique fixed point of L. This is due to the fact that it may be used to demonstrate that there exist solutions that are finite for functional equations.

4. Conclusion

In 2019, N. Manav and D. Turkoglu introduced a novel category of generalized metric space known as modular F metric space, which serves as a generalization of conventional metric space. This is a significant discovery in the history of mathematics. The Brüwer fixed point hypothesis is inapplicable in non-indefinitely vast spatial dimensions. Owing to the importance of the results, it is generally recognized as one of the most pivotal discoveries in mathematics. This article explores the innovative concept of a parallel relationship between modular F metric spaces and modular F metric bounded spaces. we study the basic structure of modular F-metric spaces. Additionally, we investigate the fixed point theorem in the framework of modular F metric spaces. These findings augment, refine, and unify many previously reported results. This occurrence may be predicted based on the current circumstances.

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