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# **Some Results on Regular Fuzzy Graphs**

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#### Abstract

In this paper, Regular fuzzy graph, Totally Regular fuzzy graph, Partially Regular fuzzy graph, Full Regular fuzzy graph, Edge Regular fuzzy graph, K-totally edge regular fuzzy graph, Odd degree Fuzzy graph, Even degree Fuzzy graph, Cubic Fuzzy graphs are discussed.

Keywords: Regular fuzzy graph, Odd degree Fuzzy graph, Even degree Fuzzy graph, Cubic Fuzzy graph.

#### 1. Introduction

In this paper, we consider only finite, simple, connected graphs. We denote the vertex set and the edge set of a graph G by V(G) and E(G) respectively. The degree of a vertex v is the number of edges incident at v, and it is denoted by d(v). A graph G is regular if all its vertices have the same degree. The notion of fuzzy sets was introduced by Zadeh as a way of representing uncertainity and vagueness [24]. The first definition of fuzzy graph was introduced by Haufmann in 1973. In 1975, A. Rosenfeld introduced the concept of fuzzy graphs [5]. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas. Irregular fuzzy graphs play a central role in combinatorics and theoretical computer science. Nagoorgani and Radha introduced the concept of degree, total degree, regular fuzzy graphs in 2008 [3]. Nagoorgani and Latha introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2012 [4]. N.R.Santhi Maheswari and C.Sekar introduced (2, k) –regular fuzzy graphs and totally (2, k) –regular fuzzy graphs, (r, 2, k) –regular fuzzy graphs, (m, k) – regular fuzzy graphs and (r, m, k) – regular fuzzy graphs [6,10,11,12]. N.R.Santhi Maheswari and C. Sekar introduced 2-neighbourly irregular fuzzy graphs and m –neighbourly irregular fuzzy graphs [17,9]. N.R.Santhi Maheswari and C.Sekar introduced an edge irregular fuzzy graphs, neighbourly edge irregular fuzzy graphs and strongly edge irregular fuzzy graph [13,7,14]. D. S. Cao, introduced 2-degree of vertex v is the the sum of the degrees of the vertices adjacent to v and it is denoted by t(v) [2]. A.Yu, M.Lu and F.Tian, introduced pseudo degree (average degree) of a vertex v is (t(v))/d(v), where d(v), is the number of edges incident at the vertex v [1]. N.R.Santhi Maheswari and C.Sekar introduced 2- degree of a vertex in fuzzy graphs, pseudo degree of a vertex in fuzzy graph and pseudo regular fuzzy graphs [8]. N.R Santhi Maheswari and M.Sutha introduced concept of pseudo irregular fuzzy graphs and highly pseudo irregular fuzzy graphs[15]. N.R.Santhi Maheswari and M.Rajeswari introduced the concept of strongly pseudo irregular fuzzy graphs [16]. Selvam Avadayappan and G.Mahadevan call this 2-degree of a vertex as support of a vertex. Selvam Avadayappan, M,Bhuvaneshwari and R.Sinthu introduced support neighbourly irregular graphs [23]. N.R.Santhi Maheswari and K.Amutha introduced support neighbourly edge irregular graphs and 1 neighbourly edge



irregular graphs, Pseudo Edge Regular and Pseudo Neighbourly edge irregular graphs [19,20,21]. N.R.Santhi Maheswari and K.Priyatharcini introduced support highly irregular graphs [22] . N.R.Santhi Maheswari and V,Jeyapratha introduced the concept of neighbourly pseudo irregular fuzzy graphs[18]. These motivate us to discuss Regular fuzzy graph, Totally Regular fuzzy graph, Partially Regular fuzzy graph, Full Regular fuzzy graph, Edge Regular fuzzy graph, K-totally edge regular fuzzy graph, Odd degree Fuzzy graph, Even degree Fuzzy graph, Cubic Fuzzy graphs and some of its properties.

### 2. Preliminaries

**Definition 2.1.** In a graph G all the vertices have same degree k then the graph G is called Regular graph or k –Regular graph.

**Definition 2.2.** In a graph *G* all the edges have the same degree k then the graph *G* is called edge regular graph or k –edge regular graph.

**Definition 2.3.** Let v be a non-empty finite set and  $E \subseteq v \times v$ . A fuzzy graph  $G: (\sigma, \mu)$  is a pair of functions  $\sigma: v \to [0,1]$  and  $\mu: E \to [0,1]$  such that  $\mu(x, y) \leq \alpha(x) \wedge \sigma(y)$  for all  $x, y \in V$  where  $\sigma$  is a fuzzy subset of an non-empty set V and  $\mu$  is a symmetric fuzzy relation on  $\sigma$ . The underlying crisp graph of  $G: (\sigma, \mu)$  is denoted by G \*: (V, E).

**Definition 2.4.** Let  $FG(\sigma, \mu)$  be a fuzzy graph on G(V, E). The degree of a vertex  $\sigma_i$  is defined by  $d_{fg}(\sigma_i) = \Sigma \mu(\sigma_i \sigma_j)$  for  $\sigma_i \neq \sigma_j$ .

**Definition 2.5.** The Maximum degree of the Vertex of  $FG(\sigma, \mu)$  is  $\Delta(FG) = \lor d_{FG}(\sigma_i) for all \sigma_i \in \sigma$ . The Minimum degree of the vertex of  $FG(\sigma, \mu)$  is  $\Delta(FG) = \land d_{FG}(\sigma_i) for all \sigma_i \in \sigma$ .

**Definition 2.6.** Let  $FG(\sigma, \mu)$  be a fuzzy graph on G(V, E), The total degree of a vertex is defined by  $td_{FG}(\sigma_i) = d_{FG}(\sigma_i) + \sigma(\sigma_i)$ .

**Definition 2.7.** Let  $FG(\sigma, \mu)$  be a fuzzy graph on G(V, E) if each vertex in *G* has a same degree *K* then  $FG(\sigma, \mu)$  is said to be Regular fuzzy graph or K-Regular fuzzy graph.

**Definition 2.8.** Let  $FG(\sigma, \mu)$  be a fuzzy graph on G(V, E) if each vertex in *G* has a same total degree *K* then  $FG(\sigma, \mu)$  is said to be Totally Regular fuzzy graph or K- Totally Regular fuzzy graph.

**Definition 2.9.** If the graph G is regular then  $FG(\sigma, \mu)$  is said to be Partially Regular fuzzy graph.

**Definition 2.10.** If the fuzzy graph  $FG(\sigma, \mu)$  is satisfying both regular fuzzy graph and partially regular fuzzy graph, then  $FG(\sigma, \mu)$  is said to be a Full Regular fuzzy graph.

**Definition 2.11.** Let  $FG(\sigma, \mu)$  be a fuzzy graph on G(V, E) if each edge in  $FG(\sigma, \mu)$  has same degree *K* then  $FG(\sigma, \mu)$  is said to be an Edge Regular fuzzy graph.

**Definition 2.12.** Let  $FG(\sigma, \mu)$  be a fuzzy graph on G(V, E) if each edge in  $FG(\sigma, \mu)$  has same total degree *K* then  $FG(\sigma, \mu)$  is said to be Totally Edge Regular fuzzy graph or *K*-totally edge regular fuzzy graph.

## 3. Some Properties of Regular fuzzy graphs

**Theorem 3.1.** Let  $G: (\sigma, \mu)$  be a fuzzy graph on G : (V, E) Then  $\Sigma d_G(v) = 2S(G)$ . **Proof :** Let  $G: (\sigma, \mu)$  can be a Fuzzy graph on G(V, E). Then the degree of a vertex  $\sigma_i$  is defined by,  $d_{FG}(\sigma_i) = \Sigma \mu(\sigma_i \sigma_j) for \sigma_i \neq \sigma_j$ .  $\Sigma_{i=1}^n d_{FG}(\sigma_i) = d_{FG}(\sigma_1) + d_{FG}(\sigma_2) + d_{FG}(\sigma_3) + \dots + d_{FG}(\sigma_{n-1}) + d_{FG}(\sigma_n)$ .  $d_{FG}(\sigma_1) = \mu(\sigma_1 \sigma_2) + \mu(\sigma_1 \sigma_3) + \mu(\sigma_1 \sigma_4) + \dots + \mu(\sigma_1 \sigma_{n-1}) + \mu(\sigma_1 \sigma_n)$ .  $d_{FG}(\sigma_2) = \mu(\sigma_2 \sigma_1) + \mu(\sigma_2 \sigma_3) + \mu(\sigma_2 \sigma_4) + \dots + \mu(\sigma_2 \sigma_n - 1) + \mu(\sigma_2 \sigma_n)$ .



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$$\begin{aligned} & d_{FG}(\sigma_n) = \mu(\sigma_n \sigma_1) + \mu(\sigma_n \sigma_3) + \mu(\sigma_n \sigma_4) + \dots + \mu(\sigma_n \sigma_{n-1}). \\ & \mathbb{P}_{1=1}^{F_{1=1}} d_{FG}(\sigma_1) = \mu(\sigma_1 \sigma_2) + \mu(\sigma_1 \sigma_3) + \mu(\sigma_1 \sigma_0) + \dots + \mu(\sigma_n \sigma_{n-1}) + \mu(\sigma_2 \sigma_1) + \mu(\sigma_2 \sigma_3) + \mu(\sigma_2 \sigma_4) + \dots + \mu(\sigma_1 \sigma_{n-1}) + \mu(\sigma_2 \sigma_3) + \mu(\sigma_1 \sigma_2) + \mu(\sigma_1 \sigma_3) + \mu(\sigma_1 \sigma_2) + \mu(\sigma_1 \sigma_3) + \dots + \mu(\sigma_1 \sigma_{n-1}) + \mu(\sigma_2 \sigma_3) + \dots + 2\mu(\sigma_2 \sigma_{n-1}) + 2\mu(\sigma_2 \sigma_{n-1}) + \mu(\sigma_1 \sigma_{n-1}) + \mu(\sigma_2 \sigma_3) + \mu(\sigma_2 \sigma_3) + \dots + \mu(\sigma_1 \sigma_{n-1}) + \mu(\sigma_1 \sigma_{n-1}) + \mu(\sigma_2 \sigma_3) + \mu(\sigma_2 \sigma_3) + \dots + \mu(\sigma_1 \sigma_{n-1}) + \mu(\sigma_2 \sigma_{n-$$

(2)  $FG(\sigma, \mu)$  is an totally edge regular Fuzzy graph.



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**Proof**: Let  $FG(\sigma, \mu)$  be a fuzzy graph on G(V, E). Suppose that  $\mu = c$  is a constant function. Then  $\mu(\sigma_i \sigma_j) = c$  for every,  $\sigma_i \sigma_j \in \mu$  where *c* is a constant.

(1)  $\Rightarrow$  (2) Assume that  $FG(\sigma, \mu)$  is a  $k_1$ - edge regular Fuzzy graph. To show  $FG(\sigma, \mu)$  is a totally edge regular fuzzy graph.  $d_{FG}(\sigma_i\sigma_j) = k_1$  for all  $\sigma_i\sigma_j \in \mu$ ,  $td_{FG}(\sigma_i\sigma_j) = d_{FG}(\sigma_i\sigma_j) + \mu(\sigma_i\sigma_j)$  for all  $\sigma_i\sigma_j \in \mu$   $td_{FG}(\sigma_i\sigma_j) = k_1 + c$  for all  $\sigma_i\sigma_j \in \mu$  Hence *G* is a  $(k_1 + c)$  –totally edge regular fuzzy graph.

(2)  $\Rightarrow$  (1) Now suppose that  $FG(\sigma, \mu)$  is a  $k_2$  -totally edge regular fuzzy graph. Then  $td_{FG}(\sigma_i\sigma_j) = k_2$  for all  $\sigma_i\sigma_j \in \mu$  To show that  $FG(\sigma, \mu)$  is the edge regular fuzzy graph.  $d_{FG}(\sigma_i\sigma_j) + \mu(\sigma_i\sigma_j) = k_2$  for all  $\sigma_i\sigma_j \in \mu$ ,  $d_{FG}(\sigma_i\sigma_j) = k_2 - \mu(\sigma_i\sigma_j)$  for all  $\sigma_i\sigma_j \in \mu$ ,  $d_{FG}(\sigma_i\sigma_j) = k_2 - c$  for all  $\sigma_i\sigma_j \in \mu$  Hence  $FG(\sigma, \mu)$  is a  $k_2$  - edge regular fuzzy graph. Hence (1) and (2) are equivalent.

Conversely, Assume that (1) and (2) are equivalent.  $FG(\sigma, \mu)$  is an edge regular Fuzzy graph if and only if  $FG(\sigma, \mu)$  is an totally edge regular Fuzzy graph. To prove that  $\mu$  is a constant function. Suppose  $\mu$  is not a constant function. Then  $\mu(\sigma_i \sigma_j) \neq \mu(\sigma_k \sigma_l)$  for at least one pair of edges  $\sigma_k \sigma_l, \sigma_i \sigma_j \in \mu$ . Let  $FG(\sigma, \mu)$  be a k - edge regular Fuzzy graph.  $d_{FG}(\sigma_i \sigma_j) = d_{FG}(\sigma_k \sigma_l) = k$  for all  $\sigma_i \sigma_j, \sigma_k \sigma_l \in$  $\mu$ .  $td_{FG}(\sigma_i \sigma_j) = d_{FG}(\sigma_i \sigma_j) + \mu(\sigma_i \sigma_j)$  for all  $\sigma_i \sigma_j \in \mu$ .  $td_{FG}(\sigma_i \sigma_j) = k + \mu(\sigma_i \sigma_j)$  for all  $\sigma_i \sigma_j \in$  $\mu$ .  $td_{FG}(\sigma_k \sigma_l) = d_{FG}(\sigma_k \sigma_l) + \mu(\sigma_k \sigma_l)$  for all  $\sigma_k \sigma_l \in \mu$ . Since  $\mu(\sigma_i \sigma_j) \neq \mu(\sigma_k \sigma_l)$  so  $td_{FG}(\sigma_i \sigma_j) \neq$  $td_{FG}(\sigma_i \sigma_j) + \mu(\sigma_i \sigma_j) = d_{FG}(\sigma_k \sigma_l) - \mu(\sigma_k \sigma_l)$  is not a totally edge regular fuzzy graph. Which is a contradiction to our assumption. Now let  $FG(\sigma, \mu)$  is a totally edge regular fuzzy graph.  $td_{FG}(\sigma_i \sigma_j) = td_{FG}(\sigma_k \sigma_l) - \mu(\sigma_k \sigma_l)$  $\mu(\sigma_k \sigma_l). d_{FG}(\sigma_i \sigma_j) - d_{FG}(\sigma_k \sigma_l) \neq 0$  [since  $\mu(\sigma_i \sigma_j) \neq \mu(\sigma_k \sigma_l)$ ] Then  $d_{FG}(\sigma_i \sigma_j) \neq d_{FG}(\sigma_k \sigma_l)$ .  $FG(\sigma, \mu)$  is not a edge regular fuzzy graph.

**Theorem 3.5.** If a fuzzy graph  $FG(\sigma, \mu)$  is both edge regular and totally edge regular, then  $\mu$  is a constant function.

**Proof**: Let  $FG(\sigma, \mu)$  be a fuzzy graph,  $k_1$  –edge regular fuzzy graph and  $k_2$  totally edge regular fuzzy graph. Then  $d_{FG}(\sigma_i\sigma_j) = k_1$  for all  $\sigma_i\sigma_j \in \mu$ . and  $td_{FG}(\sigma_i\sigma_j) = k_2$  for all  $\sigma_i\sigma_j \in \mu.td_{FG}(\sigma_i\sigma_j) + \mu(\sigma_i\sigma_j) = k_2$  for all  $\sigma_i\sigma_j \in \mu.\mu(\sigma_i\sigma_j) = k_2 - k_1$  for all  $\sigma_i\sigma_j \in \mu$ . Hence  $\mu$  is a constant function.

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