Solving Fully Fuzzy Linear Programming Problem Using Triangular Fuzzy Number By Gauss Jordan Elimination Method

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ABSTRACT

In this paper presents a method for solving Fully Fuzzy Linear Programming Problems (FFLPP) using triangular fuzzy numbers (TFNs) and the Gauss-Jordan Elimination Method. The fuzzy coefficients in the constraints and objective function are represented as TFNs, capturing the uncertainty inherent in real-world data. The method involves defuzzification of the fuzzy numbers to convert them into crisp values, followed by the application of Gauss-Jordan elimination to solve the resulting system of equations. A numerical example demonstrates the effectiveness of this approach in providing optimal solutions for decision-making problems under uncertainty.

Keywords: Fully fuzzy linear systems, triangular fuzzy numbers, elementary row operations

1. Introduction

Linear system of equations are vital for contemplating and comprehending an expansive extent of the issues in connected science, ordinarily is numerous applications a portion of the parameters in our issues are spoken to fuzzy numbers Zadeh [17] presented the ideas of fuzzy numbers and fuzzy math, fuzzy numbers number-crunching is connected and valuable in calculation of direct frameworks.

Duba's and Prade [8] proposed two meaning of fuzzy straight arrangement of conditions. Buckley and Qu [4] broadened a few techniques for tackling completely fuzzy direct arrangement of condition. Friedman et al. [10] proposed a general model for comprehending a fuzzy straight frame work whose coefficient lattice is fresh and the correct hand side section is a discretionary fuzzy vector. Numerous specialists [2,3,5] presented a few calculations for unraveling completely fuzzy straight frameworks [2,3] proposed arrangement of a fuzzy direct framework by utilizing alterative Gauss Jacobin and Gauss Seidel techniques. Dehghan et al. [5] proposed some new strategies for illuminating completely fuzzy straight frameworks based a the cramer 's rule Gaussian disposal and LU decay strategy from direct polynomial math and direct programming Dehghan et al. [6] broadened the A domain Decomposition technique[3] to locate the positive fuzzy vector arrangement of completely fluffy direct frame work. Abbas Banday etal.[1] utilized LU disintegration strategy for fathoming fuzzy straight arrangement of condition when the coefficient network is certain positive. Muzzioli et al. [13] grew completely fuzzy straight arrangement of the frame $A_1x + b_1 = A_2x + b_2$ where A_1, $A_2$are square grids of fluffy coefficients, b_1, b_2 are fuzzy numbers.

Mosleh et al. [12] acquainted a technique with discover the arrangement of completely fuzzy direct arrangement of conditions of the frame Ax +b =Cx +d where A, Care square frame works of fuzzy coefficients and b, d are fuzzy number vectors Nasseri et al. [15] utilized a specific disintegration strategies for the coefficient framework for illuminating completely fuzzy straight arrangement of conditions. Nasseri et al. [16] proposed Huang technique for registering a non negative arrangement of the completely fuzzy straight arrangement of conditions Nasseri et al. [14] presented a direct arrangement of conditions with trapezoidal fuzzy numbers as an expansion of the fuzzy straight framework.

In this paper, our point is to solve $A \otimes x = \tilde{b}$ where A is a fuzzy lattice and x and $\tilde{b}$ are fuzzy vectors (of triangular fuzzy numbers) with proper sizes. We sorted out this paper as pursues; In segment 2 we first give some fundamental ideas of fuzzy set hypothesis and some essential definitions on fuzzy numbers are checked on. In section 3, we characterize completely fuzzy direct frameworks of conditions and Gauss Jordan Elimination strategy comprehending completely fuzzy straight frameworks (FFLS). In section 4, numerical precedent is given to analyze a proposed technique. We close in area 5.

2. PRELIMINARIES

In these sections, some basic definitions are reviewed [7,11].

Definition2.1

A fuzzy subset $\tilde{A}$ of R is characterized by its participation work: $\mu_{\tilde{A}}: R \rightarrow [0,1]$ which assigns out a real number $\mu_{\tilde{A}}$in the interim [0,1], to every component $x \in R$ where the estimation of $\mu_{\tilde{A}}$at x demonstrates the review enrollment of x in A .

Definition2.2

A fuzzy set $\tilde{A}$ characterized on the wide spread arrangement of real number R, is said to be a fuzzy number if its enrollment work has the accompanying qualities.

(i) $\tilde{A}$ is convex (ie) □$_A$ (□x_1□(1□□)x_2) □□□minimum (□$_A$(x_1), □$_A$(x_2)) □x_1,x_2□R,□□□[0,1]

(ii) $\tilde{A}$ is normal (ie) there exist $x_0 \in R$ such that $\mu_{\tilde{A}}(x_0)=1$

(iii)$\mu_{\tilde{A}}$ is piecewise continuous.

Definition2.3

A fuzzy number $\tilde{A}$ is said to be non-negative fuzzy number if and only if $\mu_{\tilde{A}}(x) = 0$ for all x < 0.

Definition2.4

A fuzzy number $\tilde{A} = (p,q,r)$ is said to be Triangular fuzzy number, if its membership function has the following forms:

$$\mu_A(x) = \begin{cases} 1 - \dfrac{p-x}{q}, & p-q \le x \le p, q > 0 \\ 1 - \dfrac{x-p}{r}, & p \le x \le p+r, r > 0 \\ 0 & , \textit{otherwise} \end{cases}$$

Definition2.5

A fuzzy number = (p, q, r) is said to be Triangular fuzzy number, if its enrollment work has the accompanying structures: (i.e.) $\tilde{A} \geq 0$if and only if p -q □ 0.

Definition2.6

A triangular fuzzy number $\tilde{A}$= $(p,q,r)$is said to be zero triangular fuzzy number if and only if p = 0, q = 0, r = 0.

Definition2.7

Two fuzzy numbers $\tilde{A}$= $(p,q,r)$and $\tilde{B}$= $(s,m,n)$are said to be equal if and only if p = s, q= m and r =n.

Definition 2.8

A matrix $\tilde{A} = \left(\tilde{a}_{ij}\right)$ is known as a fuzzy if every component of $\tilde{A}$ is a fuzzy number. $\tilde{A}$ fuzzy network $\tilde{A}$ will be certain and signified by $\tilde{A}$ >0, if every component of $\tilde{A}$ is sure. We speak to nxn fuzzy grid $\tilde{A} = \left(\tilde{a}_{ij}\right)_{nxn}$ such an extent that $\tilde{a} = \left(a_{ij}, q_{ij}, r_{ij}\right)$ with new documentation $\tilde{A} = \left(A, M, N\right)$ where $A = \left(a_{ij}\right)$ $M = \left(q_{ij}\right)$and $N = \left(r_{ij}\right)$are three nxn fresh network.

Arithmetic operations an triangular fuzzy numbers

Numerous creators have surveyed the essential tasks a triangular fuzzy numbers. In this subsection, we explored the tasks a triangular fuzzy numbers [7].

Let $\tilde{A}$ = $(p,q,r) and \tilde{B}$=(s,m,n)be two triangular fuzzy numbers.

1. Addition: $\tilde{A}\oplus\tilde{B} =(p,q,r)\oplus(s,m,n)=(p+s,q+m,r+n)$
2. Subtraction: $\tilde{A}\ominus\tilde{B}=(p,q,r)\ominus(s,m,n) =(p-s,q-m,r-n)$
3. Multiplication : If $\tilde{A}\geq 0$ *and* $\tilde{B} \geq 0$ then

$$\tilde{A}\otimes\tilde{B}=(p,q,r)\otimes(s,m,n)=(ps,pm+sq,pn+sr)$$

3. Direct method

Assuming that A is a nonsingular crisp matrix, we can write
(Ax, Ay+Mx, Az+Nx) = (b,h, g).
Thus we have

$$\left.\begin{aligned} Ax &= b \\ Ay + Mx &= h \\ Az + Nx &= g \end{aligned}\right\} \qquad (3.1)$$

In other words we have

$$\left.\begin{aligned} Ax &= b \\ Ay &= h - Mx \\ Az &= g - Nx \end{aligned}\right\} \qquad (3.2)$$

So we easily get

$$Ax = b \Rightarrow x = A^{-1}b \qquad (3.3)$$

and then by this representation in the second and third equations, we have

$$y = A^{-1}h - A^{-1}Mx, \qquad (3.4)$$

and

$$y = A^{-1}h - A^{-1}Mx, \qquad (3.5)$$

3.1. Numerical Example

Consider the following FFLS solve it Direct method.

(1,2,3)□(x_1,y_1,z_1)□(2,3,4)□(x_2,y_2,z_2)□(1,2,3)□(x_3,y_3,z_3)=(4,5,6)

(3,3,4)□(x_1,y_1,z_1)□(3,4,5)□(x_2,y_2,z_2)□(2,3,4)□(x_3,y_3,z_3)=(6,7,8)

(3,4,5)□(x_1,y_1,z_1)□(1,2,3)□(x_2,y_2,z_2)□(4,5,6)□(x_3,y_3,z_3)=(8,9,10)

Solution

We mean that the given FFLS may be written as

$$\begin{bmatrix} (1,2,3) & (2,3,4) & (1,2,3) \\ (2,3,4) & (3,4,5) & (2,3,4) \\ (3,4,5) & (1,2,3) & (4,5,6) \end{bmatrix} \begin{bmatrix} (x_1,y_1,z_1) \\ (x_2,y_2,z_2) \\ (x_3,y_3,z_3) \end{bmatrix} = \begin{bmatrix} (4,5,6) \\ (6,7,8) \\ (8,9,10) \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & 1 & 4 \end{bmatrix}, M = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}, N = \begin{bmatrix} 3 & 4 & 3 \\ 4 & 5 & 4 \\ 5 & 3 & 6 \end{bmatrix}$$

$$b = \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}, \; h = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}, g = \begin{bmatrix} 6 \\ 8 \\ 10 \end{bmatrix}$$

So with equation 3.3 we get

$$A^{-1} = \frac{1}{|A|} \; adjA$$

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & 1 & 4 \end{vmatrix}$$

$$|A| = -1$$

$$adj\, A = \begin{bmatrix} 10 & -7 & 1 \\ -2 & 1 & 0 \\ -7 & 5 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -10 & 7 & -1 \\ 2 & -1 & 0 \\ 7 & -5 & 1 \end{bmatrix}$$

$$x = A^{-1}b = \begin{bmatrix} -10 & 7 & -1 \\ 2 & -1 & 0 \\ 7 & -5 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}$$

$$x = \begin{bmatrix} -6 \\ 2 \\ 6 \end{bmatrix}$$

we have $X_1 = -6$, $X_2 = 2$, $X_3 = 6$,
Similarly by equations 3.4 and 3.5 we have

$$y = A^{-1}h - A^{-1}Mx = \begin{bmatrix} -10 & 7 & -1 \\ 2 & -1 & 0 \\ 7 & -5 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix} - \begin{bmatrix} -10 & 7 & -1 \\ 2 & -1 & 0 \\ 7 & -5 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 8 \\ 10 \end{bmatrix}$$

$$y = A^{-1}h - A^{-1}Mx = \begin{bmatrix} -10 \\ 3 \\ 9 \end{bmatrix} - \begin{bmatrix} -14 \\ 4 \\ 12 \end{bmatrix}, \quad y = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$$

we have $y_1 = 4$, $y_2 = -1$, $y_3 = -3$
and

$$z = A^{-1}g - A^{-1}Nx = \begin{bmatrix} -10 & 7 & -1 \\ 2 & -1 & 0 \\ 7 & -5 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 8 \\ 10 \end{bmatrix} - \begin{bmatrix} -10 & 7 & -1 \\ 2 & -1 & 0 \\ 7 & -5 & 1 \end{bmatrix}\begin{bmatrix} 8 \\ 10 \\ 12 \end{bmatrix}$$

$$z = A^{-1}g - A^{-1}Nx = \begin{bmatrix} -14 \\ 4 \\ 12 \end{bmatrix} - \begin{bmatrix} -22 \\ 6 \\ 8 \end{bmatrix}, \quad z = \begin{bmatrix} 8 \\ -2 \\ -6 \end{bmatrix}$$

we have

$z_1 = 8$, $z_2 = -2$, $z_3 = -6$

Thus the solution is

$x_1 = (x_1, y_1, z_1) = (-6, 2, 6)$ $x_2 = (x_2, y_2, z_2) = (4, -1, -3)$ $x_3 = (x_3, y_3, z_3) = (8, -2, -6)$

4. Proposed Method

Consider nxn fuzzy linear system of equations

$$\left(a_{11} \otimes x_1\right) \oplus \left(a_{12} \otimes x_2\right) \oplus \ldots\ldots.. \oplus \left(a_{1n} \otimes x_n\right) = \tilde{b}_1$$

$$\left(a_{21} \otimes x_1\right) \oplus \left(a_{22} \otimes x_2\right) \oplus \ldots\ldots.. \oplus \left(a_{2n} \otimes x_n\right) = \tilde{b}_2$$

$$\vdots \qquad \vdots\ldots\ldots\ldots \qquad \vdots \quad \vdots$$

$$\left(a_{n1} \otimes x_1\right) \oplus \left(a_{n2} \otimes x_2\right) \oplus \ldots\ldots.. \oplus \left(a_{nn} \otimes x_n\right) = \tilde{b}_n$$

The matrix of the above equation is

$$A \otimes x = \tilde{b}$$

Where the co-efficient of matrix $A = \left(a_{ij}\right), 1 \le i, j \le n$ is a n x n fuzzy matrix and $x, \tilde{b} \in F\left(R\right)$ This system is called a fully fuzzy linear system (FFLS).

In this paper, we obtain a positive solution of fully fuzzy linear systems $A \otimes x = \tilde{b}$

Consider $A = \left(A, M, N\right) \ge 0$, $x = \left(x, y, z\right) \ge 0$

and

$$\tilde{b} = (b, h, g) \geq 0$$

The FFLS $A \otimes x = \tilde{b}$ can be written as

$$A = (A, M, N) \otimes (x, y, z) = (b, h, g)$$

using arithmetic operations

$$(Ax, Ay + Mx, Az + Nx) = (b, h, g)$$

Using definition 2.7,

$$\left.\begin{aligned} Ax &= b \\ Ay + Mx &= h \\ Az + Nx &= g \end{aligned}\right\} \qquad (4.1)$$

our proposal method is a modification of Gauss Elimination method called Gauss Jordan method,

In this strategy, the coefficient grid A of the fresh direct arrangement of condition Ax = b, Ay + Mx = h and Az + Nx = g is conveyed to a unit lattice by making the enlarged network upper triangular as well as lower triangular (i.e.) by making all components above and beneath the main corner to corner of expanded framework as zeros.

Step1: Construct the increased lattices (A, b),(A, h–Mx)and(A, g–Nx) from condition 2.1.

Step 2: Compute the estimations of x_i, y_i and z_i by utilizing the basic row tasks.

Step3: The expanded networks (A, b), (A, h – Mx) and (A, g – Nx) diminished to push decreased frame.

Step4: The arrangement of fully fuzzy straight frame work (FFLS) will be spoken to by x_i□(x_i,y_i,z_i) for all i=1,2,...,n.

4.1 Numerical Example

Consider the following FFLS solve it by proposed method.

(1,2,3)□(x_1,y_1,z_1)□(2,3,4)□(x_2,y_2,z_2)□(1,2,3)□(x_3,y_3,z_3)=(4,5,6)

(3,3,4)□(x_1,y_1,z_1)□(3,4,5)□(x_2,y_2,z_2)□(2,3,4)□(x_3,y_3,z_3)=(6,7,8)

(3,4,5)□(x_1,y_1,z_1)□(1,2,3)□(x_2,y_2,z_2)□(4,5,6)□(x_3,y_3,z_3)=(8,9,10)

Solution

The given FFLS may be written as

$$\begin{bmatrix} (1,2,3) & (2,3,4) & (1,2,3) \\ (2,3,4) & (3,4,5) & (2,3,4) \\ (3,4,5) & (1,2,3) & (4,5,6) \end{bmatrix} \begin{bmatrix} (x_1, y_1, z_1) \\ (x_2, y_2, z_2) \\ (x_3, y_3, z_3) \end{bmatrix} = \begin{bmatrix} (4,5,6) \\ (6,7,8) \\ (8,9,10) \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & 1 & 4 \end{bmatrix}, M = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 4 & 3 \\ 4 & 2 & 5 \end{bmatrix}, N = \begin{bmatrix} 3 & 4 & 3 \\ 4 & 5 & 4 \\ 5 & 3 & 6 \end{bmatrix}$$

$$b = \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}, \; h = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}, g = \begin{bmatrix} 6 \\ 8 \\ 10 \end{bmatrix}$$

The augmented matrix

$$(A,b)=\begin{bmatrix}1&2&1&4\\2&3&2&6\\3&1&4&8\end{bmatrix}$$

Applying elementary row operation on metric (A, b)

First $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$ we get $\begin{bmatrix}1&2&1&4\\0&-1&0&-2\\0&-5&1&-4\end{bmatrix}$

Again we apply elementary operations in sequence

$R_2 \to R_2(-1), R_3 \to R_3 + 5R_2, R_1 \to R_1 - 2R_2,\ R_1 \to R_1 - R_3$

Finally we get

$$\begin{bmatrix}1&0&0&-6\\0&1&0&2\\0&0&1&6\end{bmatrix}$$

From this row reduced from of augmented matrix (A, b)

we have $X_1 = -6$, $X_2 = 2$, $X_3 = 6$,

The augmented matrix

$$(A,h-Mx)=\begin{bmatrix}1&2&1&-1\\2&3&2&-1\\3&1&4&-1\end{bmatrix}$$

$R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1, R_2 \to R_2(-1), R_3 \to R_3 + 5R_2$

$R_1 \to R_1 - 2R_2, R_1 \to R_1 - R_3$

Finally we get

$$\begin{bmatrix}1&0&0&4\\0&1&0&-1\\0&0&1&-3\end{bmatrix}$$

From this row reduced from of augmented matrix $(A,h-Mx)$

we have

$y_1 = 4$, $y_2 = -1$, $y_3 = -3$

Similarly applying elementary row operations on augmented matrix

$$(A,g-Nx)=\begin{bmatrix}1&2&1&-2\\2&3&2&-2\\3&1&4&-2\end{bmatrix}$$

$R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1,\ R_2 \to R_2(-1), R_3 \to R_3 + 5R_2$

$R_1 \to R_1 - 2R_2, R_1 \to R_1 - R_3$

Finally we get

$$\begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -6 \end{bmatrix}$$

From this row reduced from of augmented matrix $(A, g - Nx)$

we have

$z_1 = 8$, $z_2 = -2$, $z_3 = -6$

Substituting the values of x_i□(x_i, y_i, z_i) for all i=1,2,...,n. x_i, y_i, z_i where i=1,2,...,n in the FFLS solution $x_i = (x_i, y_i, z_i)$, for all i=1,2,...,n we get

$$\tilde{x}_1 = (x1, y1, z1) = (-6, 2, 6)$$

$$\tilde{x}_2 = (x2, y2, z2) = (4, -1, -3)$$

$$\tilde{x}_3 = (x3, y3, z3) = (8, -2, -6)$$

we have the same solutions with this method as the Direct method solution.

5. Conclusion

In this conclusion, the method presented for solving Fully Fuzzy Linear Programming Problems (FFLPP) using triangular fuzzy numbers (TFNs) and the Gauss-Jordan Elimination Method effectively handles uncertainty in decision-making. By transforming fuzzy coefficients into crisp values through defuzzification, the approach allows the application of standard linear programming techniques to obtain optimal solutions. The numerical example demonstrated the method's efficiency and applicability, making it a valuable tool for solving real-world problems involving imprecise data. Our proposed method is easy to determine the solution of FFLS.

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