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Signed Product Cordial Labeling with Some New Graphs

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ABSTRACT:
In this paper , we investigate about the concept of signed product cordial labelling. Also, we introduced some new types of graph like Fan graph (f_n), Dual fan (Df_n) ,Corona product, Degree corona product, Bispl(G) and $DS'(G)$. Finally we conclude these all graphs admits signed product cordial labeling.

1. INDRODUCTION:

The origin of graph labeling can be attributed to Rosa. E.Sampthkumar [1,2] introduced the concept of splitting graph and duplicate graph. Gallian provide the literature on survey of different types of graph labeling. The idea of signed product cordial labeling was introduced by J.BaskarBabujee [4] and he proved that many graphs admits signed product cordial labeling. R.Vikrama Prasad, R.Dhavaseelan and S.Abhirami [5] have proved the splitting graphs on even sum cordial labeling of graphs. P.Lawrence Rozario Raj and S.Koilraj [3] have proved the cordial labeling for the splitting graph of some standard graphs

2. PRELIMINARIES:

2.1 SIGNED PRODUCT CORDIAL LABELING

A vertex labeling of graph $G = f: V(G) \rightarrow \{-1,1\}$ with induced edge labeling $f^* : E(G) \rightarrow \{-1,1\}$ defined by $f^*(uv) = f(u).f(v)$ is called a signed Product cordial labeling if $|v_f(-1) - v_f(1)| \leq 1$ and $|e_{f^*}(-1) - e_{f^*}(1)| \leq 1$, where $v_f(-1)$ is the number of vertices labeled with '-1', $v_f(1)$ is the number of vertices labeled with '1', $e_{f^*}(-1)$ is the number of edges labeled with '-1' and $e_{f^*}(1)$is the number of edges labeled with '1'.

2.2 FAN GRAPH (F_n)

The Fan f_n , $n \geq 2$, is obtained by joining all vertices of P_n to a further vertex called the centre and contains n+1 vertex and 2n-1 edges. i.e., $f_n=P_n+K_1$

2.3 DUAL FAN GRAPH (Df_n)

The Dual Fan Df_n consists of two Fan graph that have a common path.
In other words $DF_n = P_n + K_2$

2.4 CORONA PRODUCT ($G_1 \odot G_2$)

The corona $G_1 \odot G_2$ of two graphs G_1 (with n_1 vertices and m_1 edges) and G_2 (with n_2 vertices and m_2 edges) is defined as the graph obtained by taking one copy of G_1 and n_1 copies of G_2, and then joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.

2.5 DEGREE CORONA PRODUCT ($G \odot_d H$)

Let G and H be two graphs. The degree corona product ($G \odot_d H$) is obtained by taking one copy of G and $|v(G)|$ copies of H, and by joining equal degree of the i^{th} copy of H to the i^{th} vertex of G.

2.6 DUAL SPILT GRAPH (DS'(G))

For a graph G , the Dual spilt graph **DS'(G)** of a graph G is obtained by adding two new vertices $v'v''$ corresponding to each vertex $v(G)$.

2.7 DUAL FRIENDSHIP GRAPH (DF_n)

The Dual friendship graph ($\boldsymbol{DF_n}$) which consists of $2n$ copies of cycle C_3 by joining u and v..

2.8 BISPLIT GRAPH (BSPL(G))

Let G and G'' be two graphs each vertex from G(V(G)) joining each vertex of V(G'), we obtain the graph bisplit denoted by Bspl(G).

3. MAIN RESULTS:

THEOREM 3.1

STATEMENT

The fan graph F_n admits a signed product cordial labeling graph

PROOF

Let us consider the path $P_n(n \geq 2)$ and joining all the vertices of path P_n to a further vertex.
We obtain a new graph denoted by Fan graph.

Let $f : V(G) \rightarrow \{-1,1\}$ and $f^*(uv) = \begin{cases} -1 & if\ u\ and\ v\ \ have\ different\ sign \\ 1 & if\ u\ and\ v\ \ have\ same\ sign \end{cases}$

as follows

Case(i): when n is even

$v_f(-1)=(n+2)/2$; $v_f(1)=n/2$;

$e_{f^*}(-1)=n$; $e_{f^*}(1)=n-1$;

Case(ii): when n is odd

$v_f(-1) = v_f(1) = (n+1)/2$;

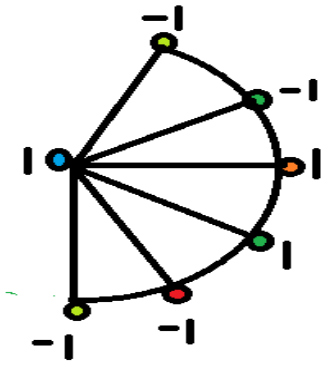
$e_{f^*}(-1)=n$; $e_{f^*}(1)=n-1$;

By the above labeling pattern, we have

$|v_f(-1) - v_f(1)| \leq 1$ $|e_{f^*}(-1)- e_{f^*}(1)| \leq 1$

Therefore, F_n admits signed product cordial labeling graph.

EXAMPLE: f_3 (when n is odd) is a signed product cordial labeling graph.



THEOREM 3.2

STATEMENT

DF_n admits signed product cordial labeling graph.

PROOF:

let us consider the paths P_n and $P_n{}'$, $(n \geq 2)$ and joining all vertex of P_n and $P_n{}'$ to a further vertex. Two paths P_n and $P_n{}'$ with n vertices connecting by a new edge 'e' with its center.

we obtain a new graph denoted Dual Fan DF_n .

let f:V(G) $\rightarrow$ {-1,1} and

$$f^*(uv) = \begin{cases} -1 & if\ u\ and\ v\ \ have\ different\ sign \\ 1 & if\ u\ and\ v\ \ have\ same\ sign \end{cases}$$

as follows

$v_f(-1) - \frac{n+2}{2} = v_f(1)/$

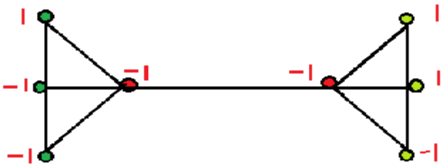
$e_{f^*}(-1)=n$; $e_{f^*}(1)= n-1$;

By the above labeling pattern ,we have

$|v_f(-1) - v_f(1)| \leq 1$ $|e_{f^*}(-1)- e_{f^*}(1)| \leq 1$

Therefore, Df_n is a signed product cordial labeling graph.

EXAMPLE: Df_6 (when n=6) is a signed product cordial labeling graph.



THEOREM 3.3

$G = (\, p_n \odot p_n{}'), (n \geq 2)$ admits signed product cordial labeling graph**.**

PROOF

Let P_n be a path with n vertices obtained by joining n copies of $p_n{}'$.

We consider the path P_n and joining each vertex of the i[th] copy to $p_n{}'$ to the i^{th} vertex of P_n, where $1 \leq i \leq |V(pi)|$. Then we obtained $G = (\, p_n \odot p_n{}'), (n \geq 2)$

The total number of vertices in this graph G , $V(G) = n(n+1)$.

The number of edges in G is , $E(G) = 2n^2 - 1$.

Define $f : V(G) \longrightarrow \{-1,1\}$

$$f^*(uv) = \begin{cases} -1 & if\ u\ and\ v\ \ have\ different\ sign \\ 1 & if\ u\ and\ v\ \ have\ same\ sign \end{cases}$$

By the above labelling pattern, we obtain

$$V_f(-1) = V_f(-1) = \frac{n(n+1)}{2}$$

$$e_{f^*}(-1) = n^2 \ \ e_{f^*}(1) = n^2 - 1$$

By the above labeling pattern , we have

$|v_f(-1) - v_f(1)| \leq 1$ $|e_{f^*}(-1)- e_{f^*}(1)| \leq 1$

Hence, $G = (\, p_n \odot p_n{}')$ admits signed product cordial labeling graph.

THEOREM 3. 4

STATEMENT

$G = (\, C_n \odot P_n), n \geq 3$ admits signed product cordial labeling graph.

PROOF

Let $C_n$be a cycle with n vertices obtained by joining n copies of P_n.

We consider the cycle C_n and by joining each vertex of the i^{th} copy to P_n to the i^{th} vertex of C_n, where $1 \leq i \leq |V(Ci)|$

Then we obtained $G = (C_n \odot P_n)$..

By the above graph the number of vertices $V(G) = n(n+1)$ and the number of edges of G is $E(G) = 2n^2$

Define f:V(G)→{-1,1} and

$$f^*(uv) = \begin{cases} -1 & if\ u\ and\ v\ have\ different\ sign \\ 1 & if\ u\ and\ v\ have\ same\ sign \end{cases}$$

By the above labelling pattern, we obtain

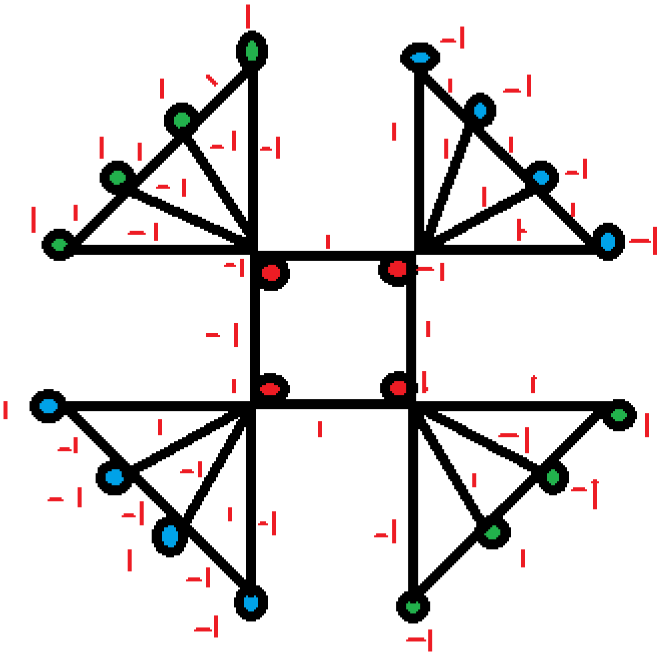
$$V_f(-1) = V_f(-1) = \frac{n(n+1)}{2}$$

$$e_{f^*}(-1) = n^2 \ \ e_{f^*}(1) = n^2$$

Finally , we obtain $|v_f(-1) - v_f(1)| \leq 1$ $|e_{f^*}(-1) - e_{f^*}(1)| \leq 1$

Hence, $G = (C_n \odot P_n)$ admits a signed product cordial labeling graph

EXAMPLE: $(C_4 \odot P_4)$ is a signed product cordial labeling graph



THEOREM 3.5

$G = (C_n \odot C_n')$ $n \geq 3$ admits signed product cordial labelling graph.

PROOF:

Let C_n be a cycle with n vertices obtained by joining n copies of C_n'.

We consider the cycle C_n and by joining each vertex of the i^{th} copy to C_n'.to the i^{th} vertex of C_n, where $1 \leq i \leq |V(C_i)|$

Then we obtained $G = (C_n \odot C_n')$

By the above graph the number of vertices $V(G) = n(n+1)$ and the number of edges of G is $E(G) = n(2n+1)$

Define f:V(G)→{-1,1} and

$$f^*(uv) = \begin{cases} -1 & if\ u\ and\ v\ have\ different\ sign \\ 1 & if\ u\ and\ v\ have\ same\ sign \end{cases}$$

By the above labelling pattern, we obtain

Case (i) : When n is odd

$$V_f(-1) = V_f(-1) = \frac{n(n+1)}{2}$$

$$e_{f^*}(-1) = \frac{2n^2+n+1}{2} \quad e_{f^*}(1) = \frac{2n^2+n-1}{2}$$

Finally , we obtain $|V_f(-1)-v_f(1)| \leq 1$ and $|e_{f^*}(-1)-e_{f^*}(1)| \leq 1$

Case(ii); when n is even

$$V_f(-1) = V_f(-1) = \frac{n(n+1)}{2}$$

$$e_{f^*}(-1) = e_{f^*}(1) = \frac{2n^2+n}{2}$$

By the above labeling pattern , we have

Finally , we obtain $|v_f(-1) - v_f(1)| \leq 1$ $|e_{f^*}(-1)- e_{f^*}(1)| \leq 1$

Hence, $G = (C_n \odot C_n{}')$ is a signed product cordial labeling graph

THEOREM 3.6

$G = (P_n \odot C_n)$, $n \geq 3$ admits signed product cordial labeling graph.

PROOF:

Let P_n be a path with n vertices obtained by joining n copies of C_n.

We consider the path P_n and by joining each vertex of the I^{th} copy to C_n to the i^{th} vertex of P_n, where $1 \leq i \leq |V(Pi)|$. Then we obtained $G = (P_n \odot C_n)$

By using corona product, The number of vertices in G is $n(n+1)$ and the number of edges $V = 2n^2 + n - 1$.

Define $f: V(G) \rightarrow \{-1,1\}$ and

$$f^*(uv) = \begin{cases} -1 & if\ u\ and\ v\ have\ different\ sign \\ 1 & if\ u\ and\ v\ have\ same\ sign \end{cases}$$

case(i); when n is odd

$$V_f(-1) = V_f(-1) = \frac{n(n+1)}{2}$$

$$e_{f^*}(-1) = e_{f^*}(1) = \frac{2n^2+n-1}{2}$$

case (ii) when n is even

$$V_f(-1) = V_f(-1) = \frac{n(n+1)}{2}$$

$$e_{f^*}(-1) = \frac{2n^2+n}{2}$$

$$e_{f^*}(1) == \frac{2n^2+n-2}{2}$$

By the above labeling pattern, we have

Finally , we obtain $|v_f(-1) - v_f(1)| \leq 1$ $|e_{f^*}(-1)- e_{f^*}(1)| \leq 1$

Hence, $G = (P_n \odot C_n)$ is admits signed product cordial labeling graph

EXAMPLE: (($P_3 \odot C_3$) is a signed product cordial labeling graph.

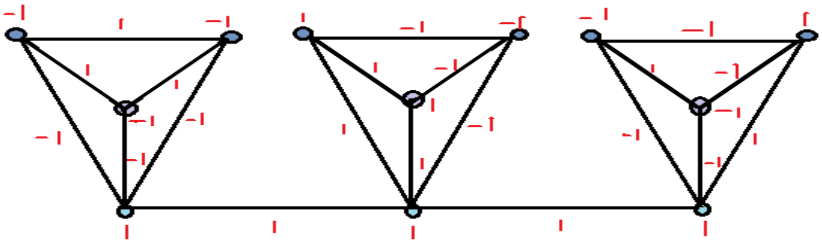


Fig 6

THEOREM 3.7

Dual split of path $P_n, (n \geq 2)$[Duspl (G)] is signed product cordial labeling graph.

PROOF:

let us consider the path $P_n, (n \geq 2)$ and each vertex v of a graph P_n' take a new vertex v' and v''.
Join v' and v'' to all vertices of P_n adjacent to v.
we obtain a new graph denoted Dual split as Duspl (G).
Define f:v(G)→{-1,1]} and

$$f^*(uv) = \begin{cases} -1 & if\ u\ and\ v\ \ have\ different\ sign \\ \ \ 1 & if\ u\ and\ v\ \ have\ same\ sign \end{cases}$$

Case(i): when n is even

$$V_f(-1) = V_f(-1) = \frac{3n}{2}$$

$$e_{f^*}(-1) = \frac{5n-4}{2}$$

$$e_{f^*}(1) = \frac{5n-6}{2}$$

Case(ii) when n is odd

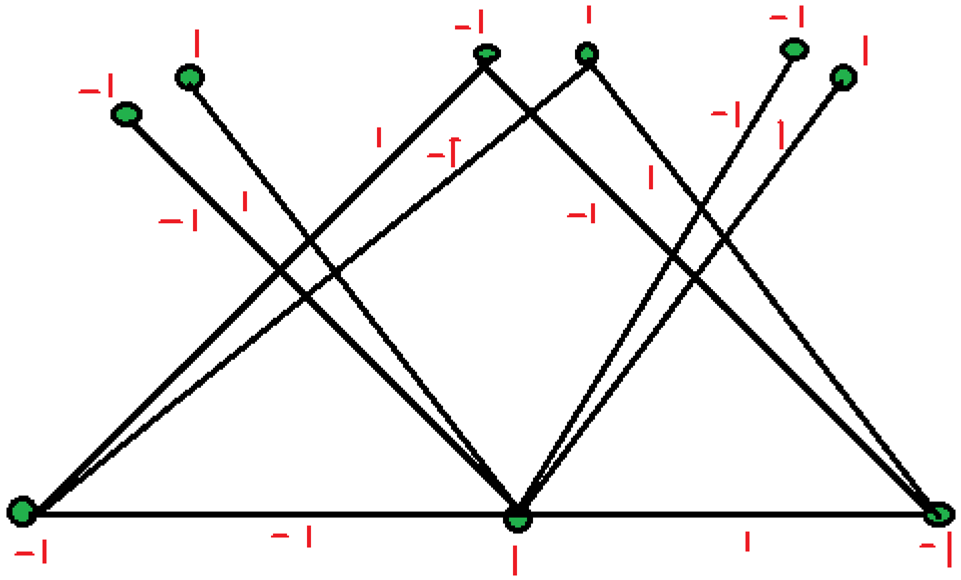
$$V_f(-1) = V_f(1) = \frac{3n-1}{2}$$

$$V_f(1) = \frac{3n+1}{2}$$

$$e_{f^*}(-1) = e_{f^*}(1) = \frac{5(n-1)}{2}$$

By the above labeling pattern, we have
$|v_f(-1) - v_f(1)| \leq 1$ $|e_{f^*}$ (-1)- $e_{f^*}(1)| \leq 1$
Therefore, Duspl (G) is signed product cordial labeling graph.

EXAMPLE:



Conclusion

In this paper, we proved that signed product cordial labeling of corona product $(G_1 \odot G_2)$, $(G_1 \odot_d G_2)$,Bispl(G), Duspl(G), f_n, $D(f_n)$ and Dual friendship graph D(F_n).Through this work motivated to present more articles relevant to different types of labeling in future.

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