International Journal for Multidisciplinary Research (IJFMR)

E-ISSN: 2582-2160 ● Website: www.ijfmr.com ● Email: editor@ijfmr.com

Odd Vertex Magic Labeling of Cyclic and Path Graphs

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Abstract

In this paper we discussed about connected and simple graphs only which is also simple and undirected. The usual notations of a Vertex set and Edge set are $V(G), E(G)$ in orderly of the graph G. Also $|V(G)|, |E(G)|$ means that the number of vertices and number of edges of the graph. Then k_0, k_e are odd vertex magic constants of cyclic graphs when p is odd and even respectively. The odd vertex magic constants k_{p_0}, k_{p_e} are for path graphs when p is odd and even respectively.

1. Introduction

All graphs in this paper are simple and connected. Also we discussed about the Odd-Vertex Magic Labeling and Vertex Magic Constants. We discussed the cyclic and path graphs are odd vertex magic labeling graphs. Here the odd vertex magic constants are differ when it is maximum. When it is value is minimum are briefly discussed in this paper [when p is odd then the odd vertex magic constant $k_0 = 3p + 2$ when G is cyclic and $k_{p_0} = 3p - 2$ when G is a path. Also we discussed the odd vertex magic constants how to find when p is even and p is odd. We use to general method of odd vertex magic labeling by symmetric group. If G is cyclic with p-points, then the odd vertex magic labeling $k_0 = 3p + 2$ when p is odd and $k_e = 3p + 3$ when p is even. Also G is a path graph with p-points then the odd vertex magic constant $k_{p_0} = 3p - 2$ when p is odd and $k_{p_e} = 3p - 1$ when p is even.

2. PRELIMINARY

Definition 2.1 (Labeled Graph)

A graph G is called Labeled if its p-points are distinguished from one another by names such as $v_1, v_2, \dots, v_p$.

Definition 2.2 (Cyclic Graph)

A Cyclic Graph is a connected graph whose every vertex is adjacent to two other distinct vertices.

Definition 2.3 (Walk)

A walk of a graph G is an alternating sequence of points and lines $v_0 x_1 x_2 v_2 \dots, x_n v_n$ .... beginning and ending with points such that each line x_i is incident with v_{i-1} and v_i.

Definition 2.4 (Path)

A walk is called a path if all its points and distinct.

Definition 2.5 (Symmetric Group)

The symmetric group s_n is the group of permutations on n objects. Usually the objects are labeled $\{1,2, \dots, n\}$ and elements of s_n are given by bijective functions.

Definition 2.6 (Transposition)
Transposition is a permutation that exchanges two elements and leaves all others fixed.
Definition 2.7 (Vertex-Magic Graph)
If a graph G with p-vertices and q-edges is labeled with the numbers $\{1,2,3,\dots,p+q\}$ such that every vertex and its incident edges adds up to same numbers. Then G is a vertex- magic graph. The same number K is called vertex-magic number.
Definition 2.8 (Odd-Vertex Magic Graph)
If a graph $G=(p,q)$ the p-vertices of the graph G are labeled with the numbers $\{1,3,\dots 2p-1\}$ and the q-edges are labeled with the numbers $\{2,4,6,\dots,2q\}$ such that every vertex and it's incident edges adds upto same number then G is called a Odd-Vertex Magic Graph. And the same number K is called Odd-vertex Magic Constant.

3. Main Results

In this section, we discuss the graph of an odd vertex magic constant and general method of the odd-vertex magic labeling of cyclic graph and path graph

3.1 Finding Odd-Vertex Magic Constant of a Graph

Let G be a (p,q) graph and f be an odd vertex magic Labeling of G. Let $V(G)=\{v_1,v_2,\dots,v_p\}$. A symmetric martic $\chi_A=(a_{ij})$, where $i,j=1,2,\dots,p$ is called a label adjacency matric of G.
if $(a_{ij})=f(v_iv_j)$ by the definition of Odd-Vertex Magic Labeling

$$K=f(v_1)+a_{12}+a_{13}+\cdots+a_{1p}$$
$$K=f(v_2)+a_{21}+a_{23}+\cdots+a_{2p}$$
$$\vdots$$
$$K=f(v_p)+a_{p1}+a_{p2}+\cdots+a_{p-1p}$$

The sum of all entries in row i together with $f(v_i)$ will be equal to magic constant k where $f(v_i)$ means that the vertex v_i is labeled with numbers for each $i=1,2,\dots,p$ we have

$$ps=[f(v_1)+f(v_2)+\cdots+f(v_p)]+2[a_{12}+a_{13}+\cdots+a_{p-1p}]$$

Where $f(v_1)+f(v_2)+\cdots+f(v_p)$ are vertex label and $a_{12}+a_{13}+\cdots+a_{p-1p}$ are edge label of the graph G.

The vertices are labeled with the number $\{1,3,5,\dots 2p-1\}$ and the edges are labeled with number $\{2,4,6,\dots 2q\}$
Case (i)

$$ps=\{1+3+5+\cdots+2p-1\}+2\{2+4+6+\cdots+2q\}$$
$$=p^2+2[q(q+1)]$$
$$=p^2+2q^2+2q \qquad (3.1)$$
$$s=\frac{1}{p}[p^2+2q^2+2q].$$

Theorem 3.1 $G=(p,q)$ be a cyclic graph with p-point and q-lines then the odd-vertex magic constant $K_0=3p+2$
Proof: Given $G=(p,q)$ be cyclic graph. We know that the odd-vertex magic constant
$s=\frac{1}{p}[p^2+2q^2+2q]. \qquad (3.2)$

Given graph G is cyclic then we know that $p = q$ (3.3)
From (3.2) and (3.3)

$$k_o = 3p + 2.$$

Note 3.2: The odd-vertex magic constant of cyclic graph $G = (p, q)$ is $k = 3p + 2$ is exist only p is odd because by the definition of odd-vertex magic labeling. Vertices are labeled with odd numbers and two incidents edges are labeled with even numbers.

So, we get an odd number, i.e., the odd-vertex magic constant of cyclic graph is an odd-number. So, $3p + 2$ is an odd number. where p is an odd.

Theorem 3.3: The odd vertex magic constant of a cyclic graph with p-points and p is even then the odd-vertex magic constant

$$k_e = 3p + 3.$$

Proof: Given G is cyclic graph with p-points and also p is even then, we get the magic constant k_e, We eliminate an odd number vertex label such that the vertex sum is equal to edge sum. Add p to the vertex sum. We get the odd-vertex magic constant

$$pk_e = p^2 + p + 2q^2 + 2q$$

$$k_e = \frac{p^2 + p + 2p^2 + 2p}{p}$$

$$k_e = \frac{1}{p}[3p^2 + 3p]$$

$$k_e = [3p + 3].$$

Examples 3.4

1. G is cyclic graph with 3-points and 3-lines. The odd vertex-magic constant $k_0 = 3p + 2$

$$k_0 = 3(3) + 2 = 11$$

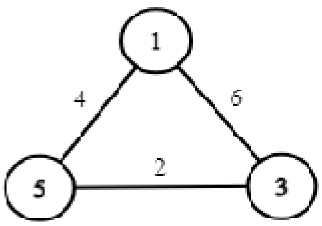

Fig 3.4.1

2. G is cyclic graph with 5-points.
Then odd vertex-magic constant $k_0 = 17$

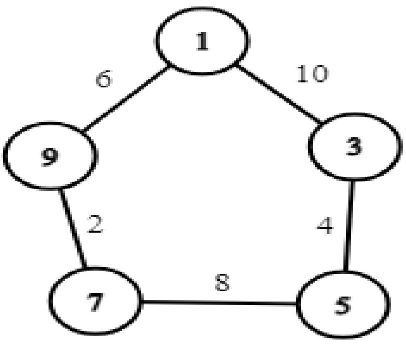

Fig 3.4.2

3. G is cyclic graph with 7-points.
Then odd vertex-magic constant $k_0 = 23$

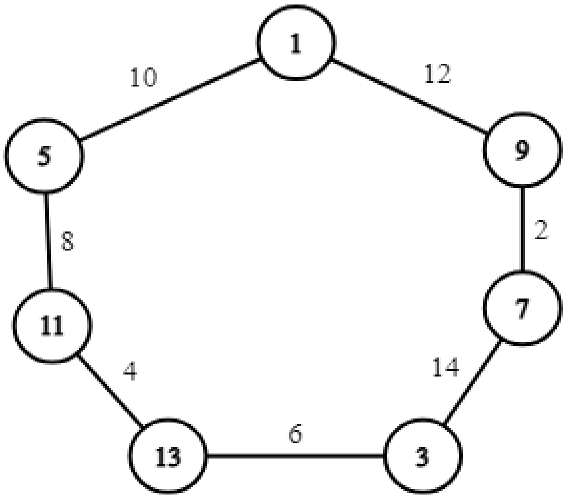

Fig 3.4.3

3.2 The General Method of the Odd-Vertex Magic Labeling of Cyclic Graph

Suppose G is cyclic graph with p-points and also p is odd. The edges are labeled with the number $\{2,4,6,\dots,2q\}$ and vertices are labeled with the numbers $\{1,3,5,\dots,2p-1\}$. We know that the magic constant $k = 3p + 2$. We form a symmetric group of the edge labeling as follows $(2,4,6,\dots 2p)$. First starting the term $\frac{2p}{2} + 1$ is as follows. i.e $\frac{2p}{2} + 3, \frac{2p}{2} + 5, \dots, 2p$. Again we strart $2,4,6,\dots$ after the term $\frac{2p}{2} + 1$ and finished all the term we get a cycle of the symmetric.

The each transposition is incident two edges on a vertex. The first transposition of the vertex is $2p - 1$ and last transposition of the vertex label is $2p - 2$. The second and third …. Are 1,3,5,… So on. We get the odd-vertex magic labeling of cyclic graph as follows

$$\begin{pmatrix} 2 & 4 & \dots & \frac{2p}{2}+1 & \dots & 2p \\ \frac{2p}{2}+1 & \frac{2p}{2}+3 & \dots & 2p & \dots & \frac{2p}{2}-1 \end{pmatrix}$$

The product of transpositions we $\left(2, \frac{2p}{2} + 1\right), \left(\frac{2p}{2} + 1, 2p\right), \dots, \left(\frac{2p}{2} + 3, 2\right)$

The vertex label are $\left(2, \frac{2p}{2} + 1\right)$ is $2p - 1$ and $\left(\frac{2p}{2} + 1, 2\right)$ is $2p - 3$. The second, third … are labeled with the odd numbers $1,3,5,7,\dots$ so on.

Example 3.5

1. Consider the cyclic graph $G = (11,11)$. The odd vertex magic labeling as follows.

$$\begin{pmatrix} 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 & 20 & 22 \\ 12 & 14 & 16 & 18 & 20 & 22 & 2 & 4 & 6 & 8 & 10 \end{pmatrix}$$

The product of transpositions are

Edge Labeling	(2,12)	(12,22)	(22,10)	(10,20)	(20,8)
Vertex Labeling	21	1	3	5	7

Edge Labeling	(8,18)	(18,6)	(6,16)	(16,4)	(4,14)	(14,2)
Vertex Labeling	9	11	13	15	17	19

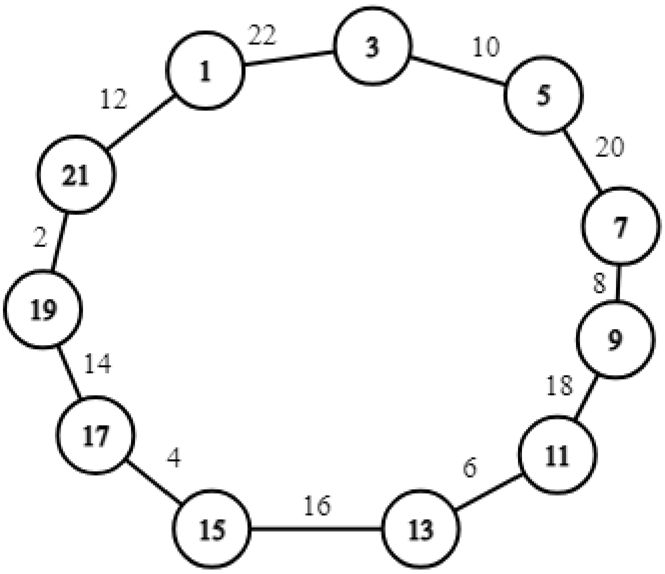

Fig 3.5.1

2. If G is cyclic graph with 4-points then the odd-vertex magic constant

$$K_0 = 3p + 3 = 3(4) + 3 = 15$$
$$V_{sum} = 1 + 3 + 7 + 9$$
$$= 20$$
$$E_{sum} = 2 + 4 + 6 + 8$$
$$= 20$$

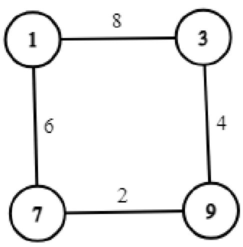

Fig 3.5.2

3. If G is a cyclic graph with 6-points then the odd-vertex magic constant\

$$K_0 = 3p + 3 = 3(6) + 3 = 21$$
$$V_{sum} = 1 + 3 + 5 + 9 + 11 + 13$$
$$= 42$$
$$E_{sum} = 2 + 4 + 6 + 8 + 10 + 12$$
$$= 42$$

Vertex Label are {1,3,5,9,11,13}

Edge Label are {2,4,6,8,10,12}

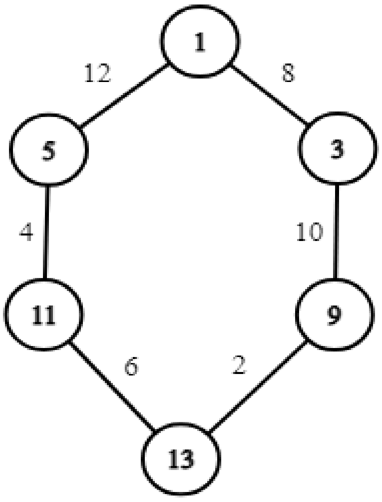

Fig 3.5.3

$K_e = 21$

Theorem 3.6: Suppose G is a path graph with p-point , p is odd and q-lines then the odd vertex magic constant is $K_p = 3p - 2$.

Proof: We know that the odd vertex magic constant of a graph G is

$$K = \frac{p^2 + 2q^2 + 2q}{p}$$

When G is a path graph with p-points. Then the number of lines of the path graph is $q = p - 1$

Now, we get

$$k_{p_0} = \frac{p^2 + 2(p-1)^2 + 2(p-1)}{p}$$

$$= \frac{p^2 + 2(p^2 - 2p + 1) + 2p - 2}{p}$$

$$= \frac{p^2 + 2p^2 - 4p + 2 + 2p - 2}{p}$$

$$= \frac{3p^2 - 2p}{p}$$

$k_{p_0} = 3p - 2.$ (3.4)

The odd vertex –magic constant is $3p - 2$.

Example 3.7

1. Consider the path with three points

$$K_{p_0} = 3p - 2 = 3(3) - 2 = 9 - 2 = 7$$

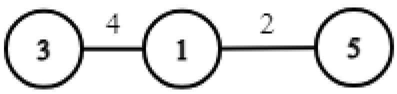

Fig 3.7.1

2. The path graph with 5-points. Then the odd-vertex magic constant

$$K_{p_0} = 3p - 2 = 3(5) - 2 = 15 - 2 = 13$$

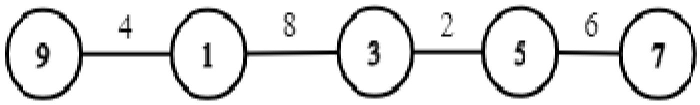

Fig 3.7.2

3.3 The General Method of Odd-Vertex Magic Labeling of path Graph

First we write the odd-numbers sequences as follows $(1,3,5,7,\ldots 2p-1)$. Next we write the relation 1 to 3, 3 to 5, we form a cycle, we eliminate the transposition $(2p-2,2p-1)$ because that point is end point of the path. Next we write the product of the transpositions as follows.$(2p-1,1),(1,3),(3,5),\ldots,(2p-5,2p-3)$,Where $(2p-1,1),(1,3)\ldots$ are edges the edge $(1,3)$ is joint the vertex label 1 and 3.

First we labeled the vertices in order $(2p-1,1,3,5,\ldots,2p-5,2p-3)$.The edges are labeled as follows. The first edge $(2p-1,1)$ is label with the number $p-1$. For the odd-vertex magic constant is $3p-2$, $3p-2-2p+1=p-1$.

The edge $(1,3)$ is labeled with the number $3p-2-p=2p-2$. The edge $(3,5)$ is labeled with the number $2p-2+3=2p+1$. Therefore $3p-2-2p-1=p-3$. And so on. We get the edge label $(p-1,2p-2,p-3,2p-4,p-5,2p-6,\ldots)$

Example 3.8

1. The path with 9-points. Then the odd-vertex magic label is

Vertex Label: $(2p-1,1,3,5,\ldots,2p-5,2p-3,\ldots)$

Where $p=9$, $(17,1,3,5,7,9,11,13,15)$.

Edge Label : $(p-1,2p-2,p-3,2p-4,p-5,2p-6,p-7,2p-8)$

Where $p=9$, $(8,16,6,14,4,12,2,10)$.

The odd vertex magic label of the path graph with 9 points is

$$k_p=3p-2=3(9)-2=25$$

So,

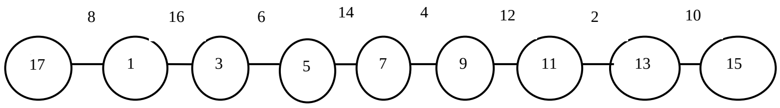

Fig 3.8.1

2. Suppose consider the path with 11-points then the vertex label is

Vertex Label:$(2p-1,1,3,5,7,9,11,\ldots,2p-5,2p-3)$

Where $p=11$, $(21,1,3,5,7,9,11,13,15,17,19)$.

Edge Label: $(p-1,2p-2,p-3,2p-4,p-5,2p-6,p-7,2p-8)$

Where $p=11$, $(10,20,8,18,6,16,4,14,2,12)$.

The odd-vertex magic label of the path graph with 11-points is

$$k_p=3p-2=33-2=31$$

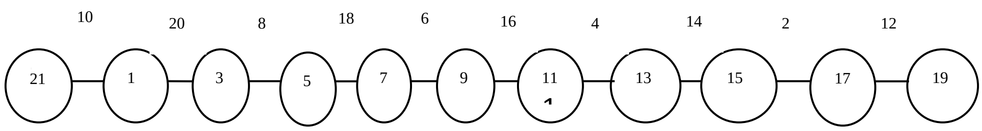

Fig 3.8.2

Theorem 3.9: If the path graph is even number of points then the odd-vertex magic constant

$k_{ep} = 3p - 1$.

Proof: We know that if the path graph having odd number of points then the odd-vertex magic constant is $k_o = 3p - 2$. In this case the magic constant k_0 is odd when p is odd. The magic constant k_0 is even. When p is even.

Now, our assumption p is even, we get the magic constant k_0 is even. But it is not possible in the path graph because by the definition of odd-vertex magic label, every vertex labeled with the odd number and every edge labeled with even numbers. The path graph every edge incident with two vertices are labeled with odd-numbers so we get the odd vertex magic constant is odd but the $k_0 = 3p - 2$. Then the magic constant is even when p is even.

It is not possible. So we equal the vertex sum and edge sum. i.e., Add p to the vertex sum. So we get

$$k_{ep} = \frac{3p^2 - 2p + p}{p} \qquad from\ (3.4)$$

$$k_{ep} = 3p - 1.$$

The odd vertex magic constant of a path graph with even number of points is $k_{ep} = 3p - 1$.

Example 3.10

1. Consider the path graph with four points the odd vertex magic constant

$$k_e = 3p - 1 = 3(4) - 1 = 11$$

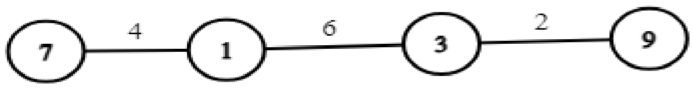

Fig 3.10.1

2. Consider the path graph with 6-points the odd vertex magic constant

$$k_e = 3p - 1 = 3(6) - 1 = 17$$

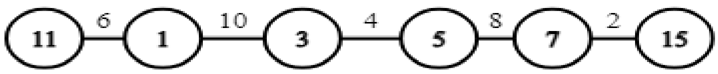

Fig 3.10.2

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