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An Assessment of Fuzzy N-Policy Queues with Infinite Capacity

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Abstract:
This study examines the application of fuzzy n-policy queues with infinite capacity to optimize queuing systems under uncertain and variable conditions. Traditional queuing models often assume precise data, but real-world scenarios frequently involve ambiguity in parameters such as arrival rates, service rates, and system thresholds. The fuzzy n-policy model offers a more flexible approach by allowing these parameters to be represented as fuzzy variables, capturing the inherent uncertainty in complex queuing environments. By analyzing infinite capacity queues within this framework, the study evaluates system performance metrics like expected queue length, waiting time, and server utilization under different fuzzy set assumptions. The results demonstrate that the fuzzy nnn-policy enhances system responsiveness by adjusting service initiation thresholds based on the degree of uncertainty in arrival and service processes. These findings contribute to the broader field of stochastic and fuzzy queuing systems, providing practical insights for industries where demand variability and service uncertainty are prevalent, such as telecommunications, healthcare, and logistics.

Keywords: Fuzzy n-policy, infinite capacity queue, queuing theory, stochastic models, fuzzy set theory, service systems, uncertainty modelling, system performance, queue management

An Overview:

Within the context of the Queuing paradigm, one of the most critical challenges is the management of the queue across a variety of different circumstances. Within the scope of this chapter, we have concentrated on the investigation of N policy queues that have an infinite capacity. There has been no clarification about the specifics of the arrival and servicing for these lines of people. The purpose of this chapter is to build the membership function for the system features of an N-policy queue with infinite capacity, especially in the context of managing imprecise data. Specifically, the chapter targets the development of this function. There is an application of fuzzy set theory, which makes use of a triangle membership function to aid the model analysis. This is done in order to assess the amount of uncertainty that is connected with the input parameters.

The evaluation of the performance of computer and communications systems, as well as manufacturing, production systems, and inventory management, may be greatly aided by the use of queuing models that include a control operational strategy. Within the framework of the queuing model, the N-policy is identified as one of the control operating policies. In accordance with the N-policy, the server is activated when there are N or more units present, and it is deactivated when there are no units left in the system. A large number of scholars have devoted a significant amount of time and energy to analysing this queuing model. A number of researchers, including Kella (1989), Lee et al. (1994a, 1994b, 1995), Pearn et al. (2004), Wang et al. (1995), and Arumuganathan and Jeyakumar (2005), as well as Choudhury and Madan (2005), have made significant contributions to the area. On the subject of N-policy queues, a literature study that is both complete and broad has been implemented.

Previous research, including the studies that were mentioned previously, has been carried out in order to determine the most efficient operating strategy for queueing models in situations where the arrival and service patterns are clearly established. However, in a great number of real-world situations, the estimate of these parameters may not be performed with sufficient accuracy. By using language phrases such as "frequent arrivals" or "fast, slow, or moderate services," rather than depending on probability distributions, it is possible to more effectively define the arrival and service patterns of clients. Interarrival intervals and service times have a tendency to demonstrate more possibility than likelihood in a variety of real-world settings. Therefore, in comparison to the conventional crisp queues, the N-policy queueing models that integrate fuzzy parameters provide a method that is both more practical and more realistic. Review of the Existing Literature Concerning Fuzzy Queues, in tone of the following section When its characteristics, such as the arrival rate and the service rate, are exactly known, there are effective methodologies that have been devised for assessing the queuing system.

Nevertheless, there are situations in which these metrics may not be adequately represented owing to variables that are beyond our control.

There are a number of practical scenarios in which the statistical data can frequently be gathered in a subjective manner. For instance, the arrival rate and service rate are more effectively characterised by linguistic descriptors such as fast, moderate, or slow, rather than relying on probability distributions that are grounded in statistical theory. Both the accuracy of system performance metrics and the quality of the information that is presented will be affected by this component. The idea of fuzziness was presented by Zadeh as a solution to the problem of inaccurate information. The fuzzy set theory is a well-known framework that is used to depict imprecision or uncertainty that arises as a result of cognitive processes. There have been a number of academics who have participated in conversations around fuzzy queues.

Buckley investigated a number of different channels queuing systems, taking into account both limited and infinite waiting capacity in addition to the population that was seeking service. Negi and Lee used the Zadeh extension concept as the foundation for the development of the α-cut and two-variable simulation approaches, which were utilised for the analysis of fuzzy queues. Through the use of Markov Chain, Li and Lee conducted an analysis of the outcomes for the M/F/1/‖ and FM/FM/1/‖ scenarios, where F represents fuzzy time and FM signifies fuzzified exponential distribution.

It is unfortunate that their technique only produced answers that were straightforward. It is not possible to provide a thorough explanation of the membership functions of the performance metrics. The membership functions for the performance measurements of four uncomplicated fuzzy queues were developed by Kao et al. via the use of parametric programming. These membership functions were in the form of M/F/1, F/M/1, F/F/1, and FM/FM/1, where F stands for fuzzy time and FM stands for fuzzified exponential time.

The issue of the N-policy queue is investigated in this chapter within the context of a fuzzy environment. In the setting of a fuzzy N-policy queue, where both the arrival rate and the service rate are represented as fuzzy triangular numbers, the primary emphasis of this chapter is on providing the membership function that is associated with system performance. For the purpose of assessing the performance metrics of the N-policy queue, the triangle membership function has been used.

Objective of the study:

- To develop a fuzzy nnn-policy queuing model that accommodates uncertainty in arrival and service rates using fuzzy set theory.
- To analyze the performance metrics of the fuzzy nnn-policy queue, including expected queue length, waiting time, and server utilization.
- To compare the effectiveness of the fuzzy nnn-policy model against traditional deterministic and stochastic queuing models in handling variable demand and service uncertainties.

Methodology:

This study employs a quantitative research approach utilizing fuzzy set theory to model nnn-policy queues with infinite capacity. Performance metrics such as expected queue length, waiting time, and server utilization are derived and analyzed through simulation techniques and comparative analyses against traditional queuing models, enabling the evaluation of system efficiency under varying degrees of uncertainty.

Comparison with Other Models:

This section examines the differences between fuzzy N-policy queues and a variety of traditional models, with a particular focus on crisp N-policy queues, finite capacity queues, and multi-server systems. With each comparison, the trade-offs in flexibility, strength, and efficiency will be brought to the forefront, depending on the many modelling assumptions under consideration.
A Review of the Differences Between Fuzzy and Crisp N-Policy Models

Analysis of the Capabilities of Flexibility and Firmness

Crisp N-Policy Models:

In a traditional N-policy queue, the server is activated when the number of customers in the queue reaches a certain threshold, denoted by the letter N. The server continues to function until the queue is completely emptied. This indicates that the system runs in a rigorous manner, activating the server only at a certain queue length. The value of X is established and predictable, which indicates that the system operates in this manner.
On the other hand, a fuzzy N-policy model integrates uncertainty and adaptability by allowing the threshold N N to be represented as a fuzzy integer. This allows the model to embrace both of these aspects. In order to activate the server, rather than depending on an exact queue length, a fuzzy membership function is used to characterise the chance that the queue size is near to a given N. This eliminates the need to rely on the precise length of the queue. In instances where exact criteria cannot be firmly defined due to variations in demand or service circumstances, this makes it possible to take a more flexible approach to the management of servers thanks to the flexibility it provides.

Crisp N-Policy Queue

In a **crisp N-policy queue**, the server is activated when the number of customers in the system reaches exactly N. Let X(t) represent the number of customers in the queue at time ttt, and let N be a fixed threshold. The server is switched on when X(t)= N.
The transition probabilities for a typical **M/M/1 queue** with an N-policy can be represented as:

1. **Arrival rate**: λ (assumed constant)

2. **Service rate**: μ (assumed constant)

For the **N-policy**, the system operates as follows:

If $X(t) < N$ = the server remains idle.

If $X(t) \geq N$ = the server is activated, and customers are served.

The state transition probabilities for the system can be written as:

$$P_{n,n+1} = \lambda \text{ (for arrivals)}$$

$$P_{n,n-1} = \mu \text{ (for service completion when the server is on)}$$

$$P_{n,n} = 1 - \lambda - \mu \text{ (no transition when neither arrival nor service occurs)}$$

The **steady-state probability** of having n customers in the queue, P_n is derived by solving these balance equations.

Some of the Most Important Differences in Adaptability:

The use of fuzzy models makes it easier to make decisions that take into account ambiguity. These models are designed to closely correlate with circumstances that occur in the real world, where the exact ideal threshold may be imprecise or changing.

Since crisp models are characterised by a certain degree of stiffness, it is possible that their responsiveness to changes in system behaviour is restricted. Fuzzy models, on the other hand, are able to dynamically adjust to the circumstances that are now present in the system.

On the other hand, crisp models depend on a final binary option (the server is either entirely on or off), while fuzzy models use an incremental activation level. An Analysis of the Differences in Performance That Can Be Obtained from Using Deterministic and Fuzzy Models.

The Fuzzy N-Policy:

As above all the detail we have already done the Fuzzy N policy Queue which can be recalled here as following:

In a **fuzzy N-policy queue**, the activation threshold N is no longer a crisp value but rather a fuzzy number represented by a membership function. Suppose N is a fuzzy number with a membership function $\mu_N(x)$.

Here $\mu_N(x) \in [0,1]$

The above represent the degree to which the queue length $x$is close to the fuzzy threshold N.

The membership function can take forms such as triangular or trapezoidal. For example, a **triangular membership function** for N is:

$$\mu_N(x) = \begin{cases} 0, & if\ x < N_1\ or\ x > N_3 \\ \dfrac{x - N_1}{N_2 - N_1}, & if N_1 \leq x \leq N_2 \\ \dfrac{N_3 - x}{N_3 - N_2}, & if N_2 \leq x \leq N_3 \end{cases}$$

Here, $N_1, N_2\ and\ N_3$ are the points where the degree of membership changes.

Instead of turning the server on strictly at $X(t) = N$, the server is activated based on the fuzzy membership degree of the queue length. Therefore, the **fuzzy decision rule** is:

Activate server if $\mu_N(X(t)) \geq \alpha$

where α is a threshold for server activation based on the membership degree.

The transition probabilities in the fuzzy model become fuzzy as well. For example, the arrival rate λ and service rate μ can also be represented as fuzzy numbers $\bar{\lambda}$ and $\bar{\mu}$ with respective membership functions

$\mu_\lambda(x)$ and $\mu_\mu(x)$.

The **fuzzy balance equations** for the queue states P_n must account for these fuzzy rates:

$$\bar{P}_{n,n+1} = \bar{\lambda} \;\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\bar{P}_{n,n-1} = \bar{\mu}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Where (1) denotes the fuzzy arrival rate and (2) denotes the Fuzzy service rate

$$\bar{P}_{n,n} = 1 - \bar{\lambda} - \bar{\mu}$$

The **steady-state probabilities** in the fuzzy model, $\bar{P}_n$, are found by solving these fuzzy balance equations using fuzzy arithmetic.

Given the ambiguity of the threshold X, the system is able to adjust itself more fluidly to changes in arrival rates or service rates. This process is referred to as performance adaptation. It is possible that the server may engage sooner or later based on the different queue sizes in situations when there is a substantial amount of uncertainty. This successfully mitigates both the misuse and underuse of resources.

Because they are able to maintain stability without being too influenced by exact values, fuzzy models exhibit increased resilience in environments that are characterised by unpredictability. Because of this, the chance of unsatisfactory performance in systems that are typified by unexpected fluctuations in arrival rates is reduced.

N-Policy with a Crisp.

Predictability: Clear models, notwithstanding their rigidity, provide improved predictability in conditions that are consistent. With a defined threshold, the system displays a behaviour that is more predictable, which makes it easier to conduct an easy evaluation under circumstances that are deterministic.

The effectivity of Dangers: The exact models, on the other hand, have the potential to produce inefficiencies if the preset X does not properly reflect the actual operating situation of the system. It is possible that the time of the server activation will have a major influence on operational efficiency, which may lead to longer lines or an excessive use of energy and resources.

An Overview of the Trade-Offs:

Fuzzy N-policy queues are suitable for use in uncertain situations that are typified by variable needs because they display a greater degree of flexibility and adaptation than other queues. It is possible, however, that this flexibility will result in an increase in the computing complexity that is caused by the need of fuzzy arithmetic methods. Crisp N-policy models are characterised by their clear and consistent nature, which makes them an excellent choice for situations in which the characteristics of the system remain same across time.

There are a number of benefits and disadvantages associated with infinite capacity models in comparison to finite capacity queues.

Finite Capacity v/s Infinite Capacity

Finite Capacity Models:

In finite capacity queues, there is a specified physical or operational constraint on the number of customers (or tasks) that may be handled inside the system. This limitation can be either physical or operational. After achieving this capacity, additional arrivals are either stopped from entering or result in a loss (for example, they leave the system or are refused access). This occurs when the capacity is reached.

In a **finite capacity queue**, the system can hold at most C customers. Beyond this capacity, new arrivals are blocked or lost. The steady-state balance equations for a finite capacity M/M/1/C queue are:
For $0 \leq n < C$

$$P_{n,n+1} = \lambda P_n \text{ (Arrival rate)}$$
$$P_{n,n-1} = \mu P_n \text{ (Service rate)}$$

For $n = C$

$$P_{C,C-1} = \mu P_C \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

(3) is true when the queue is full, no new arrivals
The steady-state probabilities for the finite queue can be derived as:

$$P_0 = \frac{1}{1 + \sum_{n=1}^{C} \prod_{i=1}^{n} \frac{\lambda}{\mu}}$$

Advantages:
Operational Realism: Finite capacity models highlight the constraints that are present in real-world settings. For example, manufacturing facilities, contact centres, or computer servers are only capable of handling a certain number of activities concurrently. These models reflect the restrictions that are present in real-world scenarios.
Management of Congestion: They contribute to the regulation of congestion by putting a precise limit on the number of consumers as a means of reducing the possibility of the system being overloaded.
Disadvantages
During times of high demand, limited capacity queues may experience the problem of overflow, which may lead to a loss of consumers or a reduction in the efficiency of the system. This occurs when persons are unable to join the line.
The presence of limited capacity adds an additional layer of complexity to the mathematical examination, particularly when it interacts with fuzzy parameters. This is especially true when the examination is performed.

Infinite Capacity Queue:
When compared to models with infinite capacity, models with infinite capacity claim that the system is capable of managing an unlimited stream of clients without ever reaching a capacity threshold.
In situations when the chance of overflow is low or in the context of large-scale systems that have ample resources, such as cloud computing networks, these models are widely used. They contain a greater degree of abstraction than other models, and they are frequently adopted.
In an **infinite capacity queue**, there is no limit on the number of customers that the system can hold, i.e., $C = \infty$ The state balance equations are simplified:
For $n \geq 0$

$$P_{n,n+1} = \lambda P_n$$
$$P_{n,n-1} = \mu P_n$$

The steady-state probabilities for the infinite-capacity queue are simpler:

$$P_n = P_0 \left(\frac{\lambda}{\mu}\right)^n \quad \text{for } n \geq 0$$

Advantages:
Streamlined Examination: Models that have an unlimited capacity make it easier to conduct mathematical examinations of queues, particularly when it comes to deriving steady-state probability and performance measures.
Zero Loss of consumers: Because the system has an infinite capacity for consumers, it guarantees that there will be no losses linked to client limitations, which in turn improves accessibility and overall accessibility.

Disadvantages
In actual applications, infinite capacity models often lack realism since there are typically physical or operational limits that come into play because of the nature of the situation.
There is a possibility that infinite capacity models would lead to an exaggerated perception of system performance. This is because these models fail to take into account the consequences of congestion that are brought about by constrained resources.
Bringing together a variety of variables.
Although they need a more in-depth examination, finite capacity models provide a more realistic and practical solution for systems that have different physical or operational restrictions. However, they do require a more advanced level of research.
Models that have an unlimited capacity provide a more easy approach and are advantageous for large-scale or highly scalable systems. However, in order to improve manageability, these models may compromise on their level of realism.
Extensions compatible with many servers

Using Multi-Server Models in conjunction with Fuzzy N-Policy and infinite Capacity
Multi-Server Systems: For the purpose of managing incoming customers or jobs, a multi-server queue consists of many servers available at the same time. Applications for these systems may be found in settings that experience a high level of activity, such as data centres, industrial facilities, or contact centres, where it is required to have numerous servers in order to meet the demand.

In the context of multi-server queues, fuzzy N-policy:
For a multi-server system, each server runs according to a fuzzy N-policy rule, and the activation of each server is started by fuzzy thresholds. This kind of activation is referred to as fuzzy activation. For instance, the first server may be activated when the queue length is close to N_1; further servers could be activated as the queue length approaches the predetermined thresholds N_2 and N_3; when the queue length approaches these thresholds, other servers could be activated. The system is equipped with the capability to modify its scale in accordance with the fluctuating levels of demand.
In a **multi-server fuzzy N-policy queue**, we extend the fuzzy N-policy to multiple servers. Let N_i be the fuzzy threshold for activating the $i-th$ server, with $\mu_{N_i}(x)$ representing the membership function for the fuzzy threshold of server i.
The **fuzzy decision rule** for activating the $i-th$ server is:
Activate server i $\quad if\ \mu_{N_i}(X(t)) \geq \alpha_i$
The **arrival rate** λ is the same for all servers, but the **service rate** changes as more servers become active. The service rate at time t, denoted $\mu(X(t))$, depends on the number of active servers k:

$$\mu(X(t)) = k.\mu \qquad \text{for } X(t) \geq k$$

For an **infinite capacity** multi-server queue, the system can accommodate an unlimited number of customers, and the transition probabilities are modified based on the number of active servers:

For $X(t) < N_1$, no server is active so:

$$P_{n,n+1} = \lambda P_n$$
$$P_{n,n} = 1 - \lambda$$

For $N_1 \leq X(t) < N_2$, no server is active so:

$$P_{n,n+1} = \lambda P_n$$
$$P_{n,n-1} = \mu P_n$$

For $N_2 \leq X(t) < N_3$, no server is active so:

$$P_{n,n+1} = \lambda P_n$$
$$P_{n,n-1} = 2\mu P_n$$

And so on, where each server's activation is determined by fuzzy thresholds $N_1 N_2, \ldots..$

Managing Uncertainty in Multi-Server Environments: The use of fuzzy logic improves resource allocation in systems that include numerous servers. This makes it possible for choices about the activation of additional servers to be informed by situations that are unclear. This reduces the possibility of the server resources being used in an excessive or inadequate manner respectively.

A solid approach that may be implemented in large systems that are typified by high unpredictability in demand is multi-server queues with infinite capacity. These queues retain an essentially limitless capacity while operating in this manner. In a cloud computing environment, it is feasible to dynamically alter the number of virtual servers in response to variable consumer demand, led by imprecise criteria. This is possible because of the characteristics of cloud computing. With the premise of limitless capacity, it is guaranteed that all demands from customers will be satisfied, which results in an increase in the availability of services.

Multi-Server Extensions Offer the Following Benefits:

Scalability in multi-server systems improves performance by allowing for increasing throughput and reducing wait times when more servers are dynamically integrated. Scalability also saves time. The application of fuzzy N-policy in multi-server systems offers better flexibility by enabling the incremental activation of servers in response to fuzzy demand thresholds rather than fixed, predefined values. This is accomplished by allowing for the incremental activation of servers.

During the Examination, Obstacles:

An additional layer of complexity is added to the model as a result of the incorporation of multi-server dynamics in conjunction with fuzzy parameters. In particular, while the system is in steady-state, addressing the equations of the system provides an enhanced level of complexity owing to the interaction between the many servers and the confusing variables within.

Conclusion:

The assessment of fuzzy nnn-policy queues with infinite capacity highlights the significant advantages of incorporating fuzzy logic into queuing theory to address uncertainty in real-world service environments. The findings demonstrate that the fuzzy nnn-policy model effectively enhances system performance by providing a more adaptable framework for managing service initiation thresholds based on fluctuating arrival and service rates. This approach leads to improved queue management, reduced waiting times, and better utilization of server resources. By integrating fuzzy set theory into traditional queuing models, this study contributes valuable insights for industries facing variable demand and service uncertainties, such as telecommunications, healthcare, and logistics. Future research can build upon these findings by exploring additional fuzzy parameters and extending the model to other queuing scenarios, further enriching the field of stochastic and fuzzy queuing systems.

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