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Fair Inverse Domination in Graphs

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Abstract

Let G be a nontrivial connected simple graph. A subset S of $V(G)$ is a dominating set of G if for every $v \in V(G)\setminus S$, there exists $x \in S$ such that $xv \in E(G)$. Let D be a minimum dominating set of G. If $V(G)\setminus D$ contains a dominating set say S of G, then S is called an inverse dominating set with respect to D. A fair dominating set in a graph G (or FD-set) is a dominating set S such that all vertices not in S are dominated by the same number of vertices from S ; that is, every two vertices not in S has the same number of neighbors in S. An inverse dominating subset S of a vertex set $V(G)$ is said to be fair inverse dominating set if for every vertex $v \in V(G)\setminus S$ is dominated by the same number of the vertex in S. A fair inverse domination number is the minimum cardinality of a fair inverse dominating set S in G, denoted by $\gamma_{fd}^{(-1)}(G)$. In this paper, we initiate the study of the concept and give the fair inverse domination number of some special graphs. Further, we give the characterization of the fair inverse dominating set in the join of two nontrivial connected graphs.

Keywords: dominating set, inverse dominating set, fair dominating set, fair inverse dominating set

1. Introduction

Domination in graph was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [1]. Following an article [2] by Ernie Cockayne and Stephen Hedetniemi in 1977, the domination in graphs became an area of study by many researchers. A subset S of $V(G)$ is a dominating set of G if for every $v \in V(G)\setminus S$, there exists $x \in S$ such that $xv \in E(G)$, i.e., $N[S] = V(G)$. The domination number $\gamma(G)$ of G is the smallest cardinality of a dominating set of G. Some studies on domination in graphs were found in the papers [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

Let D be a minimum dominating set in G. The dominating set $S \subseteq V(G)\setminus D$ is called an inverse dominating set with respect to D. The minimum cardinality of inverse dominating set is called an inverse domination number of G and is denoted by $\gamma^{-1}(G)$. The concept of inverse domination in graphs is found in [15]. Some related studies of an inverse domination in graphs are found in [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26].

Other variant of domination in a graph is the fair domination in graphs is the fair domination in graphs is the fair domination in graphs [27]. A dominating subset $S\ of\ V(G)$ is a fair dominating set of G if all the vertices are not in S are dominated by the same number of vertices from S, that is, $|N(u) \cap S| = |N(v) \cap S|$ for every two distinct vertices u and v from $V(G) \setminus S$ and a subset S of $V(G)$ is a k-fair dominating set in G if for every vertex $v \in V(G) \setminus S, |N(v) \cap S| = k$. The minimum cardinality of a fair dominating set of G, denoted by $\gamma_{fd}(G)$, is called the fair domination number of G. A fair dominating set of cardinalities $\gamma_{fd}(G)$ is called γ_{fd}-set. Some related studies of a fair domination in graphs are found in [28, 29, 30, 31, 32, 33, 34, 35].

Motivated by the introduction of the fair dominating sets and the inverse dominating sets, a new variant of domination in graphs is introduced in this paper.

An inverse dominating subset S with respect to a minimum dominating set of a vertex set $V(G)$ is said to be fair inverse dominating set if for every vertex $v \in V\ (G) \setminus S$ is dominated by the same number of the vertex in S. A fair inverse domination number is the minimum cardinality of a fair inverse dominating set S in G, denoted by $\gamma_{fd}^{(-1)}(G)$. In this paper, we initiate the study of the concept and give the fair inverse domination number of some special graphs. Further, we give the characterization of the fair inverse dominating set in the join of two nontrivial connected graphs.

For the general terminology in graph theory, readers may refer to [36]. A graph G is a pair $(V(G), E(G))$, where $V(G)$ is a finite nonempty set called the vertex-set of G and $E(G)$ is a set of unordered pairs $\{u, v\}$ (or simply uv) of distinct elements from $V(G)$ called the edge-set of G. The elements of $V(G)$ are called vertices and the cardinality $|V(G)|$ of $V(G)$ is the order of G. The elements of $E(G)$ are called edges and the cardinality $|E(G)|$ of $E(G)$ is the size of G. If $|V(G)| = 1$, then G is called a trivial graph. If $E(G) = \emptyset$, then G is called an empty graph. The open neighborhood of a vertex $v \in V(G)$ is the set $N_G(v) = \{u \in V(G) : uv \in E(G)\}$. The elements of $N_G(v)$ are called neighbors of v. The closed neighborhood of $v \in V(G)$ is the set $N_G[v] = N_G(v) \cup \{v\}$. If $X \subseteq V(G)$, the open neighborhood of X in G is the set $N_G(X) = \bigcup_{v \in X} N_G(v)$. The closed neighborhood of X in G is the set $N_G[X] = \bigcup_{v \in X} N_G[v] = N_G(X) \cup X$. When no confusion arises, $N_G[x]$ [resp. $N_G(x)$] will be denoted by $N[x]\ [resp. N(x)]$.

2. Results

Definition 2.1 A simple graph G is an undirected graph with no loop edges or multiple edges.

Definition 2.2 The path $P_n = \{a_1a_2a_3 \dots a_n\}$ is the graph with $V(P_n) = \{a_1, a_2, a_3, \dots, a_n\}$ and $E(P_n) = \{a_1a_2, a_2a_3, \dots, a_{n-1}a_n\}$.

Definition 2.3 The cycle $C_n = \{a_1a_2a_3 \dots a_na_1\}$ is the graph with $V(C_n) = \{a_1, a_2, a_3, \dots, a_n\}$ and $E(C_n) = \{a_1a_2, a_2a_3, \dots, a_{n-1}a_n, a_na_1\}$.

Definition 2.4 A graph $K_n = (V(K_n), E(K_n))$ is called a complete graph of order n when xy is an edge in K_n for every distinct pair $x, y \in V(K_n)$.

Definition 2.5 A complete bipartite graph is a graph whose vertex set can be partitioned into V_1 and V_2 such that every edge joins a vertex in V_1 with a vertex in V_2, and every vertex in V_1 is adjacent with every vertex in V_2.

Remark 2.6 Let G be a special graph. For all positive integer k,

i. $if\ G = P_n$, then $\gamma_{fd}^{(-1)}(G) = \begin{cases} \frac{n+1}{3} & if\ n = 3k-1, \\ \frac{2n}{3} & if\ n = 3k, \\ \frac{n+2}{3} & if\ n = 3k+1. \end{cases}$

ii. $if\ G = C_n$, then $\gamma_{fd}^{(-1)}(G) = \begin{cases} \frac{n}{3} & if\ n = 3k, \\ \frac{n+2}{3} & if\ n = 3k+1, \\ \frac{2n-1}{3} & if\ n = 3k+2. \end{cases}$

iii. $if\ G = K_n$, then $\gamma_{fd}^{(-1)}(G) = for\ all\ n \geq 2.$

iv. $if\ G = K_{m,n}$, then $\gamma_{fd}^{(-1)}(G) = \begin{cases} n & if\ m = 1, \\ m & if\ n = 1, \\ 2 & if\ m \geq 2\ and\ n \geq 2. \end{cases}$

Definition 2.7 The join $G + H$ of two graphs G and H is the graph with vertex-set $V(G+H) = V(G) \cup V(H)$ and edge-set $E(G+H) = E(G) \cup E(H) \cup \{uv : u \in V(G), v \in V(H)\}$

The following results are needed for our theorem.

Lemma 2.8 Let G and H be connected non-complete graphs. If $S = V(G), \gamma(G) \neq 1$, and $\gamma(H) \leq 2$, then $S \subset V(G+H)$ is a fair inverse dominating set of $G+H$.

Proof: Suppose that $S = V(G)$, $\gamma(G) \neq 1$, and $\gamma(H) \leq 2$. Then S is a dominating set of $G+H$.

Case 1. If $\gamma(H) = 1$, then let $D = \{y\}$ be a dominating set of H. This implies that D is a minimum dominating set of $G+H$. Since $S \subset V(G+H)\backslash D$, S is an inverse dominating set of $G+H$ with respect to D. Further, for every $u \in V(H) = V(G+H)\backslash S$, $N_{G+H}(u) \cap S = S$. Hence, S is a fair dominating set of $G+H$. Accordingly, S is a fair inverse dominating set of $G+H$.

Case 2. If $\gamma(H) = 2$, then let $D = \{y, z\}$ be a dominating set of H. Since $\gamma(G) \neq 1$, it follows that D is a minimum dominating set of $G+H$. Using similar arguments in Case 1, S is a fair inverse dominating set of $G+H$. □

Lemma 2.9 Let G and H be connected non-complete graphs. If $S \subset V(G), \gamma(G) \neq 1$, and for each $v \in S$, $uv \in E(G)$ for all $u \in V(G)\backslash S$, then $S \subset V(G+H)$ is a fair inverse dominating set of $G+H$.
Proof: Suppose that $S \subset V(G), \gamma(G) \neq 1$, and for each $v \in S, uv \in E(G)$ for all $u \in V(G)\backslash S$. Then S is a dominating set of G. Hence, S is a dominating set of $G+H$.
Case 1. If $\gamma(H) \leq 2$, then let D be a minimum dominating set of H. The set D is also a minimum dominating set of $G+H$ considering that $\gamma(G) \neq 1$. Since $S \subset V(G+H)\backslash D$, S is an inverse dominating set of $G+H$ with respect to D. Let $u \in V(G+H)\backslash S$.
Subcase 1. If $u \in V(G)\backslash S$, then for each $v \in S$, and $uv \in E(G)$, $N_G(u) \cap S = S$. This implies that $N_{G+H}(u) \cap S = S$.
Subcase 2. If $u \in V(H)$, then $u \in V(G+H)\backslash S$ and $N_{G+H}(u) \cap S = S$.

Thus, for each $u \in V(G+H) \setminus S$, $N_{G+H}(u) \cap S = S$, that is, S is a fair dominating set of $G+H$. Since S is an inverse dominating set of $G+H$ with respect to a minimum dominating set D, it follows that S is a fair inverse dominating set of $G+H$.

Case 2. If $\gamma(H) \nleq 2$, then let $D = \{x, y\}$ such that $x \in V(G)$ and $y \in V(H)$. Since $\gamma(G) \neq 1$, it follows that D is a minimum dominating set of $G+H$. Moreover, the set $S \subset V(G+H) \setminus D$ is an inverse dominating set of $G+H$ with respect to D. Using similar arguments in Case 1, S is a fair dominating set of $G+H$. Hence, S is a fair inverse dominating set of $G+H$. □

Lemma 2.10 Let G and H be connected non-complete graphs. If $S = V(H)$, $\gamma(H) \neq 1$, and $\gamma(G) \leq 2$, then $S \subset V(G+H)$ is a fair inverse dominating set of $G+H$.

Proof: Suppose that $S = V(H)$, $\gamma(H) \neq 1$, and $\gamma(G) \leq 2$. Then S is a dominating set of $G+H$.

Case 1. If $\gamma(G) = 1$, then let $D = \{x\}$ be a dominating set of G. This implies that D is a minimum dominating set of $G+H$. Since $S \subset V(G+H) \setminus D$, S is an inverse dominating set of $G+H$ with respect to D. Further, for every $v \in V(G) = V(G+H) \setminus S$, $N_{G+H}(v) \cap S = S$. Hence, S is a fair dominating set of $G+H$. Accordingly, S is a fair inverse dominating set of $G+H$.

Case 2. If $\gamma(G) = 2$, then let $D = \{x, w\}$ be a dominating set of G. Since $\gamma(H) \neq 1$, it follows that D is a minimum dominating set of $G+H$. Using similar arguments in Case 1, S is a fair inverse dominating set of $G+H$. □

Lemma 2.11 Let G and H be connected non-complete graphs. If $S \subset V(H)$, $\gamma(H) \neq 1$, and for each $v \in S$, $uv \in E(H)$ for all $u \in V(H) \setminus S$, then $S \subset V(G+H)$ is a fair inverse dominating set of $G+H$.

Proof: Suppose that $S \subset V(H)$, $\gamma(H) \neq 1$, and for each $v \in S$, $uv \in E(H)$ for all $u \in V(H) \setminus S$. Then S is a dominating set of H. Hence, S is a dominating set of $G+H$.

Case 1. If $\gamma(G) \leq 2$, then let D be a minimum dominating set of G. The set D is also a minimum dominating set of $G+H$ considering that $\gamma(H) \neq 1$. Since $S \subset V(G+H) \setminus D$, S is an inverse dominating set of $G+H$ with respect to D. Let $u \in V(G+H) \setminus S$.

Subcase 1. If $u \in V(H) \setminus S$, then for each $v \in S$, and $uv \in E(H)$, $N_G(u) \cap S = S$. This implies that $N_{G+H}(u) \cap S = S$.

Subcase 2. If $u \in V(G)$, then $u \in V(G+H) \setminus S$ and $N_{G+H}(u) \cap S = S$.

Thus, for each $u \in V(G+H) \setminus S$, $N_{G+H}(u) \cap S = S$, that is, S is a fair dominating set of $G+H$. Since S is an inverse dominating set of $G+H$ with respect to a minimum dominating set D, it follows that S is a fair inverse dominating set of $G+H$.

Case 2. If $\gamma(G) \nleq 2$, then let $D = \{x, y\}$ such that $x \in V(G)$ and $y \in V(H)$. Since $\gamma(H) \neq 1$, it follows that D is a minimum dominating set of $G+H$. Moreover, the set $S \subset V(G+H) \setminus D$ is an inverse dominating set of $G+H$ with respect to D. Using similar arguments in Case 1, S is a fair dominating set of $G+H$. Hence, S is a fair inverse dominating set of $G+H$. □

Lemma 2.12 Let G and H be connected non-complete graphs. If $S = S_G \cup S_H$, S_G is an r-fair dominating set of G, S_H is an s-fair dominating set of H, $(\gamma(G) \neq 1$ and $\gamma(H) \neq 1)$, and $|S_G| - |S_H| = r - s$, then $S \subset V(G+H)$ is a fair inverse dominating set of $G+H$.

Proof: Let $D = \{x, y\}$, $x \in V(G)$, and $y \in V(H)$. Then D is a dominating set of $G+H$. Since $\gamma(G) \neq 1$ and $\gamma(H) \neq 1$, it follows that D is a minimum dominating set of $G+H$. The set $S \subset V(G+H) \setminus D$ is an inverse dominating set of $G+H$ with respect to D. Suppose that $S = S_G \cup S_H$. If S_G is an r-fair dominating set of G, then for all $u \in V(G)\backslash S_G$, $|N_G(u) \cap S_G| = r$. If S_H is an s-fair dominating set of H, then for all $u \in V(H)\backslash S_H$, $|N_H(u) \cap S_H| = s$. Let $u \in V(G+H)\backslash S$.

Case 1. If $u \in V(G)\backslash S_G$, then $|N_{G+H}(u) \cap S| = |N_G(u) \cap S_G| + |S_H| = r + |S_H|$.

Case 2. If $u \in V(H)\backslash S_H$, then $|N_{G+H}(u) \cap S| = |N_H(u) \cap S_H| + |S_G| = s + |S_G|$.

If $|S_G| - |S_H| = r - s$, then $r + |S_H| = s + |S_G| = |N_{G+H}(u) \cap S|$ for all $u \in V(G+H)\backslash S$ by combining Case 1 and Case 2. Let $k = r + |S_H| = s + |S_G|$. Then $|N_{G+H}(u) \cap S| = k$ for all $u \in V(G+H)\backslash S$. This implies that S is a fair dominating set of $G+H$, that is, S is a fair inverse dominating set of $G+H$. □

Theorem 2.13 Let G and H be connected non-complete graphs. The subset $S \subset V(G+H)$ is a fair inverse dominating set of $G+H$, if one of the following conditions is satisfied.

1. ($S = V(G), \gamma(G) \neq 1,$ and $\gamma(H) \leq 2$) or ($S \subset V(G), \gamma(G) \neq 1$, and for each $v \in S, uv \in E(G)$ for all $u \in V(G)\backslash S$).
2. ($S = V(H), \gamma(H) \neq 1$, and $\gamma(G) \leq 2$) or ($S \subset V(H), \gamma(H) \neq 1$, and for each $v \in S, uv \in E(H)$ for all $u \in V(H)\backslash S$).
3. $S = S_G \cup S_H$, S_G is an r-fair dominating set of G, S_H is an s-fair dominating set of H, ($\gamma(G) \neq 1$ and $\gamma(H) \neq 1$), and $|S_G| - |S_H| = r - s$.

Proof: Suppose that statement (i) is satisfied. Consider the following cases.

Case 1. If $S = V(G)$, $\gamma(G) \neq 1$, and $\gamma(H) \leq 2$, then by Lemma 2.8, $S \subset V(G+H)$ is a fair inverse dominating set of $G+H$.

Case 2. If $S \subset V(G)$, $\gamma(G) \neq 1$, and for each $v \in S$, $uv \in E(G)$ for all $u \in V(G)\backslash S$, then by Lemma 2.9, $S \subset V(G+H)$ is a fair inverse dominating set of $G+H$.

Suppose that statement (ii) is satisfied. Consider the following cases.

Case 1. If $S = V(H)$, $\gamma(H) \neq 1$, and $\gamma(G) \leq 2$, then by Lemma 2.10, $S \subset V(G+H)$ is a fair inverse dominating set of $G+H$.

Case 2. If $S \subset V(H)$, $\gamma(H) \neq 1$, and for each $v \in S$, $uv \in E(G)$ for all $u \in V(H)\backslash S$, then by Lemma 2.11, $S \subset V(G+H)$ is a fair inverse dominating set of $G+H$.

Suppose that statement (iii) is satisfied. Then $S = S_G \cup S_H$, S_G is an r-fair dominating set of G, S_H is an s −fair dominating set of H, ($\gamma(G) \neq 1$ and $\gamma(H) \neq 1$), and $|S_G| - |S_H| = r - s$. By Lemma 2.12, $S \subset V(G+H)$ is a fair inverse dominating set of $G+H$.

The following result is an immediate consequence of Theorem 2.13.

Corollary 2.14 Let G and H be connected non-complete graphs. If $D = \{x\}$ is a dominating set of G and $S = \{y\}$ is a dominating set of G, $(x \neq y)$, then $\gamma_{fd}^{(-1)}(G+H) = 1$.

Proof: Suppose that $D = \{x\}$ is a dominating set of G and $S = \{y\}$ is a dominating set of G. Then D is a minimum dominating set of $G+H$ and $S \subset V(G+H)\backslash D$ is an inverse dominating set of $G+H$ with respect to D. Clearly, $N_{G+H}(u) \cap S = S$ for all $u \in V(G+H)\backslash S$, that is, S is a fair dominating set of $G+H$. Thus, $1 \leq \gamma_{fd}^{(-1)}(G+H) \leq |S| = 1$, implies that $\gamma_{fd}^{(-1)}(G+H) = 1$. □

Corollary 2.15 Let G and H be connected non-complete graphs, $\gamma(G) \neq 1$, and $\gamma_{fd}(G) \leq \gamma_{fd}(H)$. If $D = \{x\}$ is the only trivial dominating set of H and for all $u \in V(G)\backslash S$, $uv \in E(G)$ for each $v \in S \subset V(G)$, then $\gamma_{fd}^{(-1)}(G+H) = \gamma_{fd}(G)$.

Proof : Suppose that $D = \{x\}$ is the only trivial dominating set of H. Then D is a minimum dominating set of H and of $G+H$. The subset $S \subset V(G+H)\backslash D$ is an inverse dominating set of $G+H$ with respect to D. Since for all $u \in V(G)\backslash S$, $uv \in E(G)$ for each $v \in S \subset V(G)$, the set $N_G(u) \cap S = S$ implies that S

is a fair dominating set of G. Further, $N_{G+H}(u) \cap S$ for all $u \in V(G+H)\backslash S$. Hence S is a fair dominating set of $G+H$, that is, S is a fair inverse dominating set of $G+H$. Thus, $\gamma_{fd}^{(-1)}(G+H) \leq |S|$ for all fair inverse dominating set S of $G+H$ and for all fair dominating set S of G. Choose S as a minimum fair dominating set of G, that is, $\gamma_{fd}^{(-1)}(G+H) \leq |S| = \gamma_{fd}(G)$. Since, $\gamma_{fd}(G) \leq \gamma_{fd}(H) \leq \gamma_{fd}^{(-1)}(G+H) \leq \gamma_{fd}(G)$, it follows that $\gamma_{fd}^{(-1)}(G+H) = \gamma_{fd}(G)$. □

3. Conclusion and Recommendations

In this work, we introduced a new parameter of domination in graphs - the fair inverse domination in graphs. The fair inverse domination in the join of two graphs were characterized. The exact fair inverse domination number resulting from this binary operation of two graphs were computed. This study will pave a way to new research such bounds and other binary operations of two graphs. Other parameters involving fair inverse domination in graphs may also be explored. Finally, the characterization of a fair inverse domination in graphs and its bounds is a promising extension of this study.

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