Uncovering the Power of Inquiry-Based Teaching; Enhancing Geometric Thinking Skills in Students

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Abstract

Geometry is vital for developing critical thinking and problem-solving skills, yet students often struggle with geometric reasoning. This study explored the impact of an inquiry-based teaching approach on enhancing students' geometric thinking levels, based on the van Hiele model. Conducted in two senior high schools in Ghana, the research employed a descriptive case study design and van Hiele geometry tests to assess students' reasoning before, during, and after the intervention. Before the intervention, most students operated at lower van Hiele levels, with many at level 0 (pre-visualisation) and level 1 (visualisation), reflecting limited reasoning skills. The inquiry-based approach, rooted in constructivist theories, emphasised essential questioning, student engagement, cooperative interaction, and diverse resources. Results showed significant progress, with many students advancing to levels 3 (abstraction) and 4 (deduction). Retention tests confirmed sustained improvements. The findings highlight the effectiveness of inquiry-based teaching in fostering higher-order reasoning and long-term retention of geometric concepts. This study underscores the importance of active, student-centred pedagogies in mathematics education to enhance geometric thinking and improve academic outcomes.

Keywords: Geometry, Inquiry-Based Learning, Van Hiele Model, Geometric Thinking Levels, Design-Based Approach, Retentions

INTRODUCTION

Any country's sustainable development depends heavily on its citizens' mathematical, scientific, and technological knowledge or skills, and geometry has been the foundation. Learning geometry contributes to the student's development of numeracy skills, communicational skills, problem-solving skills, critical thinking, and collaboration [1]. The concepts and skills acquired through solving geometric problems are unconsciously applied in everyday life, as geometric principles underlie many aspects of the world [2]. Among the various branches of mathematics, geometry holds a significant place within the curriculum [3]. Its applications are pervasive in daily life, aligning with Galileo's assertion that geometry is fundamental to understanding the natural world [4]. Numerous objects in our environment are characterised by geometric shapes, whether as planes or solids, underscoring their relevance.

Studies have revealed that students struggle to understand geometry concepts [5, 6] as they cannot learn and comprehend geometry since the problems and difficulties of understanding and applying the theories and concepts of geometry persist. Studies conducted in some countries such as Turkey [7], the Czech

Republic [8], Indonesia [1], South Africa [9] and Malaysia [10] resulted in the conclusion that students perform abysmally in geometry. Ghanaian students like other students struggle with the same poor performance in geometry learning that is faced by students around the world [11]. The problem of students' poor performance in geometry is linked to the level of geometric thinking ability. Studies showed that students with high geometric thinking levels performed well in mathematics and geometry-related examinations [12, 13].

This sparks the need and relevance of enhancing students' geometric thinking in the teaching and learning process. For example, studies conducted to measure students' geometric thinking levels after being taught geometry showed that students could not reach the required geometric thinking level (van Hiele level 4) to enable them to perform well in geometry [7, 10, 11, 12, 13]. To enhance the students' geometric thinking levels, there should be a paradigm shift from the teacher-centred approaches of teaching to the student-centred where students will be actively involved in the construction of their knowledge and understanding. Van Hiele provided five instructional phases (information, directed orientation, explicitation, free orientation and integration) to guide students to achieve higher geometric thinking levels. However, this study adopted inquiry-based teaching and learning as the student-centred approach to teaching and learning and an alternative to van Hiele's five instructional models.

The inquiry-based instructional approach is a student-centred teaching method designed to actively engage students in the learning process, fostering their full involvement and participation in mathematics lessons [14]. Rooted in the principles of discovery learning, this approach aligns with the constructivist philosophy of teaching, where learners actively construct knowledge rather than passively receive it [15]. Inquiry involves an active learning process driven by questioning and critical thinking, enabling students to develop deeper, more enduring understandings compared to traditional, teacher-delivered instruction [16]. The phases of inquiry-based learning include essential questions, student engagement, cooperative interaction, performance evaluation, and the use of a variety of resources. Furthermore, Marshall et al. [14] emphasise the positive impact of inquiry-based teaching on students' mathematical achievement and their geometric thinking levels, highlighting its role in fostering meaningful learning experiences.

The study seeks to address the persistent gap in students' geometric thinking levels and their resulting low performance in geometry. Research has shown that traditional teacher-centred approaches often fail to elevate students' geometric thinking to the van Hiele level 4 required for success in geometry-related examinations [7, 12]. While the van Hiele model provides a structured approach to enhance geometric thinking, there is a limited exploration of alternative methods such as inquiry-based instruction in this context. This study seeks to bridge this gap by adopting an inquiry-based instructional approach as a student-centred alternative to enhance students' geometric thinking levels, thereby addressing the deficiencies in traditional teaching methods and contributing to improved geometry performance.

LITERATURE REVIEW

Theoretical framework

The study was grounded in the constructivist theory, which offers insights into how individuals gain knowledge and learn. Constructivism posits that learners build their understanding of the world by engaging in experiences and reflecting on them, integrating new information with prior knowledge. This process may involve revising existing beliefs or disregarding new ideas that do not seem relevant [17]. Ranjan and Padmanabhan [18] highlighted that constructivism transforms students from passive recipients of information into active participants, fostering positive beliefs and attitudes toward learning. The theory emphasises the importance of the learning environment, suggesting that the context in which ideas are taught significantly affects how students create and internalise knowledge. In this study, encouraging students to actively participate in constructing their understanding of geometric concepts was expected to boost their motivation and confidence, leading to enhanced geometric thinking levels.

Furthermore, social constructivism was integrated into the study, focusing on the importance of social interaction in knowledge construction. According to Vygotsky (1978), social constructivism asserts that students develop knowledge through collaboration with peers, teachers, and the broader social environment [19]. A key tenet of social constructivism is the role of positive interactions, which create a supportive and collaborative learning environment, enhancing students' motivation and self-assurance. By adopting an inquiry-based approach aligned with this framework, the study aimed to foster dialogue, questioning, and group exploration, creating an engaging atmosphere for learning geometry [16].

The inquiry-based learning approach was also pivotal in designing and implementing the study's intervention. Grounded in constructivist principles, inquiry-based learning emphasizes discovery, where learners construct new knowledge by building on existing ideas [20]. This approach actively involves students in the learning process, fostering critical thinking and deeper conceptual understanding [21]. In-service mathematics teachers participating in the study were trained to develop and implement inquiry-based learning interventions within constructivist classrooms.

The integration of constructivism, social constructivism, and inquiry-based learning into the study's conceptual framework provided the foundation for developing and executing the intervention. (See Figure 1)

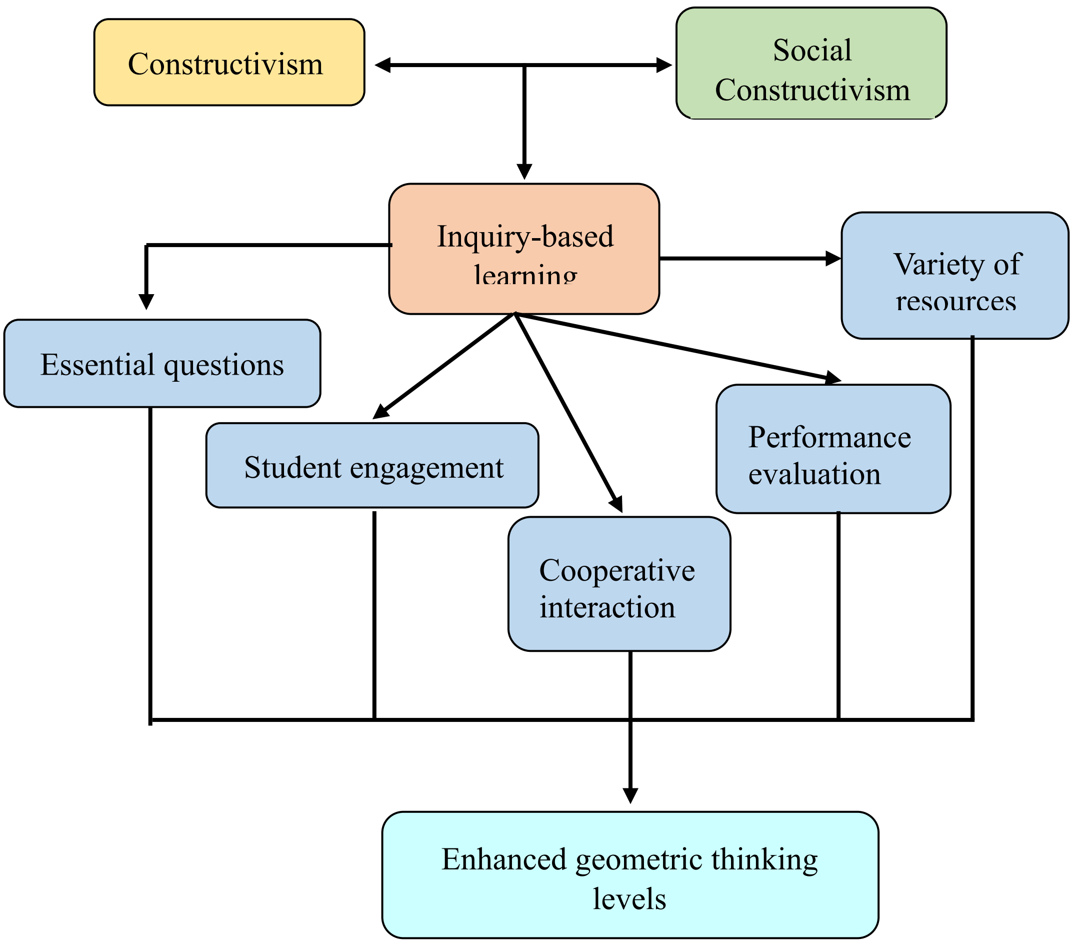

Figure 1. Conceptual framework of the study (Author's Construction, 2024)

This framework demonstrated that blending these theories in a classroom setting could empower teachers to adopt inquiry-based practices, thereby fostering improved geometric thinking levels among students. As illustrated in Figure 1, the conceptual framework underscored the potential of an inquiry-based approach in a socially interactive, constructivist learning environment to enhance students' geometric thinking levels.

Inquiry-based learning

Inquiry-based learning (IBL), developed by Dewey (1938) cited in Artigue & Blomhøj, emphasises discovery, reflection, and adaptive learning based on experiences to shape students' learning styles [22]. Defined as a student-centred and self-directed instructional method [23], IBL places learners' questions and observations at its core [24], with teachers acting as facilitators guiding students through critical questioning and collaborative processes [25]. It offers three levels structured, guided, and open inquiry tailored to classroom needs, balancing teacher control and student autonomy [26]. Studies have consistently shown that IBL enhances academic achievement, conceptual understanding, and retention compared to traditional methods [15, 16, 27]. The approach fosters deeper learning and critical thinking [21] through its five key components: essential questions, student engagement, cooperative interaction, performance evaluation, and diverse responses.

The first component, **essential questions**, involves open-ended queries posed by teachers or facilitator, students, or both, stimulating curiosity and critical thinking in line with Bloom's taxonomy [16]. The facilitator posed questions on varied concepts in geometry and the students' failure to solve these questions revealed their lack of the required knowledge in those concepts therefore, the facilitators developed lessons and activities to assist the students in developing the requisite knowledge. These questions encourage students to construct their answers through exploration and classroom activities. **Student engagement**, the second component, focuses on active involvement in learning, where students participate in activities planned and designed in the lesson. This is where students were observed evaluating and manipulating teaching and learning resources to create their understanding Research highlights that engaged students not only perform better academically but also exhibit improved school attendance, resilience, and retention [28]. **Cooperative interaction**, the third component, emphasizes collaborative learning, where students work in groups to share ideas, clarify concepts, and construct knowledge collectively [28]. This approach fosters a supportive environment, enhances communication, and builds confidence among students. **Performance evaluation**, the fourth component, the facilitator measures students' understanding through formative and summative assessments, including projects, presentations, and test items. This stage challenges students to apply their knowledge to solve complex problems, promoting higher-order thinking. The final component, a **variety of resources**, involves providing diverse teaching and learning resources such as textbooks, worksheets, and manipulatives to support conceptual understanding and active participation [29, 30]. These resources enhance the learning process, appealing to students' senses and fostering self-confidence and practical learning experiences. Together, these components position IBL as a constructivist approach that enhances students' understanding, active engagement, and retention, creating a dynamic and student-centred learning environment to enhance the students geometric thinking levels.

Students' geometric thinking levels

Studies on students' geometric thinking levels, as outlined by the Van Hiele model, reveal varying levels of attainment across educational contexts. Asemani, Asiedu-Addo, and Oppong [12] found that most Ghanaian SHS students operate at lower Van Hiele levels, with 42.5% of SHS 3 students unable to reach level 1. Similarly, Bashiru and Nyarko [13] identified that 20.95% of JHS 3 students in Ghana remained at level 0, highlighting a lack of foundational geometric understanding. Armah, Cofie, and Okpoti [31] observed that only 6% of Ghanaian pre-service teachers attained level 4, suggesting limited preparedness for teaching advanced geometry. Internationally, similar trends have been documented. Hamzeh [32] reported that most teacher candidates in Jordan achieved only level 2, while Casanova, Cantoria, and Lapinid [33] emphasized the need for substantial improvement in students' geometric reasoning on triangles in the Philippines.

International studies corroborate these trends, showing that pre-service teachers predominantly function at levels 1 and 2, with a limited understanding of higher levels [34, 35, 36]. Abdullah and Zakaria demonstrated in a quasi-experimental study that while control and experimental groups began at similar levels, the experimental group achieved significant advancement, with nearly all students reaching level 3 after the intervention [37].

Comprehensive reviews, such as [38], underline the importance of integrating metacognitive strategies with Van Hiele-based instruction to improve geometric thinking. In Turkey, [7] synthesised studies showing that students predominantly operate at levels 1 and 2, often struggling with higher-order reasoning. Mawarsari, Waluya, and Dewi similarly found that Indonesian students frequently remain at visualisation and analysis stages, requiring targeted instructional strategies to advance [39]. Guided inquiry learning models have also shown promise in enhancing geometric thinking, as noted by [40]. Furthermore, Machisi and Feza [41] demonstrated that Van Hiele-based instruction significantly improved students' competencies in geometric proofs, underscoring the model's efficacy.

MATERIAL AND METHODS

Research design

The descriptive case study design was employed as the appropriate research design for this study to enhance the students' geometric thinking levels using inquiry-based teaching and learning approach in the constructivist classroom or environment. Descriptive case study is the in-depth study and description of one or more phenomena in its real-life context that reflects the perspectives of the participant involved in the phenomena [42]. The study considered a case study because it consolidated the views of the students at the various stages of the study in order to develop a solid and firm instructional approach to promote favourable learning outcomes. Though case study allowed multiple data to understand and explain the phenomena under study, this study used only the quantitative data. A descriptive case study research design is ideal for studying and enhancing students' geometric thinking levels because it allows for an in-depth exploration of the phenomena in the real-life educational contexts, capturing the complex interplay of teaching strategies, student cognition, and classroom dynamics which provided a comprehensive and detailed understanding of the factors influencing geometric thinking. Also, this design is suitable for small samples and offered practical insights in enhancing geometric thinking levels.

Design-Based Research

Design-Based Research (DBR) is adopted as the research approach for this study. The study adopted DBR because it emphasizes the need for the design and the development of study or educational intervention that direct, educates and strengthens both practice and study in the context of education [43]. This study was conducted based on the four-stage approach of the design-based by Reeves [44]. These stages are the analysis of practical problems stage, design stage, implementation and evaluation stage and reflection and documentation stage. However, the literature had established the existence of the problem therefore, the researchers moved to the second stage of design-based research which is the design stage. At the design stage, researchers reviewed the literature to design appropriate interventions to enhance students' geometric thinking. After designing and developing the inquiry-based interventions, the researcher sought expert views and used them to refine the study intervention. This study intervention had gone through iterations to be refined. One of the characteristics of design-based research is its iterative nature [45].

At the implementation and evaluation stage, the researchers selected two senior high schools and eight in-service mathematics teachers. Professional development programmes were organised for the in-service mathematics teachers on how to design and develop inquiry-based interventions and enactment in constructivist classrooms. Professional development programmes were organised in two cohorts that was in school A and later in school B. In the first cohort or School A, the four teachers were put into two groups of two members each. The researchers took these teachers through the characteristics of an inquiry-based learning approach and how to prepare inquiry geometric lesson interventions. The researchers enacted the initial inquiry geometric lesson interventions on the teachers to experience the enactment of the interventions in the constructivist environment. After the enactment, the two groups were tasked to design and develop inquiry geometric lesson interventions.

One member of the two groups enacted their lesson interventions in micro-teachings and after the teaching, the researchers called for a focus group discussion to gain input from the other team members to refine the lesson interventions. The refined lesson interventions were enacted on the students in the real classrooms by the other team members. Before the enactment of the lesson interventions, the researchers gave a pre-test to the students and after the enactment post-tests were given in addition to retention tests three and eleven weeks after the enactment of the lesson interventions. Data were collected and analysed. All the challenges identified in the first study or cohort were resolved and a refined professional development programme was organised for the second cohort of the four teachers selected in the second school or school B as done in the first study or school A.

Participation

The study was conducted in two senior high schools in one of the municipalities in Ghana. these two senior high schools were selected randomly from 6 senior high schools in the municipality. Four in-service mathematics teachers were chosen from each school, therefore, a total of eight in-service mathematics teachers were purposively selected. Finally, four classes were selected purposively and these classes were the classes where the participating in-service mathematics teachers teach. However, only the students were the unit of the analysis therefore, the study population was 87.

Research instrument

The study adopted van Hiele's geometry test to measure students' geometric thinking levels. The test is a recognised assessment tool for measuring students' attainment of van Hiele's levels of geometric understanding. Comprising 25 multiple-choice questions divided into five blocks of five items, the VHGT evaluates levels ranging from visualisation (Level 1) to rigour (Level 5) but in the context of this study and the level of the students, the test measured up to the fourth level of van Hiele geometric thinking level (deduction) because secondary school mathematics curriculum was developed up to the fourth level. This test instrument has been used in numerous studies, such as Armah (2015) and Yazdani (2007), to assess geometric thinking and its relationship to academic performance. The study employed the scoring of one point per correct answer or using a "3 of 5 correct" success criterion to determine mastery of each level. The VHGT was administered pre-intervention, post-intervention, and as a follow-up to evaluate the retention of the students' geometric thinking levels. The retention tests were given three and eleven weeks after the implementation of the study intervention to measure the extent to which inquiry-based teaching and learning approach will enhance the students' abilities of retaining geometric knowledge.

RESULTS

The results of the student's geometric thinking based on the van Hiele geometry test as conducted before the implementation of the study intervention, after the intervention and follow-ups were presented below. The results were presented according to the phases of the study. The results of the first phase were presented in Table 1 and Figure 1.

From Table 1, the result showed that in the first phase or school A, the students performed better on the individual items in the post-test than in the pre-test. At the visualisation level (questions 1 – 5), there was a percentage increment in the accuracy of the students' answers in the post-test. For example, 16% more students answered question 1 correctly in the post-test assignment. Questions 2, 3, 4, and 5 had 14, 16, 52, and 32 percentage gains, respectively. Again, at analysis level (questions 6 – 10), the result revealed the percentage increment in the number of students who answered questions 6, 7, 8, 9 and 10 in the post-test compared to the pre-test. The result recorded a percentage gain of 36, 52, 50, 54 and 10 accuracies in answering questions 6, 7, 8, 9 and 10 respectively. For the informal deduction level (Questions 11–15), the result recorded a percentage increment in the student's performance in answering questions 11, 12, 13, 14 and 15 correctly. These percentage increments are 58, 62, 46, 88 and 56. Lastly, at the formal deduction level (Questions 16–20), the result showed a percentage increment in the student's performance in answering questions 16, 17, 18, 19 and 20. These percentage increments are 26, 50, 50, 30 and 22. The percentage gains recorded in the students' performance in the post test was as a result of the introduction of inquiry-based geometry lessons. Hence, the researcher attributed the increment or improved knowledge and understanding of the students in geometric concepts to the use of the inquiry-based teaching and learning approach.

Additionally, the two retention tests were compared to the post-test (see Table 1). At the visualization level (Questions 1–5), the results showed marked improvements from the post-test to the first retention test. Notably, there were percentage gains of 4% (Q2), 12% (Q3), 6% (Q4), and a striking 44% (Q5). This indicates that the inquiry-based approach not only facilitated understanding but also reinforced knowledge in the short term. Comparing the post-test to the second retention test, gains were recorded in Q1 (2%), Q2 (2%), and Q5 (50%), although a decline was observed in Q4. Between the first and second retention tests, there was a decrease in Q2 (-2%), Q3 (-12%), and Q4 (-18%), but gains were sustained in Q1 (+2%) and Q5 (+6%). These results demonstrate a strong retention effect, particularly for foundational concepts, despite some decline over time.

At the analysis level (Questions 6–10), the results revealed a significant retention of knowledge. From the post-test to the first retention test, substantial gains were recorded in Q6 (+40%), Q8 (+30%), Q9 (+2%), and Q10 (+2%), with a slight decline in Q7 (-6%). Comparing the post-test to the second retention test, there were gains in Q6 (+34%), Q7 (+4%), and Q8 (+24%), while Q9 remained unchanged, and Q10 saw an 8% decline. Between the first and second retention tests, a decline was noted in Q6 (-6%), Q8 (-6%), Q9 (-2%), and Q10 (-10%), while Q7 improved (+10%). These trends highlight that while retention was

strong immediately after instruction, the long-term retention was more variable across questions.
For the informal deduction level (Questions 11–15), the inquiry-based approach yielded notable initial improvements. From the post-test to the first retention test, gains were observed in Q11 (+22%), Q12 (+16%), and Q15 (+28%), though there were declines in Q13 (-8%) and Q14 (-40%). Comparing the post-test to the second retention test, Q13 improved significantly (+26%), but declines were recorded in Q11 (-16%), Q12 (-2%), Q14 (-18%), and Q15 (-28%). Between the first and second retention tests, improvements were seen in Q13 (+34%) and Q14 (+22%), while there were declines in Q11 (-38%), Q12 (-18%), and Q15 (-56%). These results suggest that while the approach was effective in fostering understanding, certain concepts may require additional reinforcement for long-term retention.
At the formal deduction level (Questions 16–20), the results demonstrated varying degrees of retention. From the post-test to the first retention test, gains were recorded in Q16 (+14%), Q17 (+12%), and Q20 (+46%), while declines were observed in Q18 (-16%) and Q19 (-10%). Comparing the post-test to the second retention test, gains were observed in Q16 (+14%), Q17 (+10%), Q19 (+12%), and Q20 (+56%), though Q18 declined (-14%). Between the first and second retention tests, Q19 (+22%) and Q20 (+10%) showed improvement, while Q18 (-2%) declined, and Q16 and Q17 remained stable or slightly decreased. These findings indicate that the inquiry-based approach had a lasting positive impact on higher-order reasoning tasks, though retention levels varied across questions.

Table 1. Students' overall performance of VHGT items in pre-, post and retention tests in School A

		Pre-test						**Post-test**						**Retention – test 1**						**Retention – test 2**				
Level 1	**Choice items**	**1**	**2**	**3**	**4**	**5**		**1**	**2**	**3**	**4**	**5**		**1**	**2**	**3**	**4**	**5**		**1**	**2**	**3**	**4**	**5**
	A	0	1	10	9	9		0	0	6	1	6		0	0	0	0	0		0	1	0	0	0
	B	**41**	0	1	**19**	9		**49**	0	0	**45**	5		**49**	0	2	**48**	0		**50**	0	3	**39**	0
	C	3	4	**36**	4	18		0	2	**43**	0	14		0	0	**48**	1	3		0	0	**43**	0	0
	D	6	**41**	2	5	**8**		1	**48**	1	0	**25**		1	**50**	0	0	**47**		0	**49**	4	11	0
	E	0	4	1	13	6		0	0	0	4	0		0	0	0	1	0		0	0	0	0	**50**
Level 2	**Choice items**	**6**	**7**	**8**	**9**	**10**		**6**	**7**	**8**	**9**	**10**		**6**	**7**	**8**	**9**	**10**		**6**	**7**	**8**	**9**	**10**
	A	9	7	**9**	11	3		11	0	**34**	1	8		1	4	**49**	1	10		2	0	**46**	1	12
	B	**8**	4	11	5	12		**26**	2	5	1	6		**46**	3	1	0	8		**43**	4	0	2	6

	C	22	12	10	**20**	13		11	2	3	**47**	16		1	1	0	**48**	11		2	1	2	**47**	16
	D	10	10	6	6	**15**		2	3	4	0	**20**		2	2	0	1	**21**		1	0	0	0	**16**
	E	1	**17**	14	8	7		0	**43**	4	1	0		0	**40**	0	0	0		2	**45**	2	0	0
Level 3	**Choice items**	**11**	**12**	**13**	**14**	**15**		**11**	**12**	**13**	**14**	**15**		**11**	**12**	**13**	**14**	**15**		**11**	**12**	**13**	**14**	**15**
	A	10	6	**3**	**4**	15		4	3	**26**	**48**	2		0	0	**22**	**28**	1		5	6	**39**	**39**	7
	B	13	**10**	0	10	**6**		2	**41**	3	1	**34**		0	**48**	4	6	**48**		9	**40**	3	3	**20**
	C	**10**	11	7	20	12		**39**	4	3	1	5		**50**	2	6	11	1		**31**	0	2	6	7
	D	6	16	4	6	10		2	1	0	0	3		0	0	3	3	0		3	3	2	1	5
	E	11	7	36	10	7		3	1	18	0	6		0	0	13	2	0		2	1	4	1	11
Level 4	**Choice items**	**16**	**17**	**18**	**19**	**20**		**16**	**17**	**18**	**19**	**20**		**16**	**17**	**18**	**19**	**20**		**16**	**17**	**18**	**19**	**20**
	A	10	17	11	13	**9**		6	8	6	7	**8**		2	2	9	7	**31**		6	2	8	3	**36**
	B	9	9	16	13	17		8	3	5	6	17		8	5	9	11	7		3	2	10	7	2
	C	**16**	**10**	16	9	10		**29**	**35**	9	8	15		**36**	**41**	10	8	7		**36**	**40**	8	1	7
	D	9	6	**4**	**14**	9		6	1	**29**	**29**	9		3	2	**21**	**24**	4		4	4	**22**	**35**	1
	E	6	8	3	1	5		1	3	1	0	1		1	0	1	0	1		1	2	2	4	4

In conclusion, the results clearly indicate that the inquiry-based teaching and learning approach effectively enhanced and retained students' geometric thinking levels across different levels of the van Hiele model. While some decline in performance was observed over time, especially in more complex tasks, the overall trends highlight the approach's success in fostering both immediate understanding and long-term retention. These findings reinforce the value of inquiry-based methods in mathematics education, particularly for developing critical thinking and problem-solving skills in geometry.

Furthermore, the result showed the geometric thinking levels attained by the students in the first phase of the study or school A (see Figure 1).

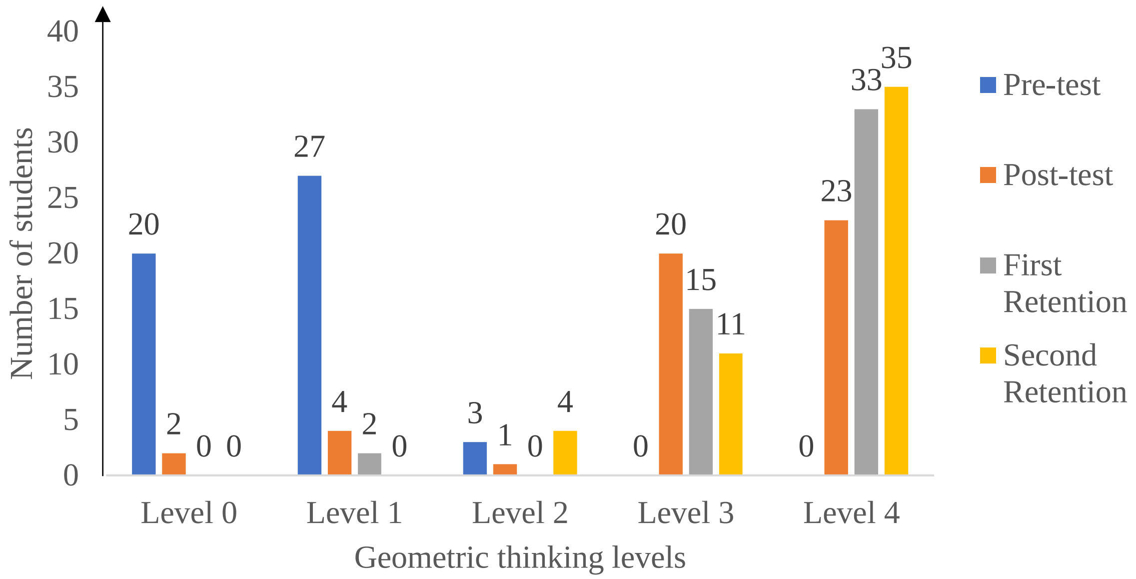

Figure 1. Students' Geometric Thinking in the Pre, Post, first and second retention in the first phase or school A.

The study revealed that introducing inquiry-based teaching significantly enhanced students' geometric thinking levels, as measured by the van Hiele model. In the pre-test, none of the 50 students attained level 3 (abstraction), and 40% were at level 0 (pre-visualisaton). A majority (54%) achieved level 1 (visualisation), while only 6% reached level 2 (analysis).

Post-test results demonstrated substantial improvement, with the percentage of students at level 0 reducing drastically to 4%. At levels 1 and 2, 8% and 2% of students, respectively, exhibited gains. Remarkably, 40% of students attained level 3, while 46% reached level 4 (deduction), a level not achieved in the pre-test. These results highlight significant progress in students' geometric thinking following the intervention.

Comparing the post-test to the first retention test, further improvements were observed. None of the students remained at level 0, and higher levels saw increases. Level 4 achievement rose from 46% in the post-test to 66% in the first retention test, indicating a 20% improvement in students reaching the highest van Hiele level.

The second retention test results revealed sustained and even greater improvement. Again, no students remained at level 0, and the number at level 1 dropped to zero. At level 2, the percentage increased to 8%. While level 3 saw a decline from 40% in the post-test to 22% in the second retention test, level 4 achievement rose to 70%, a 26% increase compared to the post-test.

When comparing the first and second retention tests, the differences were minimal. Both tests showed no students at level 0, and while level 4 saw a slight increase from 66% to 70%, these results demonstrate consistent retention of higher-order geometric thinking over time.

In conclusion, the study found that inquiry-based teaching effectively improved and sustained students' geometric thinking levels. The significant increases in students achieving level 4 in both the first and second retention tests underscore the lasting impact of this pedagogical approach.

To determine the resilience and dependability of using an inquiry-based teaching and learning approach to improve students' geometric thinking levels, the researcher repeated the study in another school or conducted the second phase of the study in another school called school B. The student's performance on the individual items of the van Hiele geometry test is presented in Table 2.

From Table 2, the results showed that at visualisation level or level 1 (Questions 1–5), the post-test results demonstrated substantial gains compared to the pre-test. Correct responses increased by 22% for Question 1, 17% for Question 2, 21% for Question 3, 62% for Question 4, and 51% for Question 5. These improvements highlight the effectiveness of the inquiry-based approach in promoting an understanding of fundamental geometric concepts. The use of hands-on activities, guided discovery, and collaborative problem-solving encouraged students to explore geometric relationships more actively, leading to enhanced comprehension. Although retention test results showed minor declines in some items, the overall performance remained higher than the pre-test, underscoring the lasting impact of this instructional method.

Again, at level 2 or analysis level (Questions 6–10), the post-test results reflected significant gains, particularly in Questions 6 (+51%), 7 (+59%), 9 (+49%), and 10 (+59%). These outcomes demonstrate how the inquiry-based approach facilitated students' ability to analyse and reason about geometric properties. While retention results showed fluctuations—such as gains in Question 8 (+65%) and Question 9 (+19%) but declines in others—these variations emphasise the need for consistent reinforcement to sustain higher-order thinking skills fostered during inquiry-based learning.

Also, at the informal deduction level or level 3 (Questions 11–15), students exhibited notable improvements in the post-test, with percentage gains ranging from 22% (Question 15) to 38% (Question 11). This level required more abstract reasoning and logical deduction, which were effectively supported by the inquiry-based approach. Retention results revealed further gains in some items, such as Questions 12 (+44%) and 13 (+30%), indicating that the inquiry-based activities helped anchor these concepts in students' memory. However, slight declines in Questions 11 (-11%) and 15 (-6%) suggest areas where ongoing support and practice are necessary.

Finally, at the formal deduction level or level 4 (Questions 16–20), the post-test results showed remarkable improvements, particularly in Questions 16 (+51%), 18 (+59%), and 20 (+15%). While retention results revealed some challenges in sustaining these gains, with declines in Questions 17 (-19%) and 18 (-38%), the overall trend underscores the transformative impact of inquiry-based learning in enhancing students' geometric reasoning abilities.

In summary, the inquiry-based teaching and learning approach was crucial in improving students' performance across all levels of the van Hiele geometric thinking test. By actively engaging students in the learning process and encouraging critical thinking, this approach facilitated a deeper understanding of geometric concepts. Although retention results highlighted areas needing reinforcement, the overall findings affirm the effectiveness of inquiry-based strategies in promoting lasting academic growth. This underscores the need for sustained implementation of such approaches to ensure long-term mastery and retention of geometric concepts.

Table 2. Students' overall performance of VHGT items in pre-, post and retention tests in School B

Pre – test Post – test													Retention – test 1						Retention test 2					
Lev el 1	Choi ce	1	2	3	4	5		1	2	3	4	5		1	2	3	4	5		1	2	3	4	5

	items																							
	A	0	2	7	5	**11**		0	2	1	0	2		0	0	0	1	2		0	0	0	0	4
	B	**27**	0	3	**14**	6		**35**	0	3	**37**	4		**35**	0	0	**33**	2		**37**	1	1	**34**	3
	C	2	5	**25**	13	17		1	0	**33**	0	9		0	1	**37**	2	6		0	2	**33**	0	1
	D	8	**29**	1	2	3		1	**35**	0	0	**22**		0	**36**	0	1	**25**		**0**	**34**	3	3	**27**
	E	0	1	1	3	0		0	0	0	0	0		2	0	0	0	2		0	0	0	0	2
Level 2	**Choice items**	**6**	**7**	**8**	**9**	**10**		**6**	**7**	**8**	**9**	**10**		**6**	**7**	**8**	**9**	**10**		**6**	**7**	**8**	**9**	**10**
	A	4	5	5	6	5		3	2	**14**	0	1		0	3	**11**	2	2		0	0	**35**	1	10
	B	**10**	6	12	9	15		**29**	1	4	2	8		**36**	2	9	2	2		**37**	10	1	1	7
	C	18	12	**11**	**16**	15		2	5	8	**34**	2		0	2	4	**28**	7		0	3	0	**35**	5
	D	4	10	6	4	**1**		3	3	3	0	**23**		1	11	5	3	**23**		0	9	1	0	**12**
	E	1	**4**	3	2	1		0	**26**	8	1	3		0	**19**	8	4	3		0	**15**	0	0	3
Level 3	**Choice items**	**11**	**12**	**13**	**14**	**15**		**11**	**12**	**13**	**14**	**15**		**11**	**12**	**13**	**14**	**15**		**11**	**12**	**13**	**14**	**15**
	A	9	16	**5**	**3**	10		4	4	**18**	**23**	4		0	2	**20**	**30**	9		3	4	**31**	**28**	7
	B	14	**9**	3	10	**8**		1	**25**	1	8	**11**		7	**27**	1	4	**15**		9	**30**	1	4	**13**
	C	**10**	4	8	13	11		**24**	5	0	2	6		**27**	4	1	0	6		**23**	0	1	4	6
	D	3	5	2	5	5		3	0	0	4	11		2	3	0	0	4		2	0	1	0	4
	E	1	3	19	6	3		5	3	18	0	5		1	1	15	3	3		0	3	3	1	7

Level 4	**Choice items**	**16**	**17**	**18**	**19**	**20**		**16**	**17**	**18**	**19**	**20**		**16**	**17**	**18**	**19**	**20**		**16**	**17**	**18**	**19**	**20**
	A	6	7	9	12	5		3	6	3	13	**11**		4	5	3	11	**25**		5	3	6	4	**28**
	B	19	9	7	2	17		1	7	3	3	4		6	4	5	10	5		5	10	4	5	2
	C	**10**	**15**	10	16	**11**		**29**	**14**	4	12	5		**17**	**25**	6	4	4		**22**	**18**	4	6	4
	D	8	5	**11**	**6**	2		4	7	**27**	**9**	16		2	1	**23**	**12**	1		2	3	**9**	**21**	0
	E	1	1	0	1	2		0	3	0	0	1		8	2	0	0	2		3	3	4	1	3

In addition, the result showed the geometric thinking levels attained by the students in the second phase of the study or school B (see Figure 2).

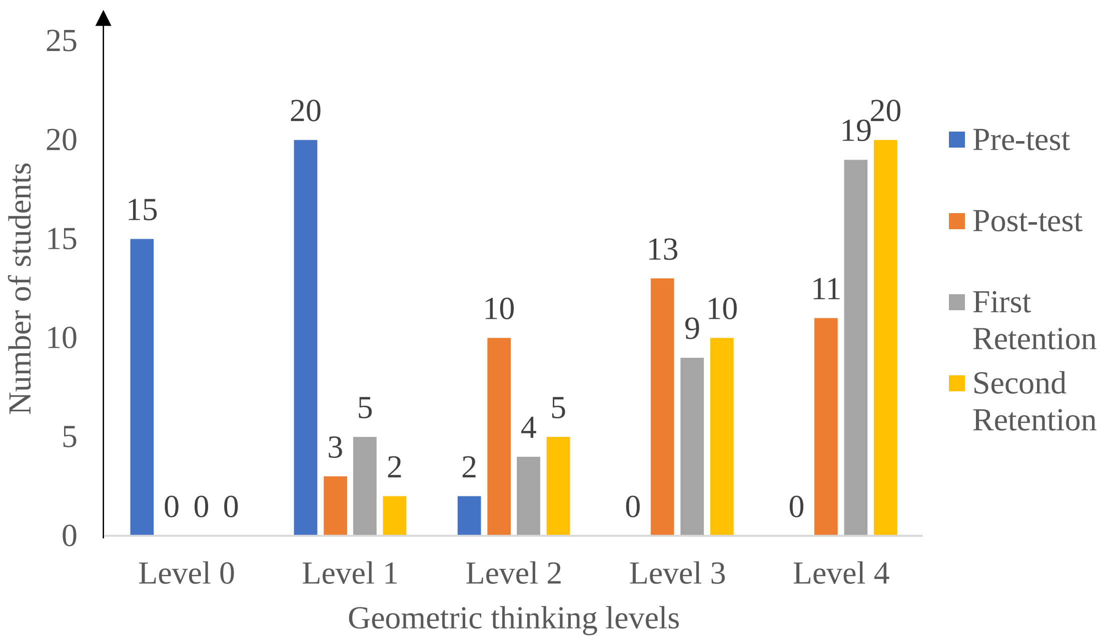

Figure 2. Figure 1. Students' Geometric Thinking in the Pre, Post, first and second retention in the second phase or school B.

The analysis of the student's attainment of van Hiele geometric thinking levels in the study's second phase (School B) revealed notable progress across all stages. In the pre-test, 15 out of 37 students (41%) were at Level 0 (pre-visualisation), unable to achieve any van Hiele level. Additionally, 20 (54%) attained Level 1, and only 2 (5%) reached Level 2. No students achieved Levels 3 (informal deduction) or 4 (formal deduction), indicating that most of the students operated at basic geometric thinking levels.

The post-test results demonstrated significant improvement. No students remained at Level 0, while only 3 students (8%) were still at Level 1 (visualisation). At Level 2 that is analysis level, the number increased to 10 students (28%), while Levels 3 and 4 saw significant gains, with 13 students (39%) and 11 students (31%), respectively. These results underscore substantial enhancement in students' geometric reasoning, particularly at advanced levels, attributed to the inquiry-based teaching approach.

Comparing the post-test and first retention test results, no students remained at Level 0, but the proportion at Level 1 increased slightly from 8.2% (3 students) to 13.5%. At Level 2, attainment decreased from 27% to 10.8%, and at Level 3, it dropped from 35.1% to 24.3%. However, Level 4 saw an increase from 29.7% in the post-test to 51.4% in the first retention test, indicating improved retention of higher-order geometric thinking.

In the second retention test, the proportion of students at Level 1 decreased further to 5.4%, with no students at Level 0. Attainment at Level 2 slightly increased to 13.5%, while Level 3 remained steady at 27%. Level 4 showed the highest improvement, with 54.1% of students achieving this level compared to 29.7% in the post-test. These findings highlight significant growth and sustained improvement in geometric thinking over time.

A comparison of the first and second retention tests revealed minor differences. While more students reached Level 4 in the second retention test (54.1% compared to 51.3%), the overall geometric thinking levels remained consistent, suggesting that the inquiry-based teaching approach effectively supports long-term knowledge retention.

In summary, the results show a significant improvement in students' geometric thinking levels from the pre-test to the post-test and retention tests. The sustained high performance across retention tests indicates that inquiry-based teaching enhances students' understanding and retention of geometric concepts, fostering long-term academic growth.

DISCUSSION

The findings of the study revealed that, before the implementation of the intervention, most students in both phases of the study were operating at level 1 (visualisation) of the van Hiele model, with a few at level 2 and none at level 3. Alarmingly, a significant proportion of the students (40% in the first phase and 41% in the second phase) remained at level 0, the pre-visualization stage. This aligns with prior research indicating that many students struggle to achieve the advanced geometric thinking required at van Hiele level 4 to excel in geometry [7, 10 - 13]. Furthermore, it corroborates studies showing that pre-service teachers often operate predominantly at levels 1 and 2, with limited progression to higher levels [33 – 35]. Similar patterns have been observed globally, including in Turkey, where Celik and Yilmaz [7] synthesised studies revealing students' challenges with higher-order reasoning, and in Indonesia, where Mawarsari et al. [39] reported that students frequently remain at the visualisation and analysis stages.

After the intervention, which utilised inquiry-based learning (IBL) approach, most students showed substantial improvement, advancing to higher van Hiele levels, particularly levels 3 (abstraction) and 4 (deduction). This significant progress underscores the transformative impact of IBL on geometric reasoning. The findings are consistent with Hardianti et al. [40], who demonstrated that guided inquiry learning models effectively enhance students' geometric thinking. Similarly, Abdullah and Zakaria [37] showed in a quasi-experimental study that, while control and experimental groups began at comparable levels, the experimental group achieved substantial advancements, with nearly all students reaching level 3 post-intervention.

These results highlight a critical issue in students' geometric reasoning abilities before the intervention, where traditional teaching methods often fail to support the hierarchical and developmental progression described by the van Hiele model [46]. The model emphasizes that students must progress sequentially through its levels of visualization, analysis, abstraction, deduction, and rigour through structured, targeted instruction [47].

The significant improvements observed after the intervention emphasize the efficacy of IBL in fostering higher-order geometric reasoning. By engaging students actively in problem-solving, critical thinking, and exploration, IBL aligns with the cognitive demands of van Hiele levels 3 and 4. Research supports this approach; for instance, Altherr et al. [48] found that inquiry-based teaching significantly enhances students' ability to connect geometric concepts and reason deductively. Similarly, Şahin and Çelikkan [49] demonstrated that students exposed to inquiry-driven tasks developed stronger spatial reasoning and analytical skills compared to those taught using traditional methods.

The fact that a substantial proportion of students initially remained at level 0 highlights a lack of foundational skills in geometric reasoning. This finding aligns with Van de Walle et al. [50], who emphasized that inadequate early experiences in geometry can impede progression to higher levels of understanding.

In summary, this study provides compelling evidence that inquiry-based approaches can bridge gaps in geometric reasoning abilities, transforming students' learning experiences from surface-level understanding to meaningful, higher-order thinking. These findings are well-supported by contemporary educational research, which consistently advocates for active, student-centred pedagogies to improve mathematical understanding and achievement [51]. Finally, the study confirms the framework of the study that the use of an inquiry-based learning approach through essential questioning, student engagement, cooperative interaction, performance evaluation and the use of a variety of resources to teach geometry in the blended constructivist and social constructivist environment enhanced students' geometric thinking levels.

CONCLUSION AND RECOMMENDATION

This study established the use of inquiry-based teaching and learning approaches through essential questions, student engagement, cooperative interaction, performance evaluation and the use of a variety of resources in the constructivist classrooms or environment enhanced student's geometric thinking levels according to van Hiele models. in teaching geometric concepts as a critical tool for enhancing students' attitudes towards the learning of geometry. The findings of the study revealed that the students who participated in the study or experienced inquiry-based intervention saw a progression in their geometric thinking levels.

The study, therefore, recommended that teachers adopt inquiry-based teaching and learning approaches, especially during the teaching of geometric concepts since it significantly improves student's geometric thinking levels.

CONFLICT OF INTEREST

There is no conflict of interest arising from this research.

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