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Johan Coloring of Neighbourhood Corona of Graphs

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Abstract

Johan coloring or J-Coloring is the proper vertex coloring in which every vertices of a graph should belongs to rainbow neighbourhood . In a given graph G,a vertex v is said to be rainbow neighbourhood if every color class of G consists of atleast of one vertex of G from the closed neighbourhood. The Neighbourhood Corona is obtained by taking one copy of G_1 and n_1 copies of G_2 and by joining each neighbour i^{th} copy of G_1 to each and every vertex of i^{th} copy of G_2. In this paper we determine Johan Chromatic number of Neighbourhood Coronna of Graphs.

Keywords: Neighbourhood Corona, Johan Coloring, Johan Chromatic Number.

1. Introduction

Let H = (V, E) be a simple, finite, connected, and undirected graph, where V and E represent the graph's vertex and edge sets, respectively. The graph H's vertex a's minimum degree and maximum degree are represented by the symbols $\delta(H)$, $\Delta(H)$ respectively. The greatest number of edges joining the vertex in a graph is the maximum degree. And the least number of edges joining the vertex in a graph is called the minimum degree.

The method of giving colors to the graph's vertices is known as coloring of graph. According to the definition of graph coloring, no two adjacent vertices should share the same color. It is also known as vertex coloring. The Chromatic number is the minimum number of colors used to color the graph in which no two adjacent vertices have the same color. There are many types of vertex coloring and edge coloring of graphs subject to certain conditions [1]. There are various applications in graph coloring some are Assignment, Scheduling ,Time Tabling and Job Allocation Problems.

Johan Kok introduced the concept of Johan Coloring. Johan coloring is the maximal coloring of G if every vertex of G belongs to rainbow neighbourhood[2]. A rainbow neighbourhood is the closed neighbourhood of a vertex which contains atleast one colored vertex of each color in the chromatic coloring[3].In this paper, we have determined the Johan coloring of Neighbourhood corona of path with path,cycle,complete and star graphs.For general definition and notations of graphs and digraphs, we refer [4].

2. Preliminaries

Neighbourhood Corona of Graph [5]

Let G and H be two graphs with x_1 and x_2 vertices and y_1 and y_2 edges. The neighbourhood corona of two graphs G and H is obtained by taking one copy og graph G and x_1 copies of graph H and by joining each neighbour of i-th vertex of G to each and every vertex of i-th copy of H. The Neighbourhood corona of G and H is denoted as G ⋆ H. The number of vertices in G ⋆ H is $x_1x_2+x_1$ vertices.

3.Main Result

In this section, we have determined the Johan Chromatic Number of Neighbourhood Corona of Path with Path, Cycle Complete Graph and Star Graph

3.1. Theorem

For a,b≥2, Johan Coloring of Neighbourhood Corona of Path graph with Path is δ+1.

Proof:

Let P_a be the path graph on a vertices and P_b be the path graph on b vertices.

The vertex set of P_a *and* P_b *is*

$V(P_a) = \{v_x: 1 \leq x \leq a\}$ and $V(P_b) = \{u_y : 1 \leq y \leq b\}$

The vertex set of Neighbourhood Corona of $P_a \star P_b$ is

$$V(P_a \star P_b) = \{v_x : 1 \leq x \leq a\} \cup \{u_{xy} : 1 \leq x \leq a \text{ and } 1 \leq y \leq b\}$$

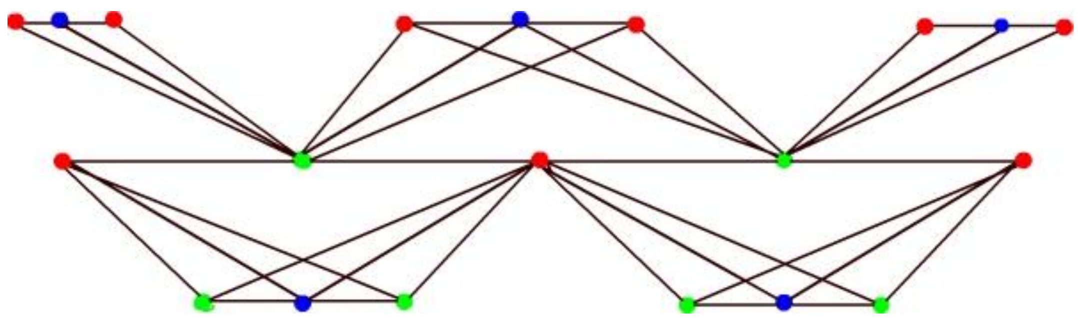
The minimum degree of $P_a \star P_b$ is 2 and maximum degree of $P_a \star P_b$ is 2b+2.

By applying Johan Coloring for Neighbourhood Corona of P_a *and* P_b , every vertex in $P_a \star P_b$ must satisfy rainbow neighbourhood condition.Thus we need δ + 1 colors.

Thus $J(P_a \star P_b)$ admits Johan coloring.

The Johan Chromatic Number of $P_a \star P_b$ is δ $(P_a \star P_b) + 1$.

Figure:1 Johan Coloring of $P_5 \star P_3$



3.2. Theorem

For a,b≥3, Johan Coloring of Neighbourhood Corona of Path with Complete Graph is δ+1.

Proof:

Let P_a be the path graph on a vertices and K_b be the complete graph on b vertices.

The vertex set of P_a *and* K_b *is*

$V(P_a) = \{v_x: 1 \leq x \leq a\}$ and $V(K_b) = \{u_y : 1 \leq y \leq b\}$

The vertex set of Neighbourhood Corona of $P_a \star K_b$ is

$$V(P_a \star K_b) = \{v_x : 1 \leq x \leq a\} \cup \{u_{xy} : 1 \leq x \leq a \text{ and } 1 \leq y \leq b\}$$

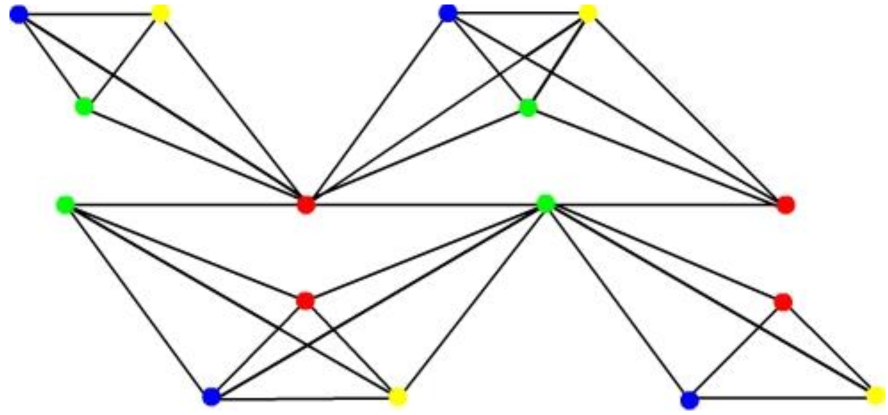
The minimum degree of $P_a \star K_b$ is b and maximum degree of $P_a \star K_b$ is 2b+2.

By applying Johan Coloring for Neighbourhood Corona of P_a *and* K_b , every vertex in $P_a \star K_b$ must satisfy rainbow neighbourhood condition.Thus we need δ + 1 colors.

Thus $J(P_a \star K_b)$ admits Johan coloring.

The Johan Chromatic Number of $P_a \star K_b$ is δ $(P_a \star K_b) + 1$.

Figure:2 Johan Coloring of $P_5 \star K_3$



3.3. Theorem

For a≥3, b≥2, Johan Coloring of Neighbourhood Corona of Path with Star Graph is δ+1.

Proof:

Let P_a be the path graph on a vertices and S_b be the Star graph on b vertices. The vertex set of P_a *and* S_b *is*

$V(P_a) = \{v_x: 1 \leq x \leq a\}$ and $V(S_b) = Z \cup \{u_y : 1 \leq y \leq b\}$

The vertex set of Neighbourhood Corona of $P_a \star S_b$ is

$$V(P_a \star S_b) = \{v_x : 1 \leq x \leq a\} \cup \{Z_x: 1 \leq x \leq a\} \cup \{u_{xy} : 1 \leq x \leq a \text{ and } 1 \leq y \leq b\}$$

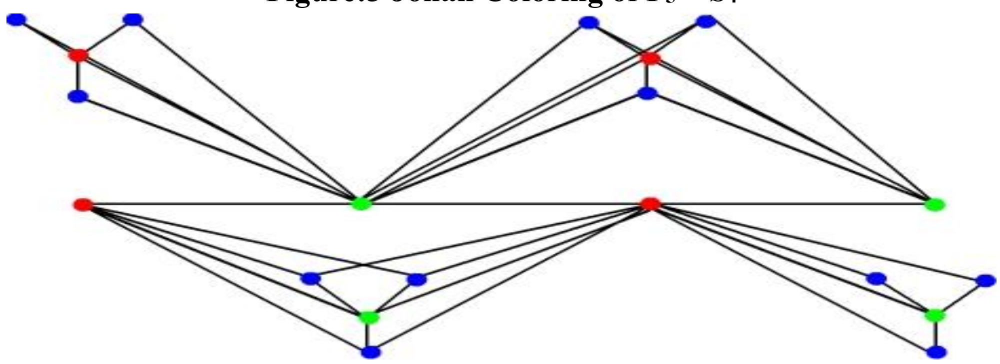
The minimum degree of $P_a \star S_b$ is 2 and maximum degree of $P_a \star S$ is 2b+2.

By applying Johan Coloring for Neighbourhood Corona of P_a *and* S_b , every vertex in $P_a \star S_b$ must satisfy rainbow neighbourhood condition.Thus we need δ + 1 colors.

Thus $J(P_a \star S_b)$ admits Johan coloring.

The Johan Chromatic Number of $P_a \star S_b$ is δ $(P_a \star S_b) + 1$.

Figure:3 Johan Coloring of $P_5 \star S_4$



3.4. Theorem

For a,b≥3, Johan Coloring of Neighbourhood Corona of Path and Cycle Graph is

$$J(P_a \star C_b) = \begin{cases} 4, & \text{When } b = 3 \\ \delta, & \text{When both } a \text{ and } b \text{ are even} \end{cases}$$

Proof:

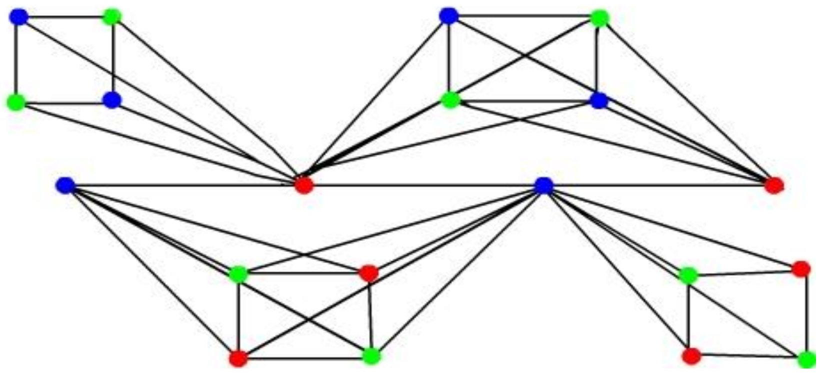
Let P_a be the path graph on a vertices and C_b be the Cycle graph on b vertices.

The vertex set of P_a *and* C_b *is*
$V(P_a) = \{v_x: 1 \leq x \leq a\}$ and $V(C_b) = \{u_y : 1 \leq y \leq b\}$
The vertex set of Neighbourhood Corona of $P_a \star C_b$ is

$$V(P_a \star C_b) = \{v_x : 1 \leq x \leq a\} \cup \{u_{xy} : 1 \leq x \leq a \text{ and } 1 \leq y \leq b\}$$

The minimum degree of $P_a \star C_b$ is 3 and maximum degree of $P_a \star C_b$ is 2b+2.
By applying Johan Coloring for Neighbourhood Corona of P_a *and* C_b , every vertex in $P_a \star C_b$ must satisfy rainbow neighbourhood condition.Thus we need δ colors.
Thus $J(P_a \star C_b)$ admits Johan coloring.
The Johan Chromatic Number of $P_a \star C_b$ is δ $(P_a \star C_b)$.

Figure:4 Johan Coloring of $P_4 \star C_4$



4. Conclusion

In this Paper, we have determined the Johan Coloring of Neighbourhood Corona of Path graph with Path, Path graph with Cycle, Path graph with Complete graph and Path graph with Star graph. We can also determine Johan Chromatic Number for other Graph Operations.

References

1. P. FORMANOWICZ , K.TANAS, "A Survey of graph coloring – It's types, methods and applications", Foundations of Computing and Decision Sciences, 2012, vol.3, No.3.
2. J. Kok, S. Naduvath, "Johan Coloring of Graph Operations", Acta Univ. Sapientiae Informatica 11,1(2019),95-108
3. J. Kok, S. Naduvath, M.K. Jamil, "Rainbow Neighbourhood Number of Graphs", Proyecciones Journal of Mathematics, 2019, vol.38, No.3.
4. D. B. West, "Introduction to graph theory", Pearson Education Inc., Delhi, 2001.
5. Gopalapillai, "The spectrum of neighborhood corona of graphs", Kragujevac Journal of Mathematics 35 (2011) 493–500