

New Approach to Fuzzy Logic Techniques in Two Space Game Theory

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Abstract

In this paper, a new approach is proposed to solve the games with imprecise entries in its payoff matrix. All these imprecise entries are assumed to be trapezoidal fuzzy numbers. Also the proposed approach provides fuzzy optimal solution of the fuzzy valued game without converting to classical version. A numerical example is provided.

Keywords: Fuzzy Logic, Two-Space Game Theory, Fuzzy Set Theory, Fuzzy Nash Equilibria Uncertainty, Strategic Interactions Player Preferences

1. Introduction

Game theory is a method for the study of decision-making in situations of conflicts and sometimes cooperation. Game theory provides a mathematical process for selecting an optimal strategy. It was developed to quantify, model and explain human behavior under conflicts between individuals and public interests. A player in a game is an autonomous decision-making unit. A strategy is a decision rule that specifies how the player will act in every possible circumstance. The mathematical treatment of the Game Theory was made available in 1944 by John Von Neumann et.al[10]through their book "Theory of Games and Economic Behavior". The Von Neumann's approach to solve the Game Theory problems was based on the principle of best out of the worst i.e., he utilized the idea of minimization of the maximum losses. Most of the competitive game theory problems can be handled by this principle. However, in real life situations, the information available is of imprecise nature and there is an inherent degree of vagueness or uncertainty present in the system under consideration. Hence the classical mathematical techniques may not be useful to formulate and solve there all world problems. In such situations ,the fuzzy sets introduced by Zadeh[11]in1965 provide effective and efficient tools and techniques to handle these problems. Many authors such as Campos[2], Sakawa et.al[6], Selvakumari et.al[8], Thirucheran et.al[9] etc., have studied fuzzygames. Sakawa et.al [6] introduced max-min solution procedure for multi-objective fuzzy games.Charilas et.al [3]collected applications of game theory in wireless networking and presents them in a layered perspective, emphasizing on which fields game theory could be effectivelyapplied. Bompard et.al [1] presented a medium run electricity market simulator based on game theory. Fiestras-Janeiro et.al [4]provides a review of the applications of cooperative game theory in the management of centralized inventory systems. Madani [5]reviewed applicability of game theory to water resources management and conflict resolution through a series of non-cooperative water resource games. His paper illustrates the dynamic structure of water resource problems and the importance of considering the game's evolution path while studying such problems.

In this paper, we have proposed a new approach based on the principle of dominance for the fuzzy optimal solution of the fuzzy valued game without converting to its equivalent crisp form.

The rest of this paper is organized as follows. In section 2, we recall the basic concepts and the results of trapezoidal fuzzy numbers and their arithmetic operations. In section 3, we have proposed a matrix method for the solution of fuzzy games. In section 4, Numerical example is provided to illustrate the efficiency of the proposed method. Section 5 gives the conclusion of this Paper.

2. PRILIMINARIES

Zadeh in 1965 first introduced fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

Definition 2.1

The characteristic function μ_A of a crisp set $A \subset X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range.

i.e) $\mu_{\tilde{A}} : X \rightarrow [0, 1]$.

The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2.2

A fuzzy set \tilde{A} , defined on the universal set of real numbers \mathbb{R} , is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0, 1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b, c]$, where $a < b < c < d$

Definition 2.3

A fuzzy number $\tilde{A} = (a, b, c, d)$ is a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \left(\frac{x-a}{b-a}\right), & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \left(\frac{x-d}{c-d}\right), & c \leq x \leq d \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.4

A fuzzy set \tilde{A} , defined on the universal set of real numbers \mathbb{R} , is said to be a generalized fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0, \omega]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
4. $\mu_{\tilde{A}}(x) = \omega$ for all $x \in [b, c]$, where $0 < \omega \leq 1$.

Ranking Function

An efficient approach for comparing the fuzzy numbers is by the use of an efficient ranking function $\mathfrak{R} : F(\mathbb{R}) \rightarrow \mathbb{R}$, by its graded man as

$$\mathfrak{R}(\tilde{A}) = \left[\left(\frac{b+c}{2} \right) + \left(\frac{\beta-\alpha}{4} \right) \right]$$

where $F(\mathbb{R})$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists i.e.,

- i. $\tilde{A} \succ \tilde{B}$ if and only if $\mathfrak{R}(\tilde{A}) > \mathfrak{R}(\tilde{B})$
- ii. $\tilde{A} \prec \tilde{B}$ if and only if $\mathfrak{R}(\tilde{A}) < \mathfrak{R}(\tilde{B})$
- iii. $\tilde{A} \approx \tilde{B}$ if and only if $\mathfrak{R}(\tilde{A}) = \mathfrak{R}(\tilde{B})$

Arithmetic Operation on Trapezoidal Fuzzy Number

The following are the four operations that can be performed on triangular fuzzy number:

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ then

Addition

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

Subtraction

$$\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$$

Multiplication

$$\tilde{A} \times \tilde{B} = (\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3))$$

Division

$$\tilde{A} / \tilde{B} = (\min(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3), a_2/b_2, \max(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3))$$

3. Matrix games

A finite two-person zero-sum game which is represented in matrix form is called a matrix game. This is a direct consequence of the fact that two opponents with exactly opposite interests play a game under a finite number of strategies, independently of his or her opponent’s action. Once both players each make

an action, their decisions are disclosed. A payment is made from one player to the other based on the outcome, such that the gain of one player equals the loss of the other, resulting in a net payoff summing to zero.

The Fuzzy Payoff Matrix

The problem that we are aiming to solve is a two player zero sum fuzzy game in which the entries in the payoff matrix A are trapezoidal fuzzy number. The fuzzy payoff matrix is

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Player A has m strategies and Player B has n strategies. If Player A chooses the ith strategies and Player B chooses jth strategies, the Player A win.

Definition 3.1. Saddle point

If the max-min value equals the mini-max value then the game is said to a saddle (equilibrium) point and the corresponding strategies are called optimum strategies. The amount of pay off at equilibrium point is known as the value of the game.

The Principle of Dominance (Dominant Strategy or Dominance Method)

Step 1. Represent the trapezoidal fuzzy numbers in the fuzzy game problem in its parametric form.

Step 2. Identify any two rows. If the elements in one row is found to be less than or equal to the corresponding elements in the other row that row dominates the other.

Step 3. Consider the elements in any column which are greater than or equal to the corresponding elements in any other column then that column is dominated.

Step 4. The dominated columns and rows are identified and deleted to form the reduced matrix.

Theorem 3.1.

For any 2x2 two person zero sum fuzzy game without any saddle point having the payoff matrix for player A as given below:

$$\begin{matrix} & B_1 & B_2 \\ A_1 & \begin{bmatrix} a_{11} & a_{12} \end{bmatrix} \\ A_2 & \begin{bmatrix} a_{21} & a_{22} \end{bmatrix} \end{matrix}$$

The optimum mixed fuzzy strategies FS_A and FS_B are determined by

$$p_1 = \frac{a_{22} - a_{21}}{a_{11} + a_{22} - (a_{12} - a_{21})}$$

$$p_2 = \frac{a_{11} - a_{12}}{a_{11} + a_{22} - (a_{12} - a_{21})}$$

$$q_1 = \frac{a_{22} - a_{12}}{a_{11} + a_{22} - (a_{12} - a_{21})}$$

$$q_2 = \frac{a_{11} - a_{21}}{a_{11} + a_{22} - (a_{12} - a_{21})}$$

and

$$\tilde{v} = \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} + a_{22} - (a_{12} - a_{21})}$$

4. Numerical Example

Consider the fuzzy game problem with payoff matrix as trapezoidal fuzzy numbers.

$$\begin{matrix} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} (1,4,5,6) & (1,2,4,5) & (3,4,7,8) & (4,5,7,8) \\ (5,10,12,17) & (8,10,11,19) & (5,7,10,14) & (7,10,11,12) \\ (-1,0,2,3) & (-1,2,3,4) & (12,14,18,20) & (8,17,21,30) \end{bmatrix} \end{matrix}$$

Representing all trapezoidal fuzzy numbers in parametric form, we have

$$\begin{matrix} & \text{Player B} & \text{Row min} \\ \text{Player A} & \begin{bmatrix} (4.5,0.5,3,1) & (3,1,1,1) & (4.5,0.5,1,3) & (6,1,1,1) \\ (11,1,5,5) & (10.5,0.5,2,8) & (8.5,1.5,2,4) & (10.5,0.5,3,1) \\ (1,1,1,1) & (2.5,0.5,3,1) & (16,2,2,2) & (19,2,9,9) \end{bmatrix} & \begin{matrix} (3,1,1,1) \\ (8.5,1.5,2,4) \\ (1,1,1,1) \end{matrix} \\ \text{Col.Max} & (11,1,5,5) & (10.5,0.5,2,8) & (16,2,2,2) & (19,2,9,9) \end{matrix}$$

Max.min= (8.5,1.5,2,4) = maximum of row minimum

Min.Max = (11,1,5,5) = minimum of column maximum and Max.Min ≠ Min.Max

It has no saddle point, so we can apply dominance method to solve the problem.

Table-1 Ranking of the trapezoidal fuzzy numbers in the fuzzy game problem

Trapezoidal Fuzzy Number	Rank of Trapezoidal Fuzzy Number
$a_{11} = (4.5,0.5,3,1)$	$R(a_{11}) = 4$
$a_{12} = (3,1,1,1)$	$R(a_{12}) = 3$
$a_{13} = (4.5,0.5,1,3)$	$R(a_{13}) = 5$
$a_{14} = (6,1,1,1)$	$R(a_{14}) = 6$
$a_{21} = (11,1,5,5)$	$R(a_{21}) = 11$
$a_{22} = (10.5,0.5,2,8)$	$R(a_{22}) = 12$
$a_{23} = (8.5,1.5,2,4)$	$R(a_{23}) = 9$
$a_{24} = (10.5,0.5,3,1)$	$R(a_{24}) = 10$
$a_{31} = (1,1,1,1)$	$R(a_{31}) = 1$
$a_{32} = (2.5,0.5,3,1)$	$R(a_{32}) = 2$
$a_{33} = (16,2,2,2)$	$R(a_{33}) = 16$

$a_{34} = (19, 2, 9, 9)$	$R(a_{34}) = 19$
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By dominance principle applied between Row 1 and Row 2, Row 1 can be omitted

$$\begin{matrix} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} (11,1,5,5) & (10.5,0.5,2,8) & (8.5,1.5,2,4) & (10.5,0.5,3,1) \\ (1,1,1,1) & (2.5,0.5,3,1) & (16,2,2,2) & (19,2,9,9) \end{bmatrix} \end{matrix}$$

By dominance principle applied between Column 1 and Column 2, Column 2 can be omitted

$$\begin{matrix} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} (11,1,5,5) & (8.5,1.5,2,4) & (10.5,0.5,3,1) \\ (1,1,1,1) & (16,2,2,2) & (19,2,9,9) \end{bmatrix} \end{matrix}$$

By dominance principle applied between Column 3 and Column 4, Column 4 can be omitted

$$\begin{matrix} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} (11,1,5,5) & (8.5,1.5,2,4) \\ (1,1,1,1) & (16,2,2,2) \end{bmatrix} \end{matrix}$$

Max.min= (8.5,1.5,2,4); Min.Max = (11,1,5,5) and Max.Min ≠ Min.Max

It has no saddle point,

The reduced fuzzy matrix represents a 2X2 two person zero sum fuzzy game without any saddle point. then theorem 3.1 we have optimum mixed fuzzy strategies as

$$p_1 = \left(\frac{15}{17.5}, 2, 5, 5\right) \quad p_2 = \left(\frac{2.5}{17.5}, 2, 5, 5\right)$$

$$q_1 = \left(\frac{7.5}{17.5}, 2, 5, 5\right) \quad q_2 = \left(\frac{10}{17.5}, 2, 5, 5\right)$$

$$\text{Strategy of Player A} = (p_1, p_2) = \left[(0,0,0,0), \left(\frac{15}{17.5}, 2, 5, 5\right), \left(\frac{2.5}{17.5}, 2, 5, 5\right) \right]$$

$$\text{Strategy of Player B} = (q_1, q_2) = \left[\left(\frac{7.5}{17.5}, 2, 5, 5\right), (0,0,0,0), \left(\frac{10}{17.5}, 2, 5, 5\right), (0,0,0,0) \right]$$

$$\begin{aligned} \text{Value of the game } \tilde{v} &= \frac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} + a_{22} - (a_{12} - a_{21})} \\ &= \left(\frac{167.5}{17.5}, 2, 5, 5\right) \end{aligned}$$

Hence the value of the game in terms $\check{\alpha} = (a_1, a_2, a_3, a_4)$ is (2.5714, 7.5714, 11.5714, 16.5714)

5. Conclusion

In this paper we have obtained the optimum solution of fuzzy game without converting to classical form by applying a new ranking method and a new fuzzy arithmetic on the parametric form of trapezoidal fuzzy numbers. It is evident from the above example that the proposed method is capable of giving fuzzy solution for the fuzzy matrix game.

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