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A Linear Programming Approach to Invest Provident Fund of Retirees

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Abstract

Old age is an unavoidable period in everyone's life. Although, it cannot be avoided everyone tries to make it easier at least by financially by proper pre-planning. Many government employees in India have pensions but the people working in private sector do not get it. But they receive a one-time Provident Fund (PF) at the time of retirement. This paper proposes a Linear Programming (LP) model to help retired individuals to strategically invest their Provident Fund in multiple asset classes. By considering factors such as risk, amount of investment and return rates, the model provides optimized investment suggestions to maximize returns without compromising financial security. The model uses LP to determine the best portfolio allocation, offering retirees a quantitative tool to enhance their investment decisions by properly verifying all the possibilities.

Keywords: Linear Programming (LP), Maximizing return, Optimization Problem, Provident Fund (PF).

1. Introduction

After retirement almost every working individual gets a provident fund (PF). The main concern of every retiree is where to invest this PF. Every retiree wants to be financially capable and independent in his old age. Most of the people working in the private sector do not have pension. Therefore, proper investment of received provident fund is necessary to them due to expenditure on medicines, physical ailments, no monthly salary due to retirement, rising inflation etc. Also, in India old people/retiree want to help to their next generation by offering some amount of money. Considering all these factors they do not want to take more risk while investing. Generally, their prior focus is to maximize profit and minimize risk. Retiree often struggle with how to allocate this fund optimally across different asset classes to maximize returns. Now a days due to inflation it is not sufficient to invest in only one traditional asset like FDs or MIS. In fact, sometimes these options have their own limitations. The paper helps such retirees to provide strategic investment suggestion that aim to maximize returns on their Provident Fund (PF) by using Linear Programming (LP) model. The LP model considers a variety of constraints such as average, the lock-up period and the total amount invested, making it adaptable to the diverse financial situations of retirees.[3]
Linear Programming, is the most widely applied mathematical technique that helps managers in decision-making and planning for the optimal allocation of limited resources. It deals with the optimization (minimization or maximization) of a function of variables known as objective function, subject to set of linear equations and/or inequalities known as constraints. By applying LP, this study aims to optimize the allocation of PF funds across various asset classes such as Saving Schemes, Fixed Deposits, Mutual Funds etc to create an investment portfolio that maximizes returns while adhering to the retiree's risk and liquidity

preferences. This research offers a model for improving the financial outcomes of retirees. By utilizing Linear Programming, retirees can make more informed, data-driven decisions, ensuring a better financial future in their post-retirement years.

2. Theory

2.1 Optimization Problem- [4]A mathematical problem is an optimization problem in which the objectives and constraints are given as mathematical functions and functional relationships. Mathematical programs of the form

Optimize $z = f(x_1, x_2, \dots, x_n)$

Subject to:

$$\left.\begin{aligned} g_1(x_1, x_2, \dots, x_n) &\le b_1 \\ g_2(x_1, x_2, \dots, x_n) &\le b_2 \\ \dots\dots\dots\dots &= \cdots \\ g_m(x_1, x_2, \dots, x_n) &\ge b_m \end{aligned}\right\} \qquad \text{---(I)}$$

Each of the m constraint relationship in (I) involves one of the three signs $\le, =, \ge$.

2.2 Linear Programs

[4]A mathematical program (I) is linear if $f(x_1, x_2, \dots, x_n)$ and each $g_i(x_1, x_2, \dots, x_n)$, $i = 1,2, \dots, m$ are linear in each of their arguments- that is, if

$$f(x_1, x_2, \dots, x_n) = c_1x_1 + c_2x_2 + \dots + c_nx_n \qquad \text{---(II)}$$

And

$$g_i(x_1, x_2, \dots, x_n) = a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \qquad \text{- - -(III)}$$

Where c_i and a_{ij}, $i = 1,2, \dots, m; j = 1,2, \dots, n$ are known constants.

Any other mathematical program is nonlinear.

2.3 Requirements for an LPP

[3] Linear programming can be used for optimization problems if the following conditions are satisfied.

- There will be a well-defined objective function which is to be either maximized or minimized and which can be expressed as a linear function of decision variables.
- 2. There must be constraints on the amount of extent of attainment of the objective and these constraints must be capable of being expressed as linear equations or inequalities in terms of variables.
- 3. There must be alternative course of action. E.g., a given product may be processed by two different machines and problem may be as to how much of the product to allocate to which machine.
- 4. Another necessary requirement is that decision variables should be interrelated and non- negative. The non-negativity condition shows that linear programming deals with real life situations for which negative quantities are generally illogical.
- 5. As stated earlier, the resources must be in a limited supply.

2.4 Problem Formulation

[4] Optimization problems most often are stated verbally. The solution procedure is to model the problem with a mathematical program and then solve the program by appropriate techniques. The following approach is recommended for transforming a word problem into a mathematical program.

Step-1) Determine the quantity to be optimized and express it as a mathematical function. Doing so serves to define the input variables.

Step-2) Identify all stipulated requirements, restrictions and limitations, and express them mathematically. These requirements constitute the constraints.

Step-3) Express any hidden conditions. Such conditions are not stipulated explicitly in the problem but are apparent from the physical situation being modelled. Generally, they involve non-negativity or integer requirements on the input variables.

2.5 Solution to LP problem

[3] Once a problem is formulated as a mathematical model, the next step is to solve the problem to get the optimal solution. There are different methods to solve LP problems.

1. Graphical method: A LPP with only two variables can be solved using graphical/geometrical method.
2. Though, in actual practice such small problems are rarely encountered.
3. Simplex method
4. We can solve LP problem using software tools like Microsoft Excel Solver, Python (libraries like Pulp, SciPy or Pomo), MATLAB, LINDO etc.

3. Methodology

[1,2] In this research paper, we have suggested the best investment plan to a retiree who wants to invest his PF to maximize profit and minimize risk by considering various investment options he has with rate of return, tenure and risk of return (in standard deviation %) for each alternative. Also, he has applied some conditions on risk of investment, maximum investment in each alternative etc. After taking all these conditions into consideration we want to suggest him final solution.

Case Study: Suppose Mr. Nathan, a 60-year-old retiree, has received a lump sum of ₹50,00,000 from his Provident Fund (PF) account upon retirement. He seeks to create an investment portfolio that will meet his goals:

- Ensure the portfolio's risk does not exceed 5% (measured as a standard deviation of return).
- Maintain liquidity by investing at least 10% of the corpus in highly liquid assets such as Fixed deposits or Debt Mutual Funds.
- No single investment option should constitute more than 30% of the total portfolio value.
- The investment amount in any individual option must be non-negative.

with conditions

- Total investment must not exceed ₹50,00,000.
- A diversified portfolio is desired to balance risk and return.

Suggest a best investment option to Mr. Nathan to maximize the total annual return of the portfolio by determining the optimal allocation for each investment option and considering all the conditions he has mentioned, distribute his ₹50,00,000.

He has following investment options.

Table 1: All possible investment alternatives

Investment Option	Rate of Return (Annual %)	Tenure (in years)	Risk (% Std. Dev)
Senior Citizens Saving Scheme (SCSS)	8.2	5(extendable by 3)	2
Post Office Monthly Income Scheme (POMIS)	7.4	5	2.5
Public Provident Fund (PPF)	7.1	15	1

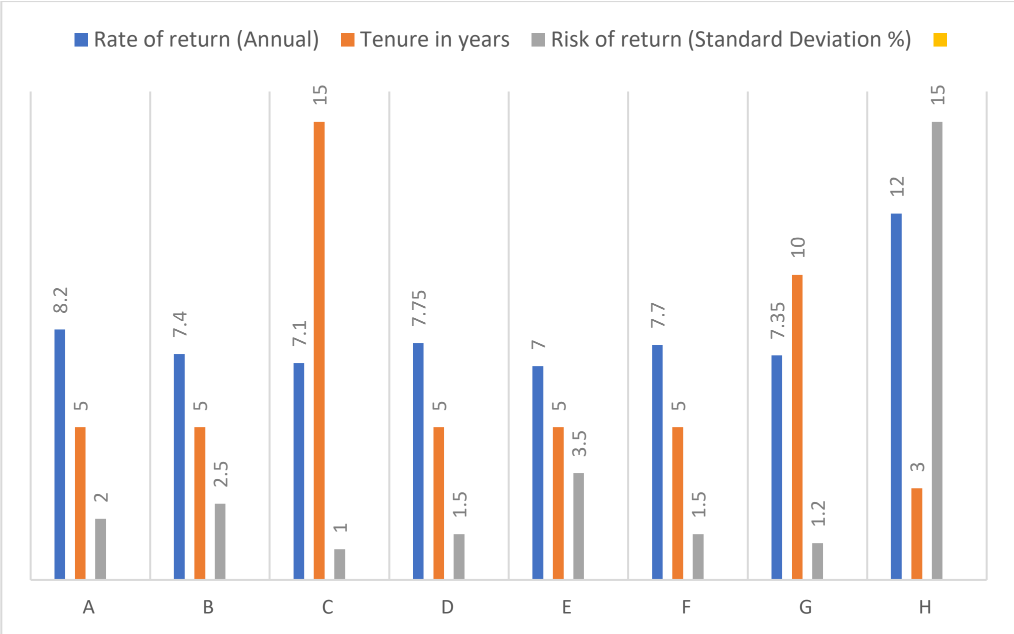
Fixed Deposits-FD (Senior Citizens)	7 to 8.5	1-10 (flexible)	1 to 2
Debt Mutual Funds	6 to 8	1-5	3.5
National Saving Certificate (NSC)	7.7	5	1.5
Government Bonds	7.2-7.5	5-10	1.2
Equity-Linked Saving Schemes (ELSS)	12 (volatile)	3	15

From above table we may observed that for FD, Debt Mutual Funds and Government Bonds there is no exact rate of return. Therefore, we have taken average of approximate values for each of them. Similarly, we have considered that period of investment in FD and Government bonds is 5 and 10 years respectively. Likewise, while calculating risk we have taken mean of lower and upper bound of risks where exact risk percentage are not given. We have labelled the investment options respectively by letters A, B, C, D, E, F, G and H.

Table 2: All investment alternatives taken into consideration

Investment Option	**Investment Option Code**	**Rate of Return (Annual %) (r_i)**	**Tenure (in years) (T_i)**	**Risk (% Std. Dev) (R_i)**
Senior Citizens Saving Scheme (SCSS)	A	8.2	5	2
Post Office Monthly Income Scheme (POMIS)	B	7.4	5	2.5
Public Provident Fund (PPF)	C	7.1	15	1
Fixed Deposits-FD (Senior Citizens)	D	7.75	5	1.5
Debt Mutual Funds	E	7	5	3.5
National Saving Certificate (NSC)	F	7.7	5	1.5
Government Bonds	G	7.35	10	1.2
Equity-Linked Saving Schemes (ELSS)	H	12	3	15

The tabular data is pictured in the next graph.



Formulation of LP Model

[3] Now we will convert the given problem into mathematical LP model as follows.

Let $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$ be the percentage amounts invested in A, B, C, D, E, F, G and H respectively. r_i be rate of return, T_i be tenure, and R_i be risk in standard deviation for each x_i; $i = 1,2,3,\dots,8$.

Our *objective* is to maximize total returns (Z).

Maximize $Z = r_1x_1 + r_2x_2 + r_3x_3 + r_4x_4 + r_5x_5 + r_6x_6 + r_7x_7 + r_8x_8$

$= 8.2x_1 + 7.4x_2 + 7.1x_3 + 7.75x_4 + 7x_5 + 7.7x_6 + 7.35x_7 + 12x_8$

The constraints are:

Constraint on total investment: $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 1$

Constraint on risk: $2x_1 + 2.5x_2 + x_3 + 1.5x_4 + 3.5x_5 + 1.5x_6 + 1.2x_7 + 15x_8 \leq 5$

Constraint on tenure/period of investment: $5x_1 + 5x_2 + 15x_3 + 5x_4 + 5x_5 + 5x_6 + 10x_7 + 3x_8 \leq 15$

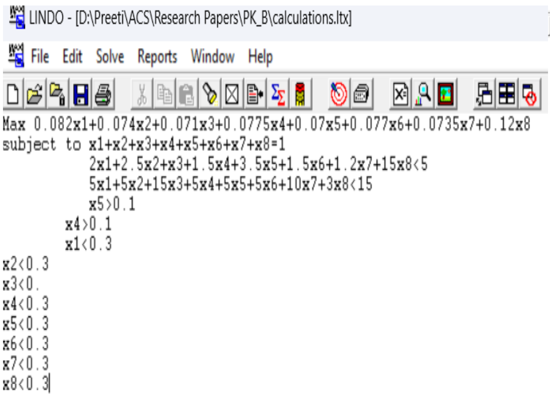
Other remaining constraints

$0 \leq x_i \leq 30\%$ of total investment, $1 \leq i \leq 8$.

i.e. $0 \leq x_i \leq 0.3$, $1 \leq i \leq 8$ and

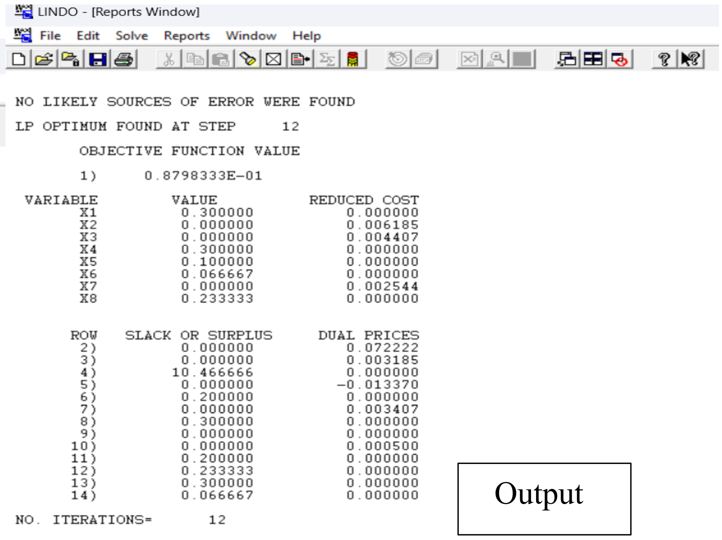
$x_4 \geq 0.1$ and $x_5 \geq 0.1$

We have used LINDO app to solve above problem. Following are the screenshots of problem and solution.



```
LINDO - [D:\Preeti\ACS\Research Papers\PK_B\calculations.ltx]
File Edit Solve Reports Window Help
Max 0.082x1+0.074x2+0.071x3+0.0775x4+0.07x5+0.077x6+0.0735x7+0.12x8
subject to x1+x2+x3+x4+x5+x6+x7+x8=1
           2x1+2.5x2+x3+1.5x4+3.5x5+1.5x6+1.2x7+15x8<5
           5x1+5x2+15x3+5x4+5x5+5x6+10x7+3x8<15
           x5>0.1
        x4>0.1
        x1<0.3
x2<0.3
x3<0.
x4<0.3
x5<0.3
x6<0.3
x7<0.3
x8<0.3
```

Input



```
LINDO - [Reports Window]
File Edit Solve Reports Window Help

NO LIKELY SOURCES OF ERROR WERE FOUND

LP OPTIMUM FOUND AT STEP     12

       OBJECTIVE FUNCTION VALUE

       1)     0.8798333E-01

  VARIABLE        VALUE          REDUCED COST
        X1         0.300000          0.000000
        X2         0.000000          0.006185
        X3         0.000000          0.004407
        X4         0.300000          0.000000
        X5         0.100000          0.000000
        X6         0.066667          0.000000
        X7         0.000000          0.002544
        X8         0.233333          0.000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          0.072222
        3)         0.000000          0.003185
        4)        10.466666          0.000000
        5)         0.000000         -0.013370
        6)         0.200000          0.000000
        7)         0.000000          0.003407
        8)         0.300000          0.000000
        9)         0.000000          0.000000
       10)         0.000000          0.000500
       11)         0.200000          0.000000
       12)         0.233333          0.000000
       13)         0.300000          0.000000
       14)         0.066667          0.000000

NO. ITERATIONS=      12
```

Output

Thus, after calculations we have the following values for $x_i's; 1 \leq i \leq 8$.

$x_1 = 0.30, x_2 = 0.00, x_3 = 0.00, x_4 = 0.30, x_5 = 0.10, x_6 = 0.066667, x_7 = 0.00, x_8 = 0.233333$

These all values represent the percentage of investment for each investment alternative mentioned earlier. Now, we will calculate actual investment per option as follow.

For alternative A: $50{,}00{,}000 \times 0.30 =$ ₹15,00,000

For alternative B: $50{,}00{,}000 \times 0.00 =$ ₹00

For alternative C: $50{,}00{,}000 \times 0.00 =$ ₹00

For alternative D: $50{,}00{,}000 \times 0.30 =$ ₹15,00,000

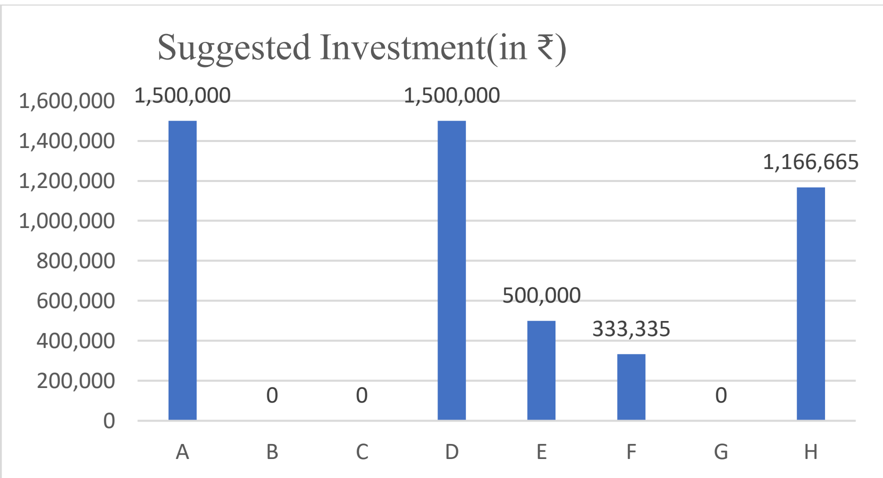
For alternative E: $50{,}00{,}000 \times 0.10 =$ ₹5,00,000

For alternative F: $50{,}00{,}000 \times 0.066667 =$ ₹3,33,335

For alternative G: $50{,}00{,}000 \times 0.00 =$ ₹00

For alternative H: $50{,}00{,}000 \times 0.233333 =$ ₹11,66,665

Thus, Mr. Nathan will receive maximum returns on his total investment of₹50,00,000 if he invest ₹15,00,000 in both Senior Citizens Saving Scheme (SCSS) and Fixed Deposits-FD (Senior Citizens), ₹5,00,000 in Debt Mutual Funds, ₹3,33,335 in National Saving Certificate (NSC) and ₹11,66,665 in Equity-Linked Saving Schemes (ELSS).



4. Conclusion and more Applications

This research paper suggests that the use of LPP in investing the PF of retirees can lead to improved investment outcomes, including higher returns and lower risk. We can modify this problem by applying more constraints like monthly income, liquidity etc.

This approach is not applicable only to retirees who want to invest their PF, but to people who want to invest in share market or Mutual funds. By applying the same procedure, we can suggest to people in which shares they should invest to get maximum profit taking minimum risk. Also, by considering annual income, monthly expenses, inflation and future financial goals, we can use this model to plan investment strategies for employees.

References:

1. A. Charnes, W. W. Cooper and M. H. Miller, "Application of Linear Programming to Financial Budgeting and the Costing of Funds", The Journal of Business; Published By: The University of Chicago Press. Vol. 32, No. 1(Jan. 1959), pp. 20-46 (27 pages)
2. Sebastian Majewski, Aleksandra Majewska, Procedia ComJan, "Behavioural portfolio as a tool supporting investment decisions". Science, Volume-207, 2022, pages 1713-1722
3. S K Gupta, D S Hira "Operations Research", S Chand and Company Limited
4. Richard Bronson, Govind Sami Nagamuthu, "Schaum's Outline of Theory and Problems of Operation Research", Second Edition - Tata McGraw-Hill Publishing Company Limited, New Delhi.