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Outer-Connected Inverse Domination in Graphs

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Abstract
Let G be a connected simple graph. A subset S of $V(G)$ is a dominating set of G if for every $v \in V(G) \setminus S$, there exists $x \in S$ such that $xv \in E(G)$. A set $D \subseteq V(G)$ is said to be an outer-connected dominating set in G if D is dominating and either $D = V(G)$ or $\langle V(G) \setminus D \rangle$ is connected. Let D be a minimum dominating set of G. A nonempty subset $S \subseteq V(G) \setminus D$ is an outer connected inverse dominating set of G, if S is an inverse dominating set with respect to D and the subgraph $\langle V(G) \setminus S \rangle$ induced by $V(G) \setminus S$ is connected. The outer connected inverse domination number of G, is denoted by $\tilde{\gamma}_c^{(-1)}(G)$, that is, the minimum cardinality of an outer connected inverse dominating set of G. In this paper, we initiate the study of the concept and give the outer-connected inverse domination number of some special graphs. Further, we give the characterization of the outer-connected inverse dominating set in the join of two nontrivial connected graphs.

Keywords: dominating set, outer-connected dominating set, inverse dominating set, outer-connected inverse dominating set

1. Introduction

Domination in graph was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [1]. Following an article [2] by Ernie Cockayne and Stephen Hedetniemi in 1977, the domination in graphs became an area of study by many researchers. A subset S of $V(G)$ is a dominating set of G if for every $v \in V(G) \setminus S$, there exists $x \in S$ such that $xv \in E(G)$, i.e., $N[S] = V(G)$. The domination number $\gamma(G)$ of G is the smallest cardinality of a dominating set of G. Some studies on domination in graphs were found in the papers [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

A set S of vertices of a graph G is an outer-connected dominating set if every vertex not in S is adjacent to some vertex in S and the sub-graph induced by $V(G) \setminus S$ is connected. The outer-connected domination number $\tilde{\gamma}_c(G)$ is the minimum cardinality of the outer-connected dominating set S of a graph G. The concept of outer-connected domination in graphs was introduced by Cyman [14]. Some related studies of outer-connected domination in graphs are found in [15, 16, 17, 18, 19, 20, 21].

Let D be a minimum dominating set in G. The dominating set $S \subseteq V(G) \setminus D$ is called an inverse dominating set with respect to D. The minimum cardinality of an inverse dominating set is called an inverse domination number of G and is denoted by $\gamma^{-1}(G)$. An inverse dominating set of cardinalities $\gamma^{-1}(G)$ is called γ^{-1} - set of G. The inverse domination in a graph was first found in the paper of Kulli [22] and can be read in the papers [23, 24, 25, 26, 27, 28, 29, 30, 31, 32].

Motivated by the introduction of the outer-connected dominating sets and the inverse dominating sets, a new variant of domination in graphs is introduced in this paper. Let D be a minimum dominating set of G. A nonempty subset $S \subseteq V(G) \setminus D$ is an outer connected inverse dominating set of G, if S is an inverse dominating set with respect to D and the subgraph $\langle V(G) \setminus S \rangle$ induced by $V(G) \setminus S$ is connected. The outer connected inverse domination number of G, is denoted by $\tilde{\gamma}_c^{(-1)}(G)$, that is the minimum cardinality of an outer connected inverse dominating set of G. In this paper, we initiate the study of the concept and give the outer-connected inverse domination number of some special graphs. Further, we show the characterization of the outer-connected inverse dominating set in the join of two nontrivial connected graphs.

For the general terminology in graph theory, readers may refer to [33]. A graph G is a pair $(V(G), E(G))$, where $V(G)$ is a finite nonempty set called the vertex-set of G and $E(G)$ is a set of unordered pairs $\{u, v\}$ (or simply uv) of distinct elements from $V(G)$ called the edge-set of G. The elements of $V(G)$ are called vertices and the cardinality $|V(G)|$ of $V(G)$ is the order of G. The elements of $E(G)$ are called edges and the cardinality $|E(G)|$ of $E(G)$ is the size of G. If $|V(G)| = 1$, then G is called a trivial graph. If $E(G) = \emptyset$, then G is called an empty graph. The open neighborhood of a vertex $v \in V(G)$ is the set $N_G(v) = \{u \in V(G) : uv \in E(G)\}$. The elements of $N_G(v)$ are called neighbors of v. The closed neighborhood of $v \in V(G)$ is the set $N_G[v] = N_G(v) \cup \{v\}$. If $X \subseteq V(G)$, the open neighborhood of X in G is the set $N_G(X) = \bigcup_{v\in X} N_G(v)$. The closed neighborhood of X in G is the set $N_G[X] = \bigcup_{v\in X} N_G[v] = N_G(X) \cup X$. When no confusion arises, $N_G[x]$ [res. $N_G(x)$] will be denoted by $N[x]$ [resp. $N(x)$].

2. Results

Definition 2.1 A simple graph G is an undirected graph with no loop edges or multiple edges.

Definition 2.2 The path $P_n = \{ a_1a_2a_3 \dots a_n\}$ is the graph with $V(P_n) = \{a_1, a_2, a_3, \dots, a_n\}$and $E(P_n) = \{ a_1a_2, a_2a_3, \dots, a_{n-1}a_n\}$.

Definition 2.3 The cycle $C_n = \{ a_1a_2a_3 \dots a_na_1\}$ is the graph with $V(C_n) = \{a_1, a_2, a_3, \dots, a_n\}$ and $E(C_n) = \{ a_1a_2, a_2a_3, \dots, a_na_1\}$.

Definition 2.4 A graph $K_n = (V(K_n), E(K_n))$ is called a complete graph of order n when xy is an edge in K_n for every distinct pair $x, y \in V(K_n)$.

Definition 2.5 A complete bipartite graph is a graph whose vertex set can be partitioned into V_1 and V_2 such that every edge joins a vertex in V_1 with a vertex in V_2, and every vertex in V_1 is adjacent with every vertex in V_2.

Remark 2.6 Let G be a special graph.

(i) if $G = C_n$, then $\tilde{\gamma}_c^{(-1)}(G) = 2 , n = 4$

(ii) if $G = P_n$, then $\tilde{\gamma}_c^{(-1)}(G) = \begin{cases} n-1, & \text{if } n = 2 \text{ or } n = 3 \\ 2, & \text{if } n = 4 \\ \text{none}, & \text{if } n \geq 5 \end{cases}$

(iii) if $G = K_n$, then $\tilde{\gamma}_c^{(-1)}(G) = 1 , \forall n \geq 2$

(iv) if $G = S_n$, then $\tilde{\gamma}_c^{(-1)}(G) = n, \forall n \geq 1$

(v) if $G = K_{m,n}$, then $\tilde{\gamma}_c^{(-1)}(G) = 2, \forall m, n \geq 2$

Definition 2.7 The join $G + H$ of two graphs G and H is the graph with vertex-set $V(G+H) = V(G) \cup V(H)$ and edge-set $E(G+H) = E(G) \cup E(H) \cup \{uv: u \in V(G), v \in V(H)\}$.

The following results are needed for our theorem.

Lemma 2.8 Let G and H be connected non-complete graphs. If $S = V(G) \setminus D_G$, where $D_G \subset V(G)$ is a minimum dominating set of $G + H$, then S is an outer-connected inverse dominating set of $G + H$.

Proof: Suppose that $S = V(G) \setminus D_G$. Then S is an inverse dominating set of $G + H$ with respect to a minimum dominating set D_G of $G + H$. Let $v \in V(G+H) \setminus S$. If $v \in V(G) \setminus S = D_G$, then $vy \in E(G+H)$ for all $y \in V(H)$. If $v \in V(H)$, then $vx \in E(G+H)$ for all $x \in D_G$. This implies that the subgraph induced by $V(G+H) \setminus S$ is connected. Hence, S is an outer-connected dominating set of $G + H$, that is, S is an outer-connected inverse dominating set of $G + H$. ■

Lemma 2.9 Let G and H be connected non-complete graphs. If $S = V(H) \setminus D_H$, where $D_H \subset V(H)$ is a minimum dominating set of $G + H$, then S is an outer-connected inverse dominating set of $G + H$.

Proof: Suppose that $S = V(H) \setminus D_H$. Then S is an inverse dominating set of $G + H$ with respect to a minimum dominating set D_H of $G + H$. Let $v \in V(G+H) \setminus S$. If $v \in V(H) \setminus S = D_H$, then $vx \in E(G+H)$ for all $x \in V(G)$. If $v \in V(G)$, then $vy \in E(G+H)$ for all $y \in D_H$. This implies that the subgraph induced by $V(G+H) \setminus S$ is connected. Hence, S is an outer-connected dominating set of $G + H$, that is, S is an outer-connected inverse dominating set of $G + H$. ■

Lemma 2.10 Let G and H be connected non-complete graphs. If $S = (V(G) \setminus D_G) \cup (V(H) \setminus \{y\})$, $y \in V(H)$, where $D_G \subset V(G)$ is a minimum dominating set of $G + H$, then S is an outer-connected inverse dominating set of $G + H$.

Proof: Suppose that $S = (V(G) \setminus D_G) \cup (V(H) \setminus \{y\})$, $y \in V(H)$. Then S is an inverse dominating set of $G + H$ with respect to a minimum dominating set $D_G \subset V(G)$ of $G + H$. Let $v \in V(G+H) \setminus S$. If $v \in V(G) \setminus S = D_G$, then $vy \in E(G+H)$. If $v \in V(H) \setminus S$, then $v = y$ and $xy \in E(G+H)$ for all $x \in D_G$. This implies that the subgraph induced by $V(G+H) \setminus S$ is connected. Hence, S is an outer-connected dominating set of $G + H$, that is, S is an outer-connected inverse dominating set of $G + H$. ■

Lemma 2.11 Let G and H be connected non-complete graphs. If $S = (V(H) \setminus D_H) \cup (V(G) \setminus x)$, $x \in V(G)$, where $D_H \subset V(H)$ is a minimum dominating set of $G + H$, then S is an outer-connected inverse dominating set of $G + H$.

Proof: Suppose that $S = (V(H) \setminus D_H) \cup (V(G) \setminus \{x\})$, $x \in V(G)$. Then S is an inverse dominating set of $G + H$ with respect to a minimum dominating set $D_H \subset V(H)$ of $G + H$. Let $v \in V(G+H) \setminus S$. If $v \in V(H) \setminus S = D_H$, then $vx \in E(G+H)$. If $v \in V(G) \setminus S$, then $v = x$ and $xy \in E(G+H)$ for all $y \in D_H$. This implies that the subgraph induced by $V(G+H) \setminus S$ is connected. Hence, S is an outer-connected dominating set of $G + H$, that is, S is an outer-connected inverse dominating set of $G + H$. ■

Lemma 2.12 Let G and H be connected non-complete graphs. If $S = (V(G) \setminus D_G) \cup A, A \subset (V(H) \setminus \{y\}), y \in V(H), A \neq \emptyset$, and $D_G \subset V(G)$ is a minimum dominating set of $G + H$, then S is an outer-connected inverse dominating set of $G + H$.

Proof: Suppose that $S = (V(G) \setminus D_G) \cup A, A \subset (V(H) \setminus \{y\}), y \in V(H), A \neq \emptyset$. Then S is an inverse dominating set of $G + H$ with respect to a minimum dominating set $D_G \subset V(G)$ of $G + H$. Let $v \in V(G + H) \setminus S$. If $v \in V(G) \setminus S = D_G$, then $vu \in E(G + H)$ for all $u \in V(H) \setminus A$. If $v \in V(H) \setminus A$, then $xv \in E(G + H)$ for all $x \in D_G$. This implies that the subgraph induced by $V(G + H) \setminus S$ is connected. Hence, S is an outer-connected dominating set of $G + H$, that is, S is an outer-connected inverse dominating set of $G + H$. ■

Lemma 2.13 Let G and H be connected non-complete graphs. If $S = (V(H) \setminus D_H) \cup B, B \subset (V(G) \setminus \{x\}), x \in V(G), B \neq \emptyset$, and $D_H \subset V(H)$ is a minimum dominating set of $G + H$, then S is an outer-connected inverse dominating set of $G + H$.

Proof: Suppose that $S = (V(H) \setminus D_H) \cup B, B \subset (V(G) \setminus \{x\}), x \in V(G), B \neq \emptyset$. Then S is an inverse dominating set of $G + H$ with respect to a minimum dominating set $D_H \subset V(H)$ of $G + H$. Let $v \in V(G + H) \setminus S$. If $v \in V(H) \setminus S = D_H$, then $vu \in E(G + H)$ for all $u \in V(G) \setminus B$. If $v \in V(G) \setminus B$, then $yv \in E(G + H)$ for all $y \in D_H$. This implies that the subgraph induced by $V(G + H) \setminus S$ is connected. Hence, S is an outer-connected dominating set of $G + H$, that is, S is an outer-connected inverse dominating set of $G + H$. ■

Theorem 2.14 Let G and H be connected non-complete graphs. The subset $S \subset V(G + H)$ is an outer-connected inverse dominating set of $G + H$, if one of the following conditions is satisfied.

1. $S \subseteq (V(G) \setminus D_G) \cup (V(H) \setminus \{y\})$, where $D_G \subset V(G)$ is a minimum dominating set of $G + H$ and $y \in V(H)$.
2. $S \subseteq (V(H) \setminus D_H) \cup (V(G) \setminus \{x\})$, where $D_H \subset V(H)$ is a minimum dominating set of $G + H$ and $x \in V(G)$.
3. $S \subseteq (V(G + H) \setminus \{x, y\}), x \in V(G), y \in V(H)$ and $\{x, y\}$ is a minimum dominating set of $G + H$.

Proof: Suppose that statement (i) is satisfied. Then $S \subseteq (V(G) \setminus D_G) \cup (V(H) \setminus \{y\})$, where $D_G \subset V(G)$ is a minimum dominating set of $G + H$ and $y \in V(H)$. Consider the following cases.

Case 1. If $S = V(G) \setminus D_G$, then by Lemma 2.8, S is an outer-connected inverse dominating set of $G + H$.

Case 2. If $S = (V(G) \setminus D_G) \cup (V(H) \setminus \{y\}), y \in V(H)$, then by Lemma 2.10, S is an outer-connected inverse dominating set of $G + H$.

Case 3. If $S = (V(G) \setminus D_G) \cup A, A \subset (V(H) \setminus \{y\}, A \neq \emptyset$, and $y \in V(H)$, then by Lemma 2.12, S is an outer-connected inverse dominating set of $G + H$.

Suppose that statement (ii) is satisfied. Then $S \subseteq (V(H) \setminus D_H) \cup (V(G) \setminus \{x\})$, where $D_H \subset V(H)$ is a minimum dominating set of $G + H$ and $x \in V(G)$. Consider the following cases.

Case 1. If $S = V(H) \setminus D_H$, then by Lemma 2.9, S is an outer-connected inverse dominating set of $G + H$.

Case 2. If $S = (V(H) \setminus S_H) \cup (V(G) \setminus \{x\}), x \in V(G)$, then by Lemma 2.11, S is an outer-connected inverse dominating set of $G + H$.

Case 3. If $S = (V(H) \setminus S_H) \cup B$, where $B \in (V(G) \setminus \{x\})$ and $x \in V(G)$, then by Lemma 2.13, S is an outer-connected inverse dominating set of $G + H$.

Suppose that statement (iii) is satisfied. Then $S \subseteq (V(G + H) \setminus \{x, y\}), x \in V(G), y \in V(H)$ and $\{x, y\}$ is a minimum dominating set of $G + H$. Consider the following cases.

Case 1. If $S = (V(G+H) \setminus \{x,y\})$, then S is an inverse dominating set of $G+H$ with respect to a minimum dominating set $\{x,y\}$. Let $v \in V(G+H) \setminus S$. If $v \in V(G) \setminus S$, then $v = x$ and $xy \in E(G+H)$. If $v \in V(H) \setminus S$, then $v = y$ and $xy \in E(G+H)$. This implies that the subgraph induced by $V(G+H) \setminus S$ is connected. Hence, S is an outer-connected dominating set of $G+H$, that is, S is an outer-connected inverse dominating set of $G+H$.

Case 2. If $S \neq (V(G+H) \setminus \{x,y\})$, then consider the following subcases.

Subcase 1. If $S = (V(G) \setminus \{x\})$, then S is an inverse dominating set of $G+H$ with respect to a minimum dominating set $\{x,y\}$. Let $v \in V(G+H) \setminus S$. If $v \in V(G) \setminus S$, then $v = x$ and $xu \in E(G+H)$ for all $u \in V(H)$. If $v \in V(H)$, then $xv \in E(G+H)$. This implies that the subgraph induced by $V(G+H) \setminus S$ is connected. Hence, S is an outer-connected dominating set of $G+H$, that is S is an outer-connected inverse dominating set of $G+H$.

Subcase 2. If $S = (V(H) \setminus \{y\})$, then S is an inverse dominating set of $G+H$ with respect to a minimum dominating set $\{x,y\}$. Let $v \in V(G+H) \setminus S$. If $v \in V(G)$, then $vy \in E(G+H)$. If $v \in V(H) \setminus S$, then $v = y$ and $uv \in E(G+H)$ for all $u \in V(G)$. This implies that the subgraph induced by $V(G+H) \setminus S$ is connected. Hence, S is an outer-connected dominating set of $G+H$, that is, S is an outer-connected inverse dominating set of $G+H$.

Subcase 3. If $S = S_G \cup S_H, S_G \subset (V(G) \setminus \{x\}), S_H \subset (V(H) \setminus \{y\}) (S_G \neq \emptyset$ and $S_H \neq \emptyset)$, then S is an inverse dominating set of $G+H$ with respect to a minimum dominating set $\{x,y\}$. Let $v \in V(G+H) \setminus S$. If $v \in V(G) \setminus S_G$, then $vu \in E(G+H)$ for all $u \in V(H) \setminus S_H$. If $v \in V(H) \setminus S_H, vu \in E(G+H)$ for all $u \in V(G) \setminus S_G$. This implies that the subgraph induced by $V(G+H) \setminus S$ is connected. Hence, S is an outer-connected dominating set of $G+H$, that is, S is an outer-connected inverse dominating set of $G+H$.

Subcase 4. If $S = S_G, S_G \subset (V(G) \setminus \{x\}), (S_G \neq \emptyset)$ and S_G is a dominating set of G, then S is a dominating set of $G+H$ and S is an inverse dominating set with respect to a minimum dominating set $\{x,y\}$. Let $v \in V(G+H) \setminus S$. If $v \in V(G) \setminus S_G$, then $vu \in E(G+H)$ for all $u \in V(H)$. If $v \in V(H), vu \in E(G+H)$ for all $u \in V(G) \setminus S_G$. This implies that the subgraph induced by $V(G+H) \setminus S$ is connected. Hence, S is an outer-connected dominating set of $G+H$, that is, S is an outer-connected inverse dominating set of $G+H$.

Subcase 5. If $S = S_H, S_H \subset (V(H) \setminus \{y\}), (S_H \neq \emptyset)$ and S_H is a dominating set of H, then S is a dominating set of $G+H$ and S is an inverse dominating set with respect to a minimum dominating set $\{x,y\}$. Let $v \in V(G+H) \setminus S$. If $v \in V(H) \setminus S_H$, then $vu \in E(G+H)$ for all $u \in V(G)$. If $v \in V(G), vu \in E(G+H)$ for all $u \in V(H) \setminus S_H$. This implies that the subgraph induced by $V(G+H) \setminus S$ is connected. Hence, S is an outer-connected dominating set of $G+H$, that is, S is an outer-connected inverse dominating set of $G+H$. ■

The following result is an immediate consequence of Theorem 2.14.

Corollary 2.15 Let G and H be connected non-complete graphs. Then

$$\tilde{\gamma}_c^{(-1)}(G+H) = \begin{cases} 1, & \text{if } A = \{x\} \text{ is a dominating set of } G \text{ (or } H) \\ & \text{and } B = \{y\} \text{is a domianting set of } G \text{ (or } H) \\ 2, & \text{if otherwise.} \end{cases}$$

Proof: Suppose that $S = V(G) \setminus D_G$, where $D_G \subset V(G)$ is a minimum dominating set of $G+H$. Then by Lemma 2.8, S is an outer-connected inverse dominating set of $G+H$. This implies that, $\tilde{\gamma}_c^{(-1)}(G+H) \leq |S|$.

Consider that $D_G = A = \{x\}$ and $B = \{y\} \subset V(G)$. Then B is also a dominating set of G, that is, B is an inverse dominating set of A of $G + H$. Let $w \in V(G + H) \setminus B$. If $w \in V(G) \setminus B$, then $wu \in E(G + H)$ for all $u \in V(H)$. If $w \in V(H)$, then $wv \in E(G + H)$, for all $v \in V(G) \setminus B$. Thus, B is an outer-connected dominating set of $G + H$, that is, B is an outer-connected inverse dominating set of $G + H$. Similarly, if B is a dominating set of H, then B is an outer-connected inverse dominating set of $G + H$. Let $S = B$. Then $1 \leq \tilde{\gamma}_c^{(-1)}(G + H) \leq |S| = |B| = 1$. Hence, $\tilde{\gamma}_c^{(-1)}(G + H) = 1$.
Suppose that $S = V(H) \setminus D_H$, where $D_H \subset V(H)$ is a minimum dominating set of $G + H$. Then by Lemma 2.9, S is an outer-connected inverse dominating set of $G + H$. Consider that $D_H = A = \{x\}$ and $B = \{y\} \subset V(H)(\text{or} V(G))$. By following similar arguments above, $\tilde{\gamma}_c^{(-1)}(G + H) = 1$.
Suppose that $S \subseteq (V(G + H) \setminus \{x, y\}), x \in V(G), y \in V(H)$, and $\{x, y\}$ is a minimum dominating set of $G + H$. By Theorem 2.14 (iii), S is an outer-connected inverse dominating set of $G + H$ with respect to a minimum dominating set $\{x, y\}$ of $G + H$. Let $S = \{v, u\}$ such that $x \neq v \in V(G)$ and $y \neq u \in V(H)$. Since $\{x, y\}$ is a minimum dominating set of $G + H$, it follows that $2 = |\{x, y\}| = \gamma(G + H) \leq \tilde{\gamma}_c^{(-1)}(G + H) \leq |S| = |\{v, u\}| = 2$, that is, $\tilde{\gamma}_c^{(-1)}(G + H) = 2$. ■

Conclusion and Recommendations

In this work, we introduced a new parameter of domination in graphs - the outer-connected inverse domination in graphs. The outer-connected inverse domination in the join of two graphs were characterized. The exact outer-connected inverse domination number resulting from this binary operation of two graphs were computed. This study will pave a way to new research such bounds and other binary operations of two graphs. Other parameters involving outer-connected inverse domination in graphs may also be explored. Finally, the characterization of an outer-connected inverse domination in graphs and its bounds is a promising extension of this study.

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