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Novel Approach for Solving An Assignment Problem Using Brute Force Method

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ABSTRACT:
This paper applies the Brute Force method to the Assignment Problem to minimize total man-Hours. By generating and evaluating all possible task assignments, we effectively find the optimal solution, demonstrating the method's capability in solving small to medium-sized problems with precision and accuracy. In this paper, we investigate the effectiveness of a Brute force method in solving the Assignment Problem.

KEYWORDS: ssignment Problem, Total Man-Hours, Brute Force Method, Optimization, Scheduling.

1. INTRODUCTION

The Assignment Problem, a fundamental concept in Operations Research and Combinatorial Optimization, aims to efficiently assign tasks to resources, minimizing total man-hours, with wide-ranging applications in workforce management, logistics, and scheduling. This study explores the application of the Brute Force method, which systematically evaluates all possible task assignments, ensuring precision and accuracy, providing exact solutions, particularly suitable for small to medium-size problems due to its computational efficiency. Despite the Assignment Problem being NP-hard, the Brute Force method offers a simple and intuitive approach, generating all possible assignments and evaluating their cost, with the optimal solution being the assignment with the minimum total cost, also serving as a benchmark to evaluate other algorithms' performance. This research investigates the computational complexity of the Brute Force method, providing insights into its effectiveness, limitations, and potential extensions, contributing to the existing literature on the Assignment Problem and its solutions.

2. LITERATURE REVIEW

The assignment problem has been a subject of extensive research since the 1950s, with the introduction of the Hungarian Method by Harold W. Kuhn (1955) [9]. This method provided an efficient solution to the assignment problem, and it has been widely used in various fields. Recent research has focused on improving the brute force algorithm for solving assignment problems, with Muhammad Bashir et al. (2020) [1] applying the brute force approach to quadratic assignment problems, and Anish Gupta, Vivek Kumar, and Anil Kumar (2021) [4] analyzing the brute force approach for assignment problems with constraints. Eranda Çela and Çağrı Güler (2011) [2] provided a comprehensive survey of assignment problems, highlighting the various techniques and algorithms used to solve them. Hong Chen and Yong Huang (2017) [3] proposed an improved brute force algorithm for assignment problems, which was further enhanced by Yong Zhang and Xiaoling Liu (2014) [18]. Yong Huang, Hong Chen, and Xueping Li (2022) [5] proposed an improved brute force algorithm for assignment problems, which has been shown to be more efficient than existing algorithms. Rakesh Jain, Vivek Kumar, and Anil Kumar (2023) [6] applied the brute force approach to quadratic assignment problems. Anil Kumar, Vivek Kumar, and Anish Gupta (2021) [7] proposed a hybrid algorithm for solving assignment problems, and Anil Kumar, Vivek Kumar, and Anish Gupta (2024) [8] proposed a new algorithm for solving assignment problems, which has been shown to be more efficient than existing algorithms. Xiaoling Liu, Yong Huang, and Hong Chen (2023) [10] proposed a new algorithm for solving quadratic assignment problems. Santosh Mishra and Anil Kumar (2020) [11] analyzed the brute force approach for assignment problems. Christos H. Papadimitriou and Kenneth Steiglitz (1982) [12] provided a comprehensive overview of combinatorial optimization, including the assignment problem. Amit Singh and Anil Kumar (2015) [13] analyzed the brute force approach for assignment problems, and Amit Singh and Anil Kumar (2018) [14] applied the brute force approach to quadratic assignment problems. Amit Singh, Anil Kumar, and Vivek Kumar (2022) [15] proposed a brute force algorithm for assignment problems with multiple objectives. Xing Wang and Jian Xu (2016) [16] proposed a brute force algorithm for assignment problems with constraints, which has been shown to be effective in solving real-world problems. Xing Wang and Jian Xu (2019) [17] proposed a brute force algorithm for quadratic assignment problems with constraints. Yong Zhang and Xiaoling Liu (2017) [19] proposed an improved brute force algorithm for quadratic assignment problems, and Yong Zhang and Xiaoling Liu (2020) [20] proposed a new algorithm for solving quadratic assignment problems.

3. RESEARCH SCOPE

The primary research scope is to review solving method of the assignment problem (Brute force method) and discuss the application of the Assignment problem in the candle sales company will provide useful information for the established data. Therefore the study optimises the man-hours required for each employee to sell candles.

4. MATHEMATICAL FORMULATION AND STRUCTURE

In this section, mathematical model of assignment problem is described. Assuming that there are 'n' jobs and 'm' persons. 'n' jobs must be performed by 'm' persons, where the cost depend on the specific assignment. Each job must be assigned to one and only one person and each person has to perform one and only one job. Let C, be the cost if the i^{th} person is assigned to the j^{th} job, the problem is to find an assignment (which job should be assigned to which person) so that the total cost for performing all jobs is minimum.

Here make an assumption that j^{th} will be completed by i^{th} person, and let

$$x_{ij} = \begin{cases} 1 \text{ if } i^{th} \text{ person is assigned to } j^{th} \text{ job} \\ 0 \text{ if } i^{th} \text{ person is assigned to } j^{th} \text{ job} \end{cases}$$

Where x_{ij} denotes that j^{th} job is to be assigned to the i^{th} person.

Then the mathematical model of assignment problem is as follows,

$Min\ z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$job

Subject to,

$\sum_{i=1}^{n} x_{ij}$= 1, i = 1,2,…..n
$\sum_{j=1}^{n} x_{ij}$= 1, j = 1,2,…..n
$x_{ij} \in [0,1]$ for i,j = 1,2,.....n
The structure of the assignment problem can be stated in the form of n×n cost matrix C_{ij} of real numbers as follows:

Persons			Jobs			
	1	2	3	…		n
1	c_{11}	c_{12}	c_{13}	…		c_{1n}
2	c_{21}	c_{22}	c_{23}	…		c_{2n}
…	…	…	…	…		…
N	c_{n1}	c_{n2}	c_{n3}	…		c_{nn}

Table 4.1 Structure of Assignment problem.

5. METHODOLOGY:

Algorithm for the proposed method

Step 1 : Formulate the objective function as a cost matrix with the number of jobs (rows) and workers (columns). Each value in the matrix represents the cost of assigning a particular worker to a job.
Min z =$\sum_{i=1}^{m} \sum_{j=1}^{n} = 1 c_{ij}\ x_{ij}$
Step 2 : Generating all possible assignments and introducing the constraints condition as
$\sum_{j=1}^{1} x_{ij}$= 1, j = 1,2, … , m i = 1,2, … , n
$\sum_{i=1}^{m} x_{ij}$= 1, j = 1,2, … , m i= 1,2, … , n
$x_{ij} = 0$ or $x_{ij} = 1$
Step 3: Calculate the Total Cost for Each Combination:
For each possible combination of job assignments calculate the total cost by summing the cost values corresponding to each assignment in the combination.
Step 4: Record the Results:
For each combination, write down the total cost of assigning the workers to the jobs in the respective combination.
Step 5. Identify the Optimal Solution:
Compare the total costs of all combinations.
Select the combination with the minimum total cost, as this represents the optimal solution.

6. NUMERICAL ANALYSIS:

A candle sales company has 4 employees (A, B, C, and D) and 4 sales areas (Area 1, Area 2, Area 3, and Area 4). The company wants to assign each employee to one sales area to minimize the total man-hours spent on sales. The man-hours required for each employee to sell candles in each area are:

Employee	Area 1	Area 2	Area 3	Area 4
A	3	5	2	4
B	6	2	8	1
C	4	3	6	2

D	1	6	5	3

Table 6.1 Basic data for the transportation problem

Step 1:

Linear Programming formulation

Min Z = $3x_{11} + 5x_{12} + 2x_{13} + 4x_{14} + 6x_{21} + 2x_{22} + 8x_{23} + 1x_{24} + 4x_{31} + 3x_{32} + 6x_{33} + 2x_{34} + 1x_{41} + 6x_{42} + 5x_{43} + 3x_{44}$

Step 2:

Enumerating all possible ways to assign the 4 employees to the 4 areas.

Subject to the Constraints:-

$x_{11} + x_{12} + x_{13} + x_{14} = 1$

$x_{21} + x_{22} + x_{23} + x_{24} = 1$

$x_{31} + x_{32} + x_{33} + x_{34} = 1$

$x_{41} + x_{42} + x_{43} + x_{44} = 1$

$x_{11} + x_{21} + x_{31} + x_{41} = 1$

$x_{12} + x_{22} + x_{32} + x_{42} = 1$

$x_{13} + x_{23} + x_{33} + x_{43} = 1$

$x_{14} + x_{24} + x_{34} + x_{44} = 1$

$x_{ij} = 0$ *or* $x_{ij} = 1$, i= 1,2,3,4 and j= 1,2,3,4

Step 3,4:

Computing the total man-hours by summing the man-hours required for the specific employee-area pair in the combination and recording the total man-hours required.

Combinations	Columns				Solution	
1	P1	P2	P3	P4	3+2+6+3	14
2	P1	P3	P4	P2	3+8+2+6	19
3	P1	P4	P2	P3	3+1+3+5	12
4	P1	P2	P4	P3	3+2+2+5	12
5	P1	P3	P2	P4	3+8+3+3	17
6	P1	P4	P3	P2	3+1+6+6	16
7	P2	P1	P3	P4	5+6+6+3	20
8	P2	P3	P4	P1	5+8+2+1	16
9	P2	P4	P1	P3	5+1+4+5	15
10	P2	P1	P4	P3	5+6+2+5	18
11	P2	P3	P1	P4	5+8+4+3	20
12	P2	P4	P3	P1	5+1+6+1	13
13	P3	P1	P2	P4	2+6+3+3	14
14	P3	P2	P4	P1	2+2+2+1	7

15	P3	P4	P1	P2	2+1+4+6	13
16	P3	P1	P4	P2	2+6+2+6	16
17	P3	P2	P1	P4	2+2+4+3	11
18	P3	P4	P2	P1	2+1+3+1	7
19	P4	P1	P2	P3	4+6+6+5	21
20	P4	P2	P3	P1	4+2+6+1	13
21	P4	P3	P1	P2	4+8+4+6	22
22	P4	P1	P3	P2	4+6+6+6	22
23	P4	P2	P1	P3	4+2+4+5	15
24	P4	P3	P2	P1	4+8+6+1	19

Table 6.2 Optimal solution for total man-hours

Step 5:

P3+P2+P4+P1= 2+2+2+1=7 & P3+P4+P2+P1=2+1+3+1=7.

The optimal solution of employees to sales areas with the minimum total man-hours is **7**.

7. CONCLUSION:

The optimal assignment of employees to different sales areas has been determined using a brute force method, considering various permutations and calculating the total man-hours required for each. By minimizing the total man-hours, the company can efficiently utilize its workforce, meet sales targets, and allocate resources effectively. The solution provides a comprehensive plan for assigning employees to sales areas, ensuring maximum capacity utilization, skill matching, and adherence to company constraints. This optimization approach enables the company to streamline its sales operations, enhance productivity, and ultimately drive business growth.

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