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Two-Dimensional Boundary-Layer Flow of Dusty Fluid Over a wedge

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Abstract:
In this paper, we investigate the laminar two-dimensional boundary-laver flow of a dusty fluid over a wedge. The mainstream flow outside the boundary- layer is considered in the form of the power of distance along the wedge surface. It is also considered that the wedge is moving with velocity U_w opposite to the mainstream flows. The interest in boundary-layer flow over a wedge has gained considerable attention because of its wide range of application in various problems of atmospheric, engineering, physiological, industry and manufacturing process fields such applica- tions include polluting city air, motion of aerosols in the upper atmosphere, trans-port of suspended powdered materials through pipes, propulsion and combustion in rockets, continuous stretching of plastic films and flow of blood in arteries are some examples. Few examples of such technological process are the boundary layer along material handling convey ears and the boundary layer along a liquid film in condensation process. So the study of two dimensional boundary layer flow a wedge has gained much interest. The influence of dust particles on viscous flows has great importance in the petroleum industry and in the purification of crude oil. Other important in the boundary layer include soil erosion by natural winds and dust entrainment in a cloud during a nuclear explosion. This modelling results into the coupled partial differential equations which are transformed into ordinary differential equations using suitable similarity transformations. Numerical method based on Keller-box is used for the coupled system for various physical parameters such as non-Newtonian fluid index parameter m, pressure gradient β, mass concentration Parameter M_c and local fluid particle interaction parameter χ. The detailed hydrodynamics behind these effects is discussed.

Keywords: Power-law fluid 1, Dusty fluid 2, Moving wedge 3, Pressure gradient 4, Mass concentration 5, non-Newtonian fluid 6, Fluid particle interaction 7, Keller-box method

Introduction

The interest in boundary layer flow (BLF) over a wedge has gained considerable attention because of its wide range of application in various problems of atmospheric, engineering, physiological, industry and manufacturing process fields such applications include polluting city air, motion of aerosols in the upper atmosphere, transport of suspended powdered materials through pipes, propulsion and combustion in rockets, continuous stretching of plastic films and flow of blood in arteries are some examples. Few examples of such technological process are the boundary layer along material handling convey ears and the boundary layer along a liquid film in condensation process. So the study of two dimensional boundary layer flow a wedge has gained much interest. The influence of dust particles on viscous flows has great importance in the petroleum industry and in the purification of crude oil. Other important in the boundary

layer include soil ersion by natural winds and dust entrainment in a cloud during a nuclear explosion. Large number of researchers are engaged in this area. In order to get a complete and fundamental laws of physics by which the most general governing equations of continuity and momentum are obtained.
Safman (1962) discussed on the stability of laminar flow of a dusty gas in which the dust particles are uniformly distributed. Alizadeh et al. (2009) studied the boundary laver equations of flow past a wedge with different angles by the Adomian de composition method. Afzal (2010) discussed the effects of the suction and injection on the laminar boundary layer flow of a viscous and incompressible fluid. Postelnicu and Pop (2011) studied of a power law fluid past a permeable stretching wedge considering variable free stream.
We shall make simplifying assumptions about the motion of the dust particles. It will be supposed that the dust particles are uniform in size and shape, and that their velocity and number density can be described fields. We also assume that the bulk concentration of the dust is very small so that the net effect of the dust on the gas is equivalent to an extra force $KN(v-u)$ per unit volume, where $u = U_w$ is the velocity of the gas and K is a constant, where it is also supposed that the Reynolds number of the relative motion of dust and gas is proportional to the relative velocity. Then with small bulk concentration and the neglect of the compressibility of the gas, the equations of motion and continuity of the gas are

$$\rho\left(\frac{\partial u}{\partial t}+(u.\nabla)u\right) = -\nabla p + \mu\nabla^2 u + KN(v-u) \quad (1)$$

$$\nabla.u = 0 \quad (2)$$

where p is the pressure, ρ and μ are the density and viscosity of the clean gas. If the dust particles are spheres of radius a, $K = 6\pi a\mu$ by the Stokes drag formula. In our study we consider the problem of Falkner-Skan equation on the laminar bound- ary layer flow of a viscous and incompressible dust fluid flow over a wedge. The effect of velocity and variables are taken in to account. Scaling group of transformations is used to present the similarity representations of the problem. Similarity equations are then solved numerically by using Keller-box method to show the effect of parameters we are considering. The basic idea of the Keller-box method is to write the governing system of coupled equations in the form of a first-order system. Using the finite different equations which are a second order accurate, we get nonlinear coupled algebraic equations that can be linearized and solved for. Then, a block-tridiagonal factorization scheme is applied on the linear system of equations and updated at each step. The governing equations are based on the boundary layer equation.

Formulation of the problem

Let us consider a two-dimensional laminar boundary layer flow of an incompressible viscous dusty fluid near an impermeable plane wall stretching with velocity U_w . The outer free stream velocity is defined in the form of power-law manner i.e., it varies as a power of a distance from the leading edge. The velocity of potential flow is given by $U(x) = U_\infty(x^m)$, where U_∞ is constant, x is measured from the stagnation point, the value of m depends on the pressure gradient in the stream direction and wedge angled denoted by $\pi\beta$. The flow considers around a wedge profile submerged in a fluid of very small viscosity. At the leading stagnation point o the thickness of the boundary layer is zero and it grows slowly towards the rear of the wedge. The pattern of streamlines and the velocity distribution deviate only slightly from those in the potential flow with the exception of the immediate vicinity of the wall. In this region we observed that a large velocity gradient are obtained. N. Datta and S. K. Mishra (1982) has developed the study of boundary layer flow of a dusty fluid over a semi- infinite flat plate. Under the usual boundary layer approximation the equations of continuity and momentum of both the fluid phase and dust phase are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\frac{\partial^2 u}{\partial y^2}+\frac{KN}{\rho\rho_d}(u_d-u) \tag{4}$$

$$\frac{\partial u_d}{\partial x}+\frac{\partial v_d}{\partial y}=0 \tag{5}$$

$$u_d\frac{\partial u_d}{\partial x}+v_d\frac{\partial u_d}{\partial y}=\frac{KN}{M\rho_d}(u-u_d) \tag{6}$$

Where x and y represents co-ordinate axes along the continuous surface in the direction of motion and perpendicular to it respectively. (u, v) and (u_d, v_d) denotes the velocity components of the fluid and particles phase along the x and y are directions respectively, ν is the kinematic viscosity of fluid, ρ is the density of the fluid phase, and ρ_d is the density of the particle phase, K is the Stokes resistance, N is the number density of dust particles, M is the mass concentration of dusty particles. Since the pressure is constant normal to the flow, it is a function of x. This means that pressure in the boundary layer is same as the pressure in the mainstream flow U_x .

The relevant boundary conditions are

$$\text{at } \ y=0: \quad u=U_w, \quad v=0$$
$$\text{as } \ y\to\infty: \quad u=u_d=U, \quad v_d\to v \tag{7}$$

This region is specified by $x>0$ and it is clearly observed that the system (3) to (6) with four unknown functions $u, v\ u_d$ and v_d are easily reduced to an equation with two unknown functions by defining the stream functions $\psi(x, y)$ and $\psi_d(x, y)$ by Datta and Mishra (1980).

$$u=\frac{\partial\psi}{\partial y}\ ,\ v=-\frac{\partial\psi}{\partial x},\ \ u_d=\frac{\partial\psi_d}{\partial y},\ \ v_d=-\frac{\partial\psi_d}{\partial x} \tag{8}$$

The continuity equations (3) and (5) for both fluid phase and particulate phase are satisfied. In terms of variables ψ and ψ_d the boundary layer equations (4) and (6) becomes

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=U\frac{dU}{dx}+v\frac{\partial^3\psi}{\partial y^3}+\frac{KN}{\rho\rho_d}\left(\frac{\partial\psi_d}{\partial y}-\frac{\partial\psi}{\partial y}\right) \tag{9}$$

$$\frac{\partial\psi_d}{\partial y}\frac{\partial^2\psi_d}{\partial x\partial y}-\frac{\partial\psi_d}{\partial x}\frac{\partial^2\psi_d}{\partial y^2}=\frac{K}{M\rho_d}\left(\frac{\partial\psi}{\partial y}-\frac{\partial\psi_d}{\partial y}\right) \tag{10}$$

with boundary conditions

$$y=0;\ \frac{\partial\psi}{\partial y}=U_w(x), \qquad \frac{\partial\psi}{\partial x}=0 \tag{11}$$

$$\text{as } y\to\infty:\ \frac{\partial\psi}{\partial y}=-\frac{\partial\psi_d}{\partial y}=U(x),\ \frac{\partial\psi}{\partial x}\to\frac{\partial\psi_d}{\partial x} \tag{12}$$

The equation and boundary conditions above are reduced from equations (9) and (10), respectively for the case of steady flow. The similar solutions of equations (9) and (10) can be obtained of the following transformation

$$\eta = \frac{y}{\delta} \text{ or } \eta = \frac{y}{\sqrt{\frac{\nu x}{U_1}}} \tag{13}$$

$$\eta = \sqrt{\frac{U}{\nu x}} yC = \sqrt{\frac{x^{m-1}U_1}{\nu}} yC \tag{14}$$

where C is the constant to be determined. The stream function in this transformation is obtained by integrating the continuity equation, we get

$$\psi = \int udy = \frac{1}{C}\sqrt{\nu U_1} x^{\frac{(m+1)}{2}} f(\eta) \tag{15}$$

$$\psi_d = \int u_d dy = \frac{1}{C}\sqrt{\nu_d U_1} x^{\frac{(m+1)}{2}} g(\eta) \tag{16}$$

$$2C^2 - 1 = m \text{ or } C = \sqrt{\frac{1+m}{2}} \tag{17}$$

from the equations (9) and (10), we get

$$f''' + ff'' + \beta(1 - f'^2) + 2M_c\chi(g' - f') = 0 \tag{18}$$

$$gg'' - \beta g'^2 + 2\chi(f' - g') = 0 \tag{19}$$

where prime denotes the differentiation with respect to η and $\beta = \frac{2m}{m+1}$, $M_c = \frac{NM}{\beta}$ is the mass concentration, $\chi = \frac{2x}{P_A(1+m)\pi U}$ is the local fluid particle interaction parameter and corresponding boundary conditions are given by

$$\text{at } \eta = 0:\ f(0) = 0,\ f'(0) = \lambda,$$

$$\text{as } \eta \to \infty;\ f'(\infty) = 1,\ g'(\infty) = 1,\ g(\infty) = f(\infty). \tag{20}$$

Introducing additional unknowns to convert the above system in to the first order coupled system like

$$f' = H \tag{21a}$$

$$H' = V \tag{21b}$$

$$g' = T \tag{21c}$$

Equations (18)-(19) becomes

$$V' + fV + \beta - \beta H^2 + 2M_c\chi(T - H) = 0 \tag{22}$$

$$BT' - \beta T^2 + 2\chi(H - T) = 0 \tag{23}$$

The relevant boundary conditions are

$$\text{at } \eta = 0:\ f(0) = 0,\ H(0) = \lambda,$$

$$\text{as } \eta \to \infty:\ H(\infty) = 1,\ T(\infty) = 1,\ g(\infty) = f(\infty). \tag{24}$$

Using central finite difference operators for the system $(21a - 21c)$, in equations (22 and (23). The system of equations is exhibit system of nonlinear algebraic equations, which becomes difficult to solve. Therefore we linearize by them by introducing, for example

$$[f]_j^{k+1} = [f]_j^k + [\delta f]_j^k$$

where $[\delta_j^k]$ has to be corrected at each step, we drop product terms like $\delta f_j, \delta V_j$ etc and also neglected square terms in δf_j. To solve equation (22) and (23) we use LU factorizing for decomposing in to a product of a lower triangular matrix L and upper triangular matrix U as follows

$$A = LU$$

The equations (22), (23) and (24) can be put in algorithm. The coding and solution of the matrix can find by using Keller-box method

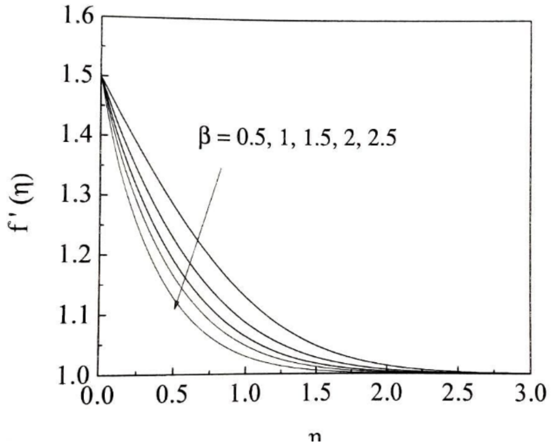

Figure 1: Variation of fluid phase velocity profiles $f'(\eta)$ with η for various values of β when $\lambda = 1.5, \chi = 0.1, Mc = 0.1$.

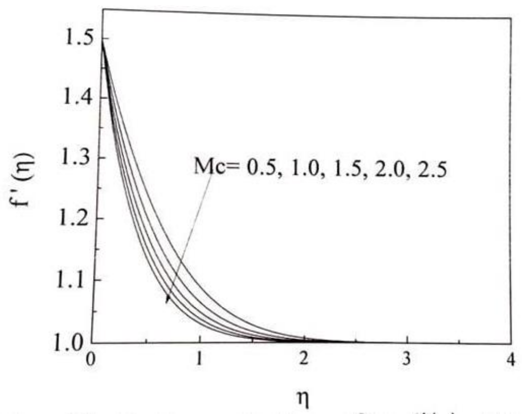

Figure 2: Variation of fluid phase velocity profiles $f'(\eta)$ with η for various values of Mc when $\lambda = 1.5, \chi = 1.0, \beta = 2.0$.

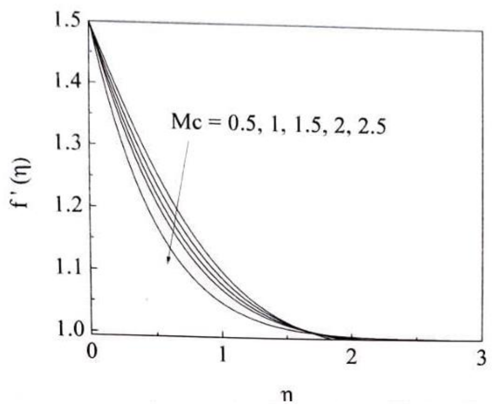

Figure 3: Variation of fluid phase velocity profiles $f'(\eta)$ with η for various values of Mc when $\lambda = 1.5, \chi = 1.0, \beta = 1$.

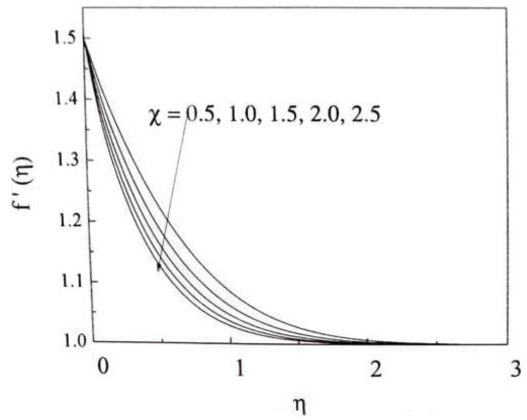

Figure 4: Variation of fluid phase velocity profiles $f'(\eta)$ with η for various values of χ when $\lambda = 1.5, Mc = 1.0, \beta = 2.0$.

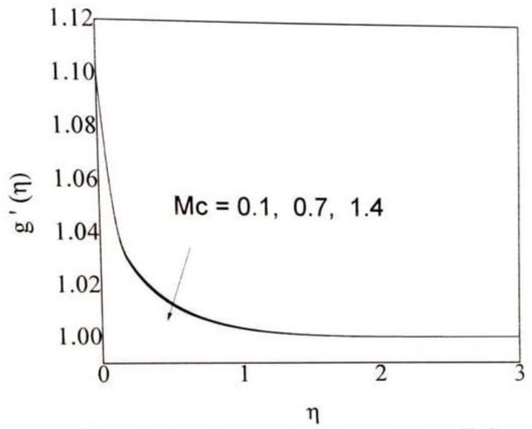

Figure 5: Variation of particle phase velocity profiles $f'(\eta)$ with η for various values of Mc when $\lambda = 1.2, \chi = 0.1, \beta = 0$.

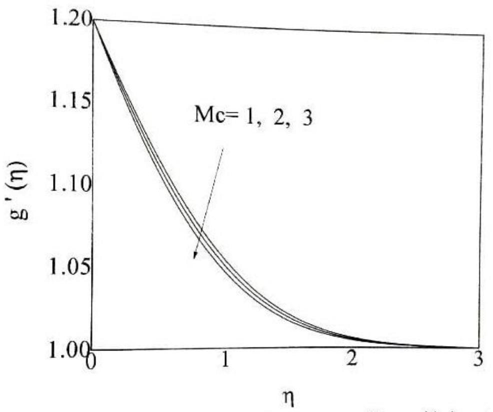

Figure 6: Variation of particle phase velocity profiles $g'(\eta)$ with η for various values of Mc when $\lambda = 1.2, \chi = 0.1, \beta = 0$.

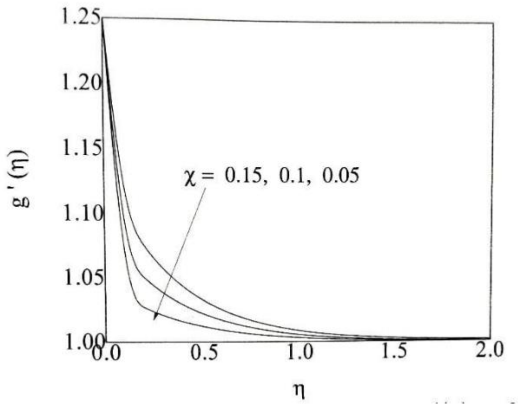

Figure 7: Variation of particle phase velocity profiles $g'(\eta)$ with η for various values of χ when $\lambda = 1.25, Mc = 0.9, \beta = 0.$

Results and discussion

In this paper, we investigate two-dimensional boundary-layer flow of a dusty fluid over a wedge. Similarity transformations are used to convert the time independent non-linear boundary-layer equations into a system of non-linear ordinary differential equations. Numerical solutions are presented for highly non-linear boundary-layer equations. The numerical solutions have been carried out by Keller-box method to study the effect of various parameters such as pressure gradient parameter β, mass concentration parameter Mc and local fluid particle interaction parameter χ. The velocity profiles $f'(\eta)$ as a function of η for different values of the pressure gradient parameter β by taking $\lambda = 1.5, \chi = 0.1$, and $Mc = 0.1$ in figure 1. It is shown that pressure gradient parameter β increases the boundary-layer velocity and hence the thickness decreases when the other parameters χ and Mc are held fixed. Also, it is noticed that all the curves approach their end-condition asymptotically, in fact quite faster for larger values of the pressure gradients. In a similar manner, from figure 2 and 3 , we observed that the mass concentration parameter Mc increases the fluid phase velocity in the boundary-layer and decreases the boundary-layer thickness. It is interesting to note that the thickness of boundary-layer decreases with increasing the values of Mc. It is further noticed that for higher values of β , the boundary layer thickness becomes thinner as is clearly observed in figure 3 for the same parameter Mc. In figure 4 the velocity profiles are shown for different values of fluid-particle interaction parameter χ. It is noticed from this figure that the velocity decreases with increasing values of χ for fluid phase in the boundary-layer. The effect of increasing value of χ is to reduce the velocity$f'(\eta)$ and thereby making the boundary-layer thinner. Also it reveals that for large values of χ i.e. the relaxation time of the dust particle decreases.

The variation of velocity profiles with $g'(\eta)$ with η for different values of mass con- centration parameter Mc for dust phase has been illustrated in the figures 5 and 6. It is found that with an increase in the mass concentration parameter, the dust phase velocity decreases. This is due to the fact that, the dust particles are increased then dust fluid velocity automatically decreases and keeping other parameters fixed.

Also the boundary-layer thickness decreases as mass concentration parameter Mc increases. Although, there is a little variation in the dust phase profile but we anticipate that there is a rather appreciable difference in the corresponding hydrodynamics. We further expect the similar solution structure and hydrodynamics on the dust phase velocity behaviour when fluid particle interaction parameter χ is varied gradually in figure 7. Even in these figures, the all the curves approach mainstream flow asymptotically.

Conclusions

This paper deals the two- dimensional boundary-layer flow of steady dusty fluid over an exponential stretching surface is considered. The similarity transformations are used the governing partial differential equations are reduced into set of non-linear ordinary differential equations are solved numerically by applying Keller-box method. The velocity of both fluid phase and dust phase profiles are obtained for various values of physical parameters like fluid interaction parameter χ, mass concentration parameter Mc and pressure gradient parameter β. The fluid particle interaction parameter decreases the velocity components in the fluid phase and increases in the dust phase.

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