On Ancient Indian Mathematics

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Abstract

Ancient Indian mathematics showcases an extraordinary evolution from practical applications tied to ritual and daily life to abstract theoretical constructs. This paper explores the development of Indian mathematical practices, tracing their roots from early proto-arithmetic in Vedic rituals to sophisticated algebraic methods exemplified by Aryabhata and others. Special emphasis is given to the use of geometry in altar construction, the encoding of numerical information in Vedic texts, and the development of astronomical calculations. Through an analysis of primary texts and their historical contexts, this study highlights the ingenuity and depth of Indian mathematical traditions while addressing the challenges posed by the absence of written numeral systems during significant early periods.

Keywords: Vedic Mathematics, Vedic Ritual, Astronomy, Śulvasūtras, Shatapatha Brahman, Maitriyani Samhita

1. Introduction:

Humanity's engagement with mathematics began through concrete, lived experiences. Archaeological evidence such as tally sticks, scratch-marked bones, and clay tokens suggests early counting and record-keeping practices. These tools evolved across cultures, reflecting the innate mathematical ability in humans (Barras, 2021; Schmandt-Besserat, 1996; Damerow, 2012). For instance, early Egyptians recorded quantities using clay bullae, and Mesopotamians invented cuneiform to represent goods and numbers (Clagett, 1999). However, on the Indian subcontinent, no archaeological evidence from 18th to 3rd century BCE directly demonstrates numerical knowledge. Instead, ancient texts like the Śulvasūtras and the Vedāṇga Jyotiṣa reveal rich mathematical practices in astronomy and altar construction.

The antiquity and advancement of Indian civilization, particularly its mathematical knowledge, have been extensively studied (Thapar, 2004; Witzel, 2001; Bryant, 2001). Mathematics, essential for science and technology, was evident in the Harappan civilization's urban planning, including grid roads and public baths. However, after its decline (1900–1700 BCE), the Vedic period (1500–500 BCE) lacks archaeological evidence but features rich textual references to mathematics in astronomy, calendrics, and sacrificial altar-making, as seen in the Śulva Sutras and Vedanga Jyotish. Despite these, questions remain about the development of mathematics in a culture without scripts, numeral systems, or zero. Modern projections onto ancient texts often misrepresent their mathematical practices. This work examines the feasibility and scope of ancient mathematics within these historical constraints.

Kenoyer (2020) suggests that a culture can survive without a writing system, though its absence influences its nature and capabilities. In the Vedic tradition, the lack of a script fostered a reliance on memorization and orality, leading to the development of poetic meters (chandas) and hymn composition for knowledge transmission. However, this limitation also affected mathematics, which relies on external symbols for numbers and procedural manipulation. Questions about numerical visualization, time tracking, astronomical calculations for calendars, and altar-related computations highlight the context-dependent and constrained nature of arithmetic in this period (Dasgupta, J., 2024). Across cultures, time-tracking has been a common activity, and the Vedic texts reflect this tendency. They describe the division of time into days, months, seasons, and years, with rituals like the year-long **gavamāyana sattra** marking significant celestial events (Satapatha Brahman [SB] 1.6.3.35–36; Tilak, 1893). The Vedic system used names, rituals, and objects like stones and bricks to represent time units, akin to tally marks in earlier cultures (Marshack, 1991; Livio, 2002). The **sadaha**, dividing months into five parts, adjusted for lunar irregularities (Taittiriya Sanhita [TS] 7.2.1; SB, pt. III, p. xxi; SB, pt. V, p. 148; Dikshit, 1969).

A sophisticated naming system for time segments provided unique designations for days, nights, and smaller units, enabling extensive time tracking (Dikshit, 1969). Numbers are also mentioned in texts like the **Rig Veda** (RV) and **Shatapatha Brahman** (Witzel, 2001; Kak, 1993). However, these systems lacked numeral symbols, relying instead on concrete representations. For example, the SB equates bricks and stones with time units like days, muhurtas, and seasons (SB 10.4.2.4–17; SB 10.5.4.5). This "proto-arithmetic" was tied to ritual and lacked the abstraction possible with numeral symbols (Seidenberg, 1981).

Despite limitations in standardization and scope, this system reflected advanced mathematical thinking compared to prehistoric times. The advent of script and numerals in later periods marked a shift towards abstract mathematics, enabling growth beyond the context-dependent practices of the Vedic period.

A few other Vedic texts, such as the *Taittiriya Samhita* (TS 4.4.11), *Kathak Samhita* (KS 17.10), and the *Maitriyani Samhita* (MS 2.8.14), also repeat the same numerical sequences, occasionally with slight variations. This raises significant questions: how were these numbers generated, and what did they signify? The initial number names—*eka* (1), *daśa* (10), *śata* (100), and *sahasra* (1,000)—indicate a decimal system. This system likely originated from the practice of counting on ten fingers, established by the Upper Paleolithic/Mesolithic period, suggesting its use in India during the 2nd millennium BCE. Comparable base-10 systems were used in Egypt by 3000 BCE. In the absence of symbolic notation, numbers were often tied to concrete objects, as seen in references to bricks and cows in both the *Yajurveda* and other Vedic texts. These objects helped contextualize large numbers within a cultural framework.

Later texts, like the Buddhist *Lalitavistāra* and the Jain *Anuyogadvara Sutra*, also engage with extraordinarily large numbers, generated through systematic operations like multiplication or squaring. For instance, *Lalitavistāra* describes numbers up to 10^{53}, while the Jain text mentions 2^{96} ($\sim 10^{29}$) as the world's human population. These numbers, often accompanied by unique names, seem designed to demonstrate mathematical ingenuity rather than practical application. Their contexts, such as measuring mythical Mount Meru or estimating sand grains in the Ganges, emphasize infinity and the metaphysical.

However, early numerical names were inconsistent. While numbers like *eka*, *daśa*, *śata*, and *sahasra* remained standard for centuries, others varied across texts and authors. For example, *ayuta* denoted 10,000 in the *Yajurveda* but a billion in *Lalitavistāra*. This lack of standardization persisted into later mathematical works and was noted by scholars like Al-Biruni (Sachau, 1910).

The constancy of smaller number names reflects their practical relevance in daily life during the Vedic period, linked to commerce, astronomy, and pastoral wealth. Numbers larger than *sahasra* gained broader societal significance only during the period of the *Arthashastra* (300 BCE–300 CE), with references to salaries and fines in the tens of thousands. In the subsequent centuries, large numbers became crucial for astronomical calculations. This trajectory illustrates the transition from numbers rooted in concrete materiality to those serving abstract and metaphysical purposes, aligning with the philosophical underpinnings of infinity in Indian thought.

Vedanga Jyotish (VJ), an astronomical text from the Vedic period, provides insights into the mathematical knowledge of its time, particularly in calendar-making. Its discussion often centers on its astronomical concepts and uncertain dating (ranging from 1400–400 BCE), while the mathematical feasibility of the described computations remains underexplored. The text outlines a 5-year yuga cycle, beginning with the conjunction of the sun, moon, and Dhanishthā nakshatra at the winter solstice. The solar year is 366 days, yielding 1830 days in a yuga, with 62 lunar synodic months (29.5 days each) and 67 lunar sidereal months (27.3 days each). This setup requires detailed fractional calculations, such as determining nakshatras for new and full moons within the yuga, and reflects knowledge of arithmetic and algebraic procedures, such as the rule of three. However, the feasibility of performing these operations without numerals, a place-value system, or zero raises questions.

Mathematical concepts like fractions may have existed since the Rig Veda, which mentions parts of Purusha, though these could reflect concrete rather than abstract fractions (Trivedi, 1954). Performing operations like adding or dividing fractions necessitates understanding complex procedures not intuitive or naturally acquired (Dehaene, 1997; Devlin, 2000; Caswell, 2007; Heath and Starr, 2022). The Arthashastra, situated between the Vedic and classical periods of Indian mathematics, indicates a progression in using unit fractions for practical purposes, although fractional calculations remained rare. Similarly, the Śulva Sutras used fractions tied to concrete practices like constructing altars with ropes and pegs (Thibaut, 1875; Dani, 2010).

Brahmi numerals, appearing in the 3rd century BCE, lacked a zero or positional value system, which limits their use in abstract calculations (Datta and Singh, 2004). The earliest evidence of fraction reduction and related operations appears in Jain texts between the 2nd–5th centuries CE and later works by Brahmagupta and Sridhar in the 7th–8th centuries CE. This historical context suggests possibilities: pre-existing knowledge of yuga cycles, the Arthashastra predating VJ, or the extant VJ being a later synthesis combining oral traditions with emerging mathematics. The silence of the Arthashastra on VJ further supports its later composition. Several possibilities emerge regarding the composition of Vedanga Jyotish (VJ): (1) the five-year yuga cycle and synodic lunar month calculations may have predated VJ; (2) the Arthashastra could have been composed earlier, as its omission of VJ and sidereal lunar positions is puzzling, especially given the role of court astrologers in calendrical computations; and (3) the extant VJ might have been compiled later, integrating earlier synodic calculations with lunar nakshatra positions when suitable mathematical methods became available. The conjunction of the sun, moon, and Dhanishthā nakshatra at the winter solstice, orally transmitted over centuries, likely served as a reference point for computations, which could be performed subsequently due to the fixed-sky model in VJ (Datta and Singh, 2004).

2. Abstract and Concrete Concepts:

Humanity's engagement with mathematics began through concrete, lived experiences. Archaeological evidence such as tally sticks, scratch-marked bones, and clay tokens suggests early counting and record-keeping practices. These tools evolved across cultures, reflecting the innate mathematical ability in humans (Barras, 2021; Schmandt-Besserat, 1996; Damerow, 2012). For instance, early Egyptians recorded quantities using clay bullae, and Mesopotamians invented cuneiform to represent goods and numbers (Clagett, 1999). However, on the Indian subcontinent, no archaeological evidence from 18th to 3rd century BCE directly demonstrates numerical knowledge. Instead, ancient texts like the Śulvasūtras and the Vedāṇga Jyotiṣa reveal rich mathematical practices in astronomy and altar construction. Indian mathematics initially centered on practical arithmetic and geometry. The *Rig Veda* (~1500 BCE) reflects early numeric concepts, with subsequent texts like the *Shatapatha Brahmana* describing proto-arithmetic tied to sacrificial rituals and calendar computations (Eggeling, 1897). Using objects like bricks and yajushmati stones, ancient practitioners performed physical operations instead of symbolic calculations. Geometry focused on altar construction using ropes and pegs, revealing early discoveries akin to the Pythagorean theorem (*Sulvasutras*, Thibaut, 1875).

2.1 Arithmetic

The Rig Veda (c. 1500 BCE) reveals early numerical knowledge, referencing numbers such as 3, 5, 7, 360, and 720 (RV 1.164.11; RV 10.62.7-8). Although lacking numerical symbols or operations, ancient texts like the Shatapatha Brahmana (SB) exhibit sophisticated counting and enumeration methods. For example, SB 10.4.2.2-17 describes dividing 720 bricks into smaller groups to represent days and nights.

These practices, embedded in rituals and societal activities, relied on tangible objects such as bricks, stones, and natural phenomena like the moon's phases. Arithmetic in this era was “proto-arithmetic,” involving operations like sorting, counting, and distributing concrete objects. This contrasts with abstract arithmetic, which manipulates written symbols independent of context.

With the advent of writing, Indian mathematics shifted toward abstraction. Aryabhata's *Aryabhatiya* (5th century CE) introduces algebraic methods for solving equations, including quadratic equations and proportions, signaling a leap in mathematical thinking (Shukla & Sarma, 1976). Later scholars like Bhaskara II expanded on these foundations, formalizing concepts of unknown variables and equations (Colebrooke, 1817). The Vedic texts, particularly the *Rig Veda* and *Shatapatha Brahmana*, encode the mathematical constant Pi. One example is a verse traditionally praising deities, which, when decrypted using the Katapayadi system, approximates Pi to 32 decimal places. This system, assigning numerical values to Sanskrit syllables, reveals a sophisticated interplay between spirituality and mathematics (Srivas Krishna Das Brahmacari, 2024). The Vedic approach to Pi rivals and, in some respects, surpasses Western approximations like Archimedes' geometric methods. Aryabhata's formula for the circumference of a circle demonstrates precision and awareness of Pi's irrationality centuries before it was formally proven in Europe by Lambert in 1761 (Plofker, 2009). Texts like the *Sulvasutras* incorporate Pi for precise architectural designs, including temples, altars, and domes. These applications underline a holistic worldview where mathematical precision intersected with cultural and spiritual life (Sarasvati Amma, 1999).

2.2 Geometry

Geometry in ancient India originated from practical needs, particularly in altar construction. The Śulvasūtras (800–300 BCE) provide instructions for constructing altars of various shapes while maintaining specific sizes, such as a 7.5 square puruṣa. Using tools like ropes and pegs, these altar-makers discovered principles now associated with geometry.

Methods included drawing perpendicular lines using ropes and marking cardinal directions with shadows (Kātyāyana Śulvasūtra 1.2-3). For example, the diagonal of a square was empirically measured as approximately √2 times its side, derived through iterative adjustments of rope lengths (Baudhāyana

Śulvasūtra 2.13). Such discoveries reflect hands-on approaches rather than theoretical deductions.
Geometry at this stage remained concrete, using physical tools and observations. The abstraction of geometry required symbolic representation and formal proofs, which emerged only after the development of writing systems.

2.3 Algebra

The transition to abstract mathematics became evident with the advent of writing. Aryabhata's *Aryabhatiya* (5th century CE) marks a milestone, covering topics like quadratic equations, arithmetic progressions, and indeterminate equations. Proportions, rooted in practical arithmetic, laid the foundation for algebra. For instance, Aryabhata uses the rule of three to solve proportional problems, representing an early form of linear equations.

While early texts lacked symbols for unknown quantities, later mathematicians like Brahmagupta introduced terms like "avyakta" (unknown) and used color-coded variables for multiple unknowns. Bhāskara II's *Bījagaṇita* formalized algebraic methods, including solving quadratic equations by completing the square.

The abstraction of algebra relied on three characteristics:

1. Generality of solutions.
2. Representation of unknown quantities.
3. Equations with balanced sides.

This evolution, dependent on symbolic representation, reflects a gradual shift from solving practical problems to manipulating abstract concepts.

The progression of Indian mathematics from concrete to abstract parallels developments in other ancient cultures like Babylon. Early practices involving tangible objects, such as bricks and ropes, evolved into abstract mathematical thinking with the advent of writing. Proportions served as a bridge, enabling the development of algebraic principles. This transition illustrates the interplay between practical needs and theoretical advancements in mathematical history.

3. Conclusion

The mathematical journey of ancient India highlights a remarkable transition from concrete, ritual-based practices to abstract theoretical constructs. Early Vedic mathematics, constrained by oral traditions, relied on tangible representations and contextual problem-solving. However, the advent of writing systems and numeral innovations facilitated a shift toward abstraction, paving the way for the sophisticated contributions of later mathematicians like Aryabhata and Brahmagupta. This evolution underscores the resilience and ingenuity of Indian mathematical traditions, which balanced practical applications with theoretical advancements. Ancient Indian mathematics not only shaped its contemporary cultural and scientific milieu but also laid foundational principles that resonate with modern mathematical practices.

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