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(S, d)Magic Labeling of Subdivision of Some Snake Graphs -Paper II

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Abstract
Let G (p, q) $be\ a$ connected, undirected, simple and non-trivial graph with p vertices and q edges. Let f be an injective function f: V(G) → $\{ s, s+d, s+2d, ... s+(q+1)d \}$ and g be an injective function g: E(G) → $\{d, 2d, 3d \ldots. 2(q-1)d\}$.Then the function f is said to be (s, d) magic labeling if $f(u) + g(uv) + f(v)$ is a constant, for all $u, v \in V(G)$ and $uv \in E(G)$. A graph G is called (s, d) magic graph if it admits (s, d) magic labeling.

Keywords: Subdivision on pentagonal snake graph, Alternate pentagonal snake graph and Quadrilateral snake graph

1. Introduction

The graphs discussed in this context are finite, undirected, and simple. The notations V(G) and E(G) represent the vertex set and edge set of a graph G, respectively, while p and q denote the number of vertices and edges in G.
In 2001, Barrientos [3] introduced the concept of KC_4-snake graphs as an extension of the triangular snake, which was earlier defined by Rosa [2]. Barrientos demonstrated that KC_4-snake graphs are graceful. A quadrilateral snake is a specific type KC_4snake graph characterized by the string (1,1, 1…,1). Gnanajothi [4] further established that quadrilateral snakes are graceful.
We introduce (s, d) Magic labeling of graphs. If G admits (s, d) Magic labeling, then G is called as (s, d) Magic graph. In this paper, a new concept of (s, d) Magic labeling has been introduced for some graphs.
[5] Let $G\ (p, q)$ be a simple, non-trivial, connected, undirected graph with p vertices and q edges. Consider the following: $f: V(G) \to \{s, s+d, s+2d, .. s+(q+1)d\}$ and
$g: E(G) \to \{d, 2d, 3d \ldots 2(q-1)d\}$ be an injective function. Then, for any $u, v \in V(G)$ and $uv \in E(G)$, $f(u) + g(uv) + f(v)$ is a constant, and the function f is said to be $(S$, d) magic labeling. If a graph G admits (S, d) magic labeling, then it is referred to as a (S, d) magic graph.

2. DEFINITIONS

Definition 2.1 A subdivision of a graph G is a graph formed by subdividing edges of G. Subdividing an edge e with end points u, v results in a graph with one new vertex w and an edge set that replaces e with two new edges uw and wv.
Notation:

1.$S'(PS_n)$ be a graph obtained from a pentagonal snake graph by subdividing only the edges on the main path of pentagonal snake graph
2. $S(PS_n)$ denotes subdivision on all the edges of PS_n
3.(i)$S'(A^1PS_n)$ denotes a subdivision of path of APS_n when n is even and the first pentagon starts from u_1 and the last ends with u_n
(ii)($S(A^1PS_n)$be a graph obtain by subdivision of APS_n when n is even and first triangle starts from $u_1$and the last ends with u_n
(iii) $S'(A^2PS_n)$Subdivision on path of APS_n when n is even and first triangle starts from u_2 and the last ends with u_{n-1}
(iv)$S(A^2PS_n)$Subdivision on path of APS_n when n is even and first triangle starts from u_2 and the last ends with u_{n-1}
(v) $S'(A^3PS_n)$Subdivision on path of $APS_n$when n is odd and first triangle starts from u_1 and the last ends with u_{n-1}
(vi) S(A^3PS_n)Subdivision on path of $APS_n$when n is odd and first triangle starts from u_1 and the last ends with u_{n-1}
4. $S'(Q_n)$be the graph obtained from a Quadrilateral snake graph by subdividing only the edges on the main path of the Quadrilateral snake graph.
5.S (Q_n) be the subdivision graph of all the edges of quadrilateral snake graph Q_n

3. Main Results

Theorem 3.1 The subdivision on pentagonal snake graph admits (S,d) magic labeling
Proof: Let G = S (PS_n).let the edges of $u_iu_{i+1,}u_iz_i, s_iu_{2i}, v_iz_i$, v_is_i are subdivided by $w_{i,}r_i$, t_i, x_i, $y_i$respectively,the following cases are
Case 1: $S'(PS_n)$ denotes as subdivision on main path of PS_n admits (S,d) magic labeling
Let $V(G) = \{u_i: 1 \le i \le n\} \cup \{w_i : 1 \le i \le n\} \cup \{v_i:1 \le i \le n-1\} \cup \{x_i, y_i: 1 \le i \le n-1\}$and $E(G) = \{u_iw_i: 1 \le i \le n-1\} \cup \{w_iu_{i+1}: 1 \le i \le n-1\} \cup \{x_iv_i: 1 \le i \le n-1\} \cup \{y_iv_i: 1 \le i \le n-1\} \cup \{x_iu_i: 1 \le i \le n-1\} \cup \{y_iu_{i+1}: 1 \le i \le n-1\}$
Here $p = 5n - 4$ and $q = 6(n-1)$
Define the function f from the vertex set to $\{s, s+d, s+2d \dots . s+(q+1)\}$,
$g: E(G) \to \{d, 2d, 3d \dots . .2(q-1)d\ \}$ to label the edges

3.1 αTable Labeling of vertices $S'(PS_n)$					
Value of i	$f(u_{i+1})$	$f(w_{i+1})$	$f(v_{i+1})$	$f(x_{i+1})$	$f(y_{i+1})$
$0 \le i \le n-1$	$s + (4i+2)d$	$-$	$-$	$-$	$-$
$0 \le i \le \frac{n}{2}$	$-$	$s + (4i + 4)d$	$s + (6i + 1)d$	$s + (6i - 1)d$	$s + (3i + 4)d$

3.1 bTable Labeling of edges $S'(PS_n)$						
Value of i	$g(u_iw_i)$	$g(w_iu_{i+1})$	$g(v_ix_i)$	$g(v_iy_i)$	$g(y_iu_{i+1})$	$g(x_iu_i)$
$1 \le i \le n-1$	$2s + 2(q-1)d - (f(u_i) + f(w_i))$	$2s + 2(q-1)d - (f(w_i) + f(u_{i+1}))$	$2s + 2(q-1)d - (f(v_i) + f(x_i))$	$2s + 2(q-1)d - (f(v_i) + f(y_i))$	$2s + 2(q-1)d - (f(y_i) + f(u_{i+1}))$	$2s + 2(q-1)d - (f(x_i) + f(u_i))$

Thus, $S'(PS_n)$ subdivision on path of (PS_n) admits (S,d) magic labeling.

Case 2:

$S(PS_n)$ denotes subdivision on all the edges of PS_n admits (S,d) magic labeling.

Let $V(G) = \{u_i: 1 \le i \le n\} \cup \{w_i : 1 \le i \le n\} \cup \{v_i: 1 \le i \le n-1\} \cup \{x_i, y_i, z_i, s_i r_i, t_i 1 \le i \le n-1\}$and $E(G) = \{u_iw_i: 1 \le i \le n-1\} \cup \{w_iu_{i+1}: 1 \le i \le n-1\} \cup \{u_ir_i: 1 \le i \le n-1\} \cup \{t_iu_{i+1}: 1 \le i \le n-1\} \cup \{z_ir_i: 1 \le i \le n-1\} \cup \{s_it_i: 1 \le i \le n-1\} \cup \{z_ix_i: 1 \le i \le n-1\} \cup \{y_is_i: 1 \le i \le n-1 \cup xv_i: 1 \le i \le n-1\} \cup \{v_iy_i: 1 \le i \le n-1\}$.Here $p = 9n-8$ and $q = 10(n-1)$

Define the function f from the vertex set to $\{s, s+d, s+2d \ldots . s+(q+1)\}$,

$g: E(G) \to \{d, 2d, 3d \ldots . .2(q-1)d\}$ to label the edges

3.1 cTable Labeling of vertices $S(PS_n)$					
Value of i	$f(u_{i+1})$	$f(w_{i+1})$	$f(r_{i+1})$	$f(t_{i+1})$	$f(z_{i+1})$
$0 \le i \le n-1$	$s + 9id$	$-$	$-$	$-$	$-$
$0 \le i \le \frac{n}{2}$	$-$	$s + (9i+8)d$	$s + (9i+1)d$	$s + (9i+7)d$	$s + (9i+2)d$
Value of i	$f(s_{i+1})$	$f(x_{i+1})$	$f(y_{i+1})$	$f(v_{i+1})$	$-$
$0 \le i \le \frac{n}{2}$	$s + (9i+6)d$	$s + (9i+3)d$	$s + (9i+5)d$	$s + (9i+4)d$	$-$

3.1 d Table Labeling of edges $S(PS_n)$					
Value of i	$g(u_iw_i)$	$g(w_iu_{i+1})$	$g(u_ir_i)$	$g(z_ir_i)$	$g(t_iu_{i+1})$
$1 \le i \le n-1$	$2s + 2(q-1)d - (f(u_i) + f(w_i))$	$2s + 2(q-1)d - (f(w_i) + f(u_{i+1}))$:	$2s + 2(q-1)d - (f(u_i) + f(r_i))$	$2s + 2(q-1)d - (f(z_i) + f(r_i))$	$2s + 2(q-1)d - (f(t_i) + f(u_{i+1}))$
Value of i	$g(s_it_i)$	$g(z_ix_i)$	$g(y_is_i)$	$g(x_iv_i)$	$g(v_iy_i)$

$1 \le i \le n-1$	$2s + 2(q-1)d - (f(s_i) + f(t_i))$	$2s + 2(q-1)d - (f(z_i) + f(x_i))$	$2s + 2(q-1)d - (f(y_i) + f(s_i))$	$2s + 2(q-1)d - (f(x_i) + f(v_i))$	$2s + 2(q-1)d - (f(v_i) + f(y_i))$

Thus, Subdivision on S (PS_n) admits $(S$,d) magic labeling

Theorem 3.2 The subdivision on alternate pentagonal snake graph admits $(S$,d) magic labeling.

Proof: Let G= S (APS_n) is obtain by subdividing all the edges of APS_n, the following cases are

Case 1: $S'(A^1PS_n)$ denotes a subdivision of path of APS_n when n is even and the first pentagon starts from u_1 and the last ends with u_n is $(S$,d) magic labeling.

Let $V(G) = \{u_i: 1 \le i \le n\} \cup \{w_i : 1 \le i \le n-1\} \cup \{v_i: 1 \le i \le \frac{n}{2}\} \cup \{x_i, y_i: 1 \le i \le \frac{n}{2}\}$ and $E(G) = \{u_iw_i: 1 \le i \le n-1\} \cup \{w_iu_{i+1}: 1 \le i \le n-1\} \cup \left\{x_iv_i: 1 \le i \le \frac{n}{2}\right\} \cup \left\{y_iv_i: 1 \le i \le \frac{n}{2}\right\} \cup \{x_{i+1}u_{2i+1}: 1 \le i \le \frac{n}{2}\} \cup \{y_iu_{2i}: 1 \le i \le \frac{n}{2}\}$.

Here $p = \frac{7n-2}{2}$ and $q = 2(2n-1)$.

Define the function f from the vertex set to $\{s, s+d, s+2d \ldots . s+(q+1)\}$,

$g: E(G) \to \{d, 2d, 3d \ldots . .2(q-1)d\ \}$ to label the edges

3.2 . aTable Labeling of vertices $S'(APS_n)$							
Value of i	$f(u_{2i+1})$	$f(u_{2(i+1)})$	$f(w_{2i+1})$	$f(w_{2(i+1)})$	$f(v_{i+1})$	$f(y_{i+1})$	$f(x_{i+1})$
$0 \le i \le \frac{n-2}{2}$	$s + 7id$	$s + (7i + 5)d$	$s + (7i + 4)d$	$s + (7i + 6)d$	$s + (7i + 2)d$	$s + (7i + 3)d$	$s + (7i + 1)d$

3.2. bTable Labeling of edges $S'(APS_n)$						
Value of i	$g(u_iw_i)$	$g(w_iu_{i+1})$	$g(v_ix_i)$	$g(v_iy_i)$	$g(y_iu_{2i})$	$g(x_{i+1}u_{2i+1})$
$1 \le i \le n-1$	$2s + 2(q-1)d - (f(u_i) + f(w_i))$	$2s + 2(q-1)d - (f(w_i) + f(u_{i+1}))$	−	−	−	−
$1 \le i \le \frac{n}{2}$	−	−	$2s + 2(q-1)d - (f(v_i) + f(x_i))$	$2s + 2(q-1)d - (f(v_i) + f(y_i))$	$2s + 2(q-1)d - (f(y_i) + f(u_{2i}))$	$2s + 2(q-1)d - (f(x_{i+1}) + f(u_{2i+1}))$

Thus, $S'(APS_n)$ subdivision on path of APS_n when n is even and the first pentogon starts from u_1 and the last ends with u_n admits (S,d) magic labeling.

Case 2: $S(A^1PS_n)$Subdivision on APS_n when n is even and the first pentogon starts from u_1 and the last ends with u_n

Subdivision of APS_n when n is even and the first pentagon starts from u_1 and the last ends with u_n admits (S,d) magic labeling

Let $V(G) = \{u_i: 1 \le i \le n\} \cup \{w_i : 1 \le i \le n-1\} \cup \{v_i: 1 \le i \le \frac{n}{2}\} \cup \{x_i, y_i z_i, s_i r_i, t_i: 1 \le i \le \frac{n}{2}\}$and $E(G) = \{u_i w_i: 1 \le i \le n-1\} \cup \{w_i u_{i+1}: 1 \le i \le n-1\} \cup \left\{x_i v_i: 1 \le i \le \frac{n}{2}\right\} \cup \left\{y_i v_i: 1 \le i \le \frac{n}{2}\right\} \cup \{r_{i+1} u_{2i+1}: 0 \le i \le \frac{n-2}{2}\} \cup \{t_i u_{2i}: 1 \le i \le \frac{n}{2}\} \cup \left\{x_i z_i: 1 \le i \le \frac{n}{2}\right\} \cup \left\{y_i s_i: 1 \le i \le \frac{n}{2}\right\} \cup \left\{z_i r_i: 1 \le i \le \frac{n}{2}\right\} \cup \left\{t_i s_i: 1 \le i \le \frac{n}{2}\right\}$.Here $p = \frac{11n-2}{2}$ and $q = 2(3n-1)$

Define the function f from the vertex set to $\{s, s+d, s+2d \dots . s+(q+1)\}$,

$g: E(G) \to \{d, 2d, 3d \dots . . 2(q-1)d \}$ to label the edges

3.2. cTable Labeling of vertices $S(A^1PS_n)$							
Value of i	$f(u_{2i+1})$	$f(u_{2(i+1)})$	$f(w_{2i+1})$	$f(w_{2(i+1)})$	$f(v_{i+1})$	$f(y_{i+1})$	$f(x_{i+1})$
$0 \le i \le \frac{n-2}{2}$	$s + 11id$	$s+(11i+5)d$	$s+(11i+8)d$	$s+(11i+10)d$	$s+(11i+4)d$	$s+(11i+5)d$	$s+(11i+3)d$
Value of i	$f(r_{i+1})$	$f(t_{i+1})$	$f(z_{i+1})$	$f(s_{i+1})$	−	−	−
$0 \le i \le \frac{n-2}{2}$	$s+(11i+1)d$	$s+(11i+7)d$	$s+(11i+2)d$	$s+(11i+6)d$	−	−	−

3.2. dTable Labeling of edges $S(A^1PS_n)$						
Value of i	$g(u_i w_i)$	$g(w_i u_{i+1})$	$g(v_i x_i)$	$g(v_i y_i)$	$g(r_{i+1} u_{2i+1})$	$g(t_i u_{2i})$
$1 \le i \le n-1$	$2s+2(q-1)d-(f(u_i)+f(w_i))$	$2s+2(q-1)d-(f(w_i)+f(u_{i+1}))$	−	−	−	−
$0 \le i \le \frac{n-2}{2}$	−	−	−	−	$2s+2(q-1)d-(f(r_{i+1})+f(u_{2i+1}))$	−

$1 \leq i \leq \frac{n}{2}$	−	−	$2s + 2(q-1)d - (f(v_i) + f(x_i))$	$2s + 2(q-1)d - (f(v_i) + f(y_i))$	−	$2s + 2(q-1)d - (f(t_i) + f(u_{2i}))$
Value of i	$g(x_iz_i)$	$g(y_is_i)$	$g(z_ir_i)$	$g(s_it_i)$		
$1 \leq i \leq \frac{n}{2}$	$2s + 2(q-1)d - (f(x_i) + f(z_i))$	$2s + 2(q-1)d - (f(y_i) + f(s_i))$	$2s + 2(q-1)d - (f(z_i) + f(r_i))$	$2s + 2(q-1)d - (f(s_i) + f(t_i))$	−	−

Thus, $S(A^1PS_n)$ subdivision on APS_n when n is even and the first pentogon starts from u_1 and the last ends with u_n admits (S,d) magic labeling.

Case 3: $S'(A^2PS_n)$ denotes a subdivision of path of APS_n when n is even and the first pentagon starts from u_2 and the last ends with u_{n-1} is (S,d) magic labeling.

Let $V(G) = \{u_i: 1 \leq i \leq n\} \cup \{w_i : 1 \leq i \leq n-1\} \cup \{v_i: 1 \leq i \leq \frac{n}{2} - 1\} \cup \{x_i, y_i: 1 \leq i \leq \frac{n}{2} - 1\}$and $E(G) = \{u_iw_i: 1 \leq i \leq n-1\} \cup \{w_iu_{i+1}: 1 \leq i \leq n-1\} \cup \left\{x_iv_i: 1 \leq i \leq \frac{n}{2} - 1\right\} \cup \left\{y_iv_i: 1 \leq i \leq \frac{n}{2} - 1\right\} \cup \{x_{i+1}u_{2i+1}: 1 \leq i \leq \frac{n}{2} - 1\} \cup \{y_iu_{2i}: 1 \leq i \leq \frac{n}{2} - 1\}$.

Here $p = \frac{6n-2}{2}$ and $q = 2(2n-3)$.

Define the function f from the vertex set to $\{s, s+d, s+2d \dots . s+(q+1)\}$,
$g: E(G) \rightarrow \{d, 2d, 3d \dots . . 2(q-1)d\}$ to label the edges

3.2 . *e*Table Labeling of vertices $S'(A^2PS_n)$							
Value of i	$f(u_{2i+1})$	$f(u_{2(i+1)})$	$f(w_{2i+1})$	$f(w_{2(i+1)})$	$f(v_{i+1})$	$f(y_{i+1})$	$f(x_{i+1})$
$0 \leq i \leq \frac{n-2}{2}$	$s + 7id$	$s + (7i + 2)d$	$s + (7i + 1)d$	−	−	−	−
$0 \leq i \leq \frac{n}{2} - 2$	−	−	−	$s + (7i + 6)d$	$s + (7i + 4)d$	$s + (7i + 5)d$	$s + (7i + 3)d$

3.2. fTable Labeling of edges $S'(A^2PS_n)$						
Value of i	$g(u_iw_i)$	$g(w_iu_{i+1})$	$g(v_ix_i)$	$g(v_iy_i)$	$g(y_iu_{2i+1})$	$g(x_iu_{2i})$
$1 \leq i \leq n-1$	$2s + 2(q-1)d - (f(u_i) + f(w_i))$	$2s + 2(q-1)d - (f(w_i) + f(u_{i+1}))$	$-$	$-$	$-$	$-$
$1 \leq i \leq \frac{n}{2}$-1	$-$	$-$	$2s + 2(q-1)d - (f(v_i) + f(x_i))$	$2s + 2(q-1)d - (f(v_i) + f(y_i))$	$2s + 2(q-1)d - (f(y_i) + f(u_{2i+1}))$	$2s + 2(q-1)d - (f(x_i) + f(u_{2i}))$

Thus, $S'(A^2PS_n)$ subdivision on path of APS_n when n is even and the first pentogon starts from u_2 and the last ends with u_{n-1} admits (S,d) magic labeling.

Case 4: $S(A^2PS_n)$Subdivision on APS_n when n is even and the first pentogon starts from u_2 and the last ends with u_{n-1} admits (S,d) magic labeling

Let $V(G) = \{u_i: 1 \leq i \leq n\} \cup \{w_i : 1 \leq i \leq n-1\} \cup \{v_i: 1 \leq i \leq \frac{n}{2} - 1\} \cup \{x_i, y_i z_i, s_i r_i, t_i: 1 \leq i \leq \frac{n}{2} - 1\}$and $E(G) = \{u_iw_i: 1 \leq i \leq n-1\} \cup \{w_iu_{i+1}: 1 \leq i \leq n-1\} \cup \left\{x_iv_i: 1 \leq i \leq \frac{n}{2} - 1\right\} \cup \left\{y_iv_i: 1 \leq i \leq \frac{n}{2} - 1\right\} \cup \{r_{i+1}u_{2i+1}: 0 \leq i \leq \frac{n}{2} - 1\} \cup \{t_iu_{2i}: 1 \leq i \leq \frac{n}{2} - 1\} \cup \left\{x_iz_i: 1 \leq i \leq \frac{n}{2} - 1\right\} \cup \left\{y_is_i: 1 \leq i \leq \frac{n}{2} - 1\right\} \cup \left\{z_ir_i: 1 \leq i \leq \frac{n}{2} - 1\right\} \cup \left\{t_is_i: 1 \leq i \leq \frac{n}{2} - 1\right\}$

Here $p = \frac{11n-16}{2}$ and $q = 2(3n-5)$

Define the function f from the vertex set to $\{s, s+d, s+2d \ldots . s+(q+1)\}$,

$g: E(G) \to \{d, 2d, 3d \ldots ..2(q-1)d\ \}$ to label the edges

3.2. gTable Labeling of vertices $S(A^2PS_n)$								
Value of i	$f(u_{2i+1})$	$f(u_{2(i+1)})$	$f(w_{2i+1})$					
$0 \leq i \leq \frac{n}{2} - 1$	$s + 11id$	$s + (11i + 2)d$	$s + (11i + 1)d$	$-$	$-$	$-$	$-$	$-$
Value of i	$f(r_{i+1})$	$f(t_{i+1})$	$f(z_{i+1})$	$f(s_{i+1})$	$f(w_{2(i+1)})$	$f(v_{i+1})$	$f(y_{i+1})$	$f(x_{i+1})$

$0 \le i \le \frac{n}{2} - 2$	$s + (11i + 3)d$	$s + (11i + 9)d$	$s + (11i + 4)d$	$s + (11i + 8)d$	$s + (11i + 10)d$	$s + (11i + 6)d$	$s + (11i + 7)d$	$s + (11i + 5)d$

3.2. hTable Labeling of edges $S(A^2PS_n)$						
Value of i	$g(u_iw_i)$	$g(w_iu_{i+1})$	$g(v_ix_i)$	$g(v_iy_i)$	$g(r_iu_{2i})$	$g(t_iu_{2i+1})$
$1 \le i \le n-1$	$2s + 2(q-1)d - \left(f(u_i) + f(w_i)\right)$	$2s + 2(q-1)d - (f(w_i) + f(u_{i+1}))$	$-$	$-$	$-$	$-$
$0 \le i \le \frac{n}{2} - 1$	$-$	$-$	$2s + 2(q-1)d - \left(f(v_i) + f(x_i)\right)$	$2s + 2(q-1)d - \left(f(v_i) + f(y_i)\right)$	$2s + 2(q-1)d - \left(f(r_i) + f(u_{2i})\right)$	$2s + 2(q-1)d - \left(f(t_i) + f(u_{2i+1})\right)$
Value of i	$g(x_iz_i)$	$g(y_is_i)$	$g(z_ir_i)$	$g(s_it_i)$		
$1 \le i \le \frac{n}{2}$-1	$2s + 2(q-1)d - \left(f(x_i) + f(z_i)\right)$	$2s + 2(q-1)d - \left(f(y_i) + f(s_i)\right)$	$2s + 2(q-1)d - \left(f(z_i) + f(r_i)\right)$	$2s + 2(q-1)d - \left(f(s_i) + f(t_i)\right)$	$-$	$-$

Thus, $S(A^2PS_n)$ subdivision on APS_n when n is even and the first pentogon starts from u_2 and the last ends with u_{n-1} admits (S,d) magic labeling.

Case 5: $S'(A^3PS_n)$ Subdivision on path of (APS_n) when n is odd and the first pentagon starts from u_1 and the last ends with u_{n-1}admits (S,d) magic labeling

Let $V(G) = \{u_i\colon 1 \le i \le n\} \cup \{w_i : 1 \le i \le n\} \cup \{v_i\colon 1 \le i \le n-1\} \cup \{x_i, y_i\colon 1 \le i \le \frac{n-1}{2}\}$and $E(G) = \{u_iw_i\colon 1 \le i \le n-1\} \cup \{w_iu_{i+1}\colon 1 \le i \le n-1\} \cup \left\{x_iv_i\colon 1 \le i \le \frac{n-1}{2}\right\} \cup \left\{y_iv_i\colon 1 \le i \le \frac{n-1}{2}\right\} \cup \{x_{i+1}u_{2i+1}\colon 1 \le i \le \frac{n-1}{2}\} \cup \{y_iu_{2i}\colon 1 \le i \le \frac{n-1}{2}\}$

Here $p = \frac{7n-5}{2}$ and $q = 4(n-1)$

Define the function f from the vertex set to $\{s, s+d, s+2d \ldots . s+(q+1)\}$,

$g\colon E(G) \to \{d, 2d, 3d \ldots . . 2(q-1)d\ \}$ to label the edges

3.2 . *i*Table Labeling of vertices $S'(A^3PS_n)$							
Value of i	$f(u_{2i+1})$	$f(u_{2(i+1)})$	$f(w_{2i+1})$	$f(w_{2(i+1)})$	$f(v_{i+1})$	$f(y_{i+1})$	$f(x_{i+1})$
$0 \leq i \leq \frac{n-1}{2}$	$s+7id$	$-$	$-$	$-$	$-$	$-$	$-$
$1 \leq i \leq \frac{n-1}{2}$	$-$	$-$	$-$	$-$	$s+(7i+2)d$	$s+(7i+3)d$	$s+(7i+1)d$
$0 \leq i \leq \frac{n-3}{2}$	$-$	$s+(7i+5)d$	$s+(7i+4)d$	$s+(7i+6)d$	$-$	$-$	$-$

3.2. *j*Table Labeling of edges $S'(A^3PS_n)$						
Value of i	$g(u_iw_i)$	$g(w_iu_{i+1})$	$g(v_ix_i)$	$g(v_iy_i)$	$g(y_iu_{2i})$	$g(x_{i+1}u_{2i+1})$
$1 \leq i \leq n-1$	$2s+2(q-1)d-\left(f(u_i)+f(w_i)\right)$	$2s+2(q-1)d-(f(w_i)+f(u_{i+1}))$	$-$	$-$	$-$	$-$
$0 \leq i \leq \frac{n-3}{2}$	$-$	$-$	$-$	$-$	$-$	$2s+2(q-1)d-(f(x_{i+1})+f(u_{2i+1}))$
$1 \leq i \leq \frac{n-1}{2}$	$-$	$-$	$2s+2(q-1)d-\left(f(v_i)+f(x_i)\right)$	$2s+2(q-1)d-\left(f(v_i)+f(y_i)\right)$	$2s+2(q-1)d-\left(f(y_i)+f(u_{2i})\right)$	$-$

Thus, $S'(A^3PS_n)$ subdivision on (APS_n) when n is odd and the first pentagon starts from u_1 and the last ends with u_{n-1} admits (S,d) magic labeling.

Case 6

$S\ (A^3PS_n)$ Subdivision of (APS_n) when n is odd and the first pentagon starts from u_1 and the last ends with u_{n-1}admits $(S$,d) magic labeling

Let $V(G) = \{u_i: 1 \le i \le n\} \cup \{w_i : 1 \le i \le n\} \cup \{v_i: 1 \le i \le n-1\} \cup \{x_i, y_i z_i, s_i r_i, t_i: 1 \le i \le \frac{n-1}{2}\}$and $E(G) = \{u_i w_i: 1 \le i \le n-1\} \cup \{w_i u_{i+1}: 1 \le i \le n-1\} \cup \left\{x_i v_i: 1 \le i \le \frac{n-1}{2}\right\} \cup \left\{y_i v_i: 1 \le i \le \frac{n-1}{2}\right\} \cup \{r_{i+1} u_{2i+1}: 0 \le i \le \frac{n-3}{2}\} \cup \{t_i u_{2i}: 1 \le i \le \frac{n-1}{2}\} \cup \left\{x_i z_i: 1 \le i \le \frac{n-1}{2}\right\} \cup \left\{y_i s_i: 1 \le i \le \frac{n-1}{2}\right\} \cup \left\{z_i r_i: 1 \le i \le \frac{n-1}{2}\right\} \cup \left\{t_i s_i: 1 \le i \le \frac{n-1}{2}\right\}$

Here $p = \frac{11n-9}{2}$ and $q = 6(n-1)$

Define the function f from the vertex set to $\{s, s+d, s+2d \dots . s+(q+1)\}$,
$g: E(G) \to \{d, 2d, 3d \dots . . 2(q-1)d\ \ \}$ to label the edges

3.2. kTable Labeling of vertices $S\ (A^3PS_n)$							
Value of i	$f(u_{2i+1})$	$f(u_{2(i+1)})$	$f(w_{2i+1})$	$f(w_{2(i+1)})$	$f(v_{i+1})$	$f(y_{i+1})$	$f(x_{i+1})$
$0 \le i \le \frac{n-1}{2}$	$s+11id$	–	–	–	–	–	–
$0 \le i \le \frac{n-3}{2}$	–	$s+(11i+5)d$	$s+(11i+8)d$	$s+(11i+10)d$	$s+(11i+4)d$	$s+(11i+5)d$	$s+(11i+3)d$
Value of i	$f(r_{i+1})$	$f(t_{i+1})$	$f(z_{i+1})$	$f(s_{i+1})$	–	–	–
$0 \le i \le \frac{n-3}{2}$	$s+(11i+1)d$	$s+(11i+7)d$	$s+(11i+2)d$	$s+(11i+6)d$	–	–	–

3.2 . lTable Labeling of edges $S\ (A^3PS_n)$						
Value of i	$g(u_i w_i)$	$g(w_i u_{i+1})$	$g(v_i x_i)$	$g(v_i y_i)$	$g(r_{i+1} u_{2i+1})$	$g(t_i u_{2i})$
$1 \le i \le n-1$	$2s+2(q-1)d-(f(u_i)+f(w_i))$	$2s+2(q-1)d-(f(w_i)+f(u_{i+1}))$	–	–	–	–

$0 \le i \le \frac{n-3}{2}$	$-$	$-$	$-$	$-$	$2s + 2(q-1)d - \left(f(r_{i+1}) + f(u_{2i+1})\right)$	$-$
$1 \le i \le \frac{n-1}{2}$	$-$	$-$	$2s + 2(q-1)d - \left(f(v_i) + f(x_i)\right)$	$2s + 2(q-1)d - \left(f(v_i) + f(y_i)\right)$	$-$	$2s + 2(q-1)d - \left(f(t_i) + f(u_{2i})\right)$
Value of i	$g(x_i z_i)$	$g(y_i s_i)$	$g(z_i r_i)$	$g(s_i t_i)$	$-$	$-$
$1 \le i \le \frac{n-1}{2}$	$2s + 2(q-1)d - \left(f(x_i) + f(z_i)\right)$	$2s + 2(q-1)d - \left(f(y_i) + f(s_i)\right)$	$2s + 2(q-1)d - \left(f(z_i) + f(r_i)\right)$	$2s + 2(q-1)d - \left(f(s_i) + f(t_i)\right)$	$-$	$-$

Thus, $S\,(A^3PS_n)$ subdivision on (APS_n) when n is odd and the first pentagon starts from u_1 and the last ends with u_{n-1} admits (S,d) magic labeling.

Theorem 3.3 Subdivision of the Quadrilateral snake graph admits (s, d) magic labeling.

Proof: Case 1:

Let G $=S'(Q_n)$be the graph obtained from a Quadrilateral snake graph by subdividing only the edges on the main path of the Quadrilateral snake graph.

$V(G) = \{v_{i,}: 1 \le i \le n, y_{j:} 1 \le j \le n-1, u_{i+1}: 0 \le i \le 2n-3\}$ and

$E(G) = \{(v_i y_{i)}, (y_i v_{i+1})(u_{2i} v_{i+1}): 1 \le i \le\ n-1\} \cup \{(u_{2i+1} v_{i+1}), (u_{2i+1} u_{2(i+1)}): 0 \le i \le n-2\}\}$.
Here $p = 4n - 3$ and $q = 5(n-1)$

Define the function f from the vertex set to $\{s, s+d, s+2d \ldots . s+(q+1)\}$,
$g: E(G) \to \{d, 2d, 3d \ldots . . 2(q-1)d\ \}$ to label the edges

3.3. a Labeling of vertices Quadrilateral snake graph $S'(Q_n)$			
Value of i	$f(v_{i+1})$	$f(y_{i+1})$	$f(u_{i+1})$
$0 \le i \le n-1$	$s + 4id$	$-$	$-$
$0 \le i \le n-2$	$-$	$s + 2(2i+1)d$	$-$
$0 \le i \le 2n-3$	$-$	$-$	$s + (2i+1)d$

3.3. b Labeling of Edges of Quadrilateral snake graph $S'(Q_n)$					
Value of i	$g(v_iy_i)$	$g(y_iv_{i+1})$	$g(u_{2i}v_{i+1})$	$g(u_{2i+1}v_{i+1})$	$g\left(u_{2i+1}u_{2(i+1)}\right)$
$0 \leq i \leq \mathrm{n} - 2$	$-$	$-$	$-$	$2s + 2(\mathrm{q}-1)d - (f(u_{2i+1}) + f(v_{i+1}))$	$2s + 2(\mathrm{q}-1)d - (f(u_{2i+1}) + f\left(u_{2(i+1)}\right)$
$1 \leq i \leq \mathrm{n} - 1$	$2s + 2(\mathrm{q} - 1)d - (f(v_i) + f(y_i))$	$2s + 2(\mathrm{q}-1)d - (f(y_i) + f(v_{i+1}))$	$2s + 2(\mathrm{q}-1)d - (f(u_{2i}) + f(v_i))$	$-$	$-$

Thus, Quadrilateral snake graph by subdividing only the edges on the main path of the Quadrilateral snake graph $S'(Q_n)$ admits (s, d) magic labeling.

Case 2:

The subdivision on all edges of quadrilateral snake graph admits (s, d) magic labeling.

Let G = S (Q_n) be the subdivision graph of all the edges of quadrilateral snake graph Q_n

Now $V(G) = \{v_i: 1 \leq i \leq n \cup y_j, w_j: 1 \leq j \leq n-1 \cup x_i: 1 \leq i \leq 2(n-1)\}$

$E(G) = \{(v_iy_i), (y_iv_{i+1}), (v_ix_{2i-1}), (v_{i+1}x_{2i}), (w_iu_{2i-1}), (w_iu_{2i}): 1 \leq i \leq n-1 \cup (x_iu_i): 1 \leq i \leq 2(n-1)\}$

Here $p = 7n - 6$ and $q = 8(n-1)$

Define the function f from the vertex set to $\{s, s+d, s+2d \ldots . s+(q+1)\}$,

$g: E(G) \rightarrow \{d, 2d, 3d \ldots . . 2(q-1)d \}$ to label the edges

3.3. c Labeling of vertices Quadrilateral snake graph $S(Q_n)$							
$f(v_n) = f(v_{n-1)} + 5d$							
$f\left(x_{2(n-1)}\right) = f(V_n) + d$							
Value of i	$f(v_{i+1})$	$f(y_{i+1})$	$f(w_{i+1})$	$f(x_{2i-1})$	$f(x_{2i})$	$f(u_{2i-1})$	$f(u_{2i})$
$0 \leq i \leq n-2$	$s + (7i + 1)d$	$s + 7id$	$s + (7i + 4)d$	$-$	$-$	$-$	$-$
$1 \leq i \leq n-1$	$-$	$-$	$-$	$s + (7i - 5)d$	$-$	$s + (7i - 4)d$	$s + (7i - 2)d$
$1 \leq i \leq n-2$	$-$	$-$	$-$	$-$	$s + (7i - 1)d$	$-$	$-$

3.3. d Labeling of vertices Quadrilateral snake graph $S(Q_n)$							
Value of i	$g(v_i y_i)$	$g(y_i v_{i+1})$	$g(v_i x_{2i-1})$	$g(v_{i+1} x_{2i})$	$g(w_i u_{2i-1})$	$g(w_i u_{2i})$	$g(x_i u_i)$
$1 \leq i \leq n-1$	$2s + 2(q-1)d - (f(v_i) + f(y_i))$	$2s + 2(q-1)d - (f(y_i) + f(v_{i+1}))$	$2s + 2(q-1)d - (f(v_i) + f(x_{2i-1}))$	$2s + 2(q-1)d - (f(x_{2i}) + f(v_{i+1}))$	$2s + 2(q-1)d - (f(w_i) + f(u_{2i-1}))$	$2s + 2(q-1)d - (f(w_i) + f(u_{2i}))$	$-$
$1 \leq i \leq 2(n-1)$	$-$	$-$	$-$	$-$	$-$	$-$	$2s + 2(q-1)d - (f(x_i) + f(u_i))$

Thus the subdivision graph of the Quadrilateral snake graph $S(Q_n)$ admits (s,d) magic labelling.

Example 3.3.b Subdivision of quadrilateral snake graph $S(Q_6)$

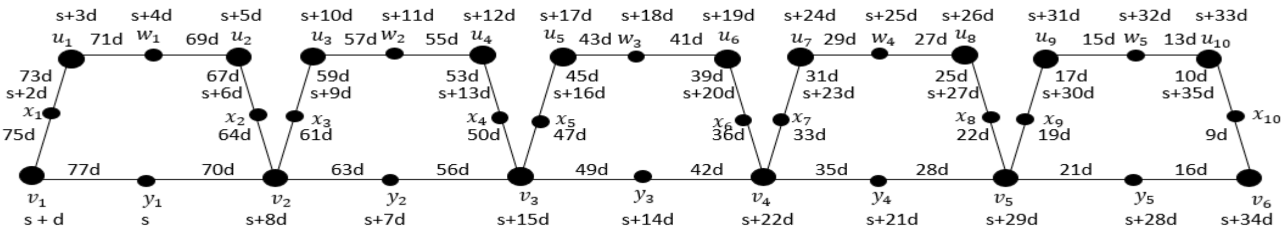

Figure 3.3. b Subdivision of quadrilateral snake graph $S(Q_6)$

4. Conclusion:

In this study, a (s,d) Magic Labeling has been discovered for a few graphs such as Subdivision on pentagonal snake graph, Alternate pentagonal snake graph and Quadrilateral snake graph Future research will examine the (s,d) Magic labeling of additional graphs and some graph families.

5. References

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