On the Forgotten Eccentricity Connectivity Topological Index of Pseudo-Regular Graphs

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Abstract
For a graph $G = (V, E)$, the second degree of a vertex v in a graph G is the number of its second neighbors, that is the number of vertices in G having distance 2 to v. In this manuscript we have computed the Forgotten eccentricity connectivity topological index of graph G is defined as, $F\xi^c(G) = \sum_{v\in V(G)} d^3(v).e(v)$ and $F\xi^c(G) = \sum_{v\in V(G)} d_2^3(v).e(v)$, where, $d^3(v)$ and $d_2^3(v)$ are the degree of the vertices u and v and e(v) is a eccentricity of the vertices u and v. In this paper the Forgotten eccentricity connectivity topological index of pseudo-regular graphs are determined.

Keywords: Forgotten topological index, Connectivity, Eccentricity index.

1. Introduction

Throughout this paper, by $G = (V, E)$ we mean an undirected simple graph with vertex set V and edge set E. As usual, we denote the number of vertices and edges in a graph G by n and m respectively. The distance $d_G(u, v)$ between any two vertices $u, v \in V$ of G is equal to the length of a shortest path between u and v. For a vertex v of G, the eccentricity of v is
$e(v) = max\{d(v,u): u \in V(G)\}$. The diameter of G is $diam(G) = max\{e(v): v \in V(G)\}$ and the radius of G is $rad(G) = min\{e(v): v \in V(G)\}$.
A topological index of a G is a graph invariant number calculated from G.Various topological indices represent molecule structures and have got greater applications in chemistry. The Zagreb indices have been introduced, more than fifty years ago by Gutman and
Trinajestic [8], in 1972, and studied by various authors in [4, 2, 5, 9, 10, 17, 11, 3, 6]. Followed by the first and second Zagreb indices, Furtula and Gutman (2015) introduced forgotten topological index (also called F-index) which was defined as.

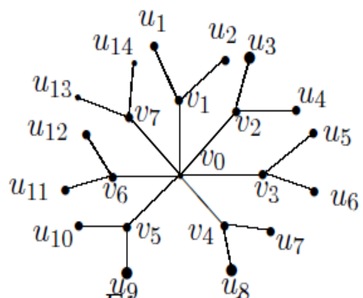
$$F(G) = \sum_{v\in V(G)} d^3(v) = \xi^c(G) = \sum_{u,v\in V(G)} d^2(u) + d^2(v),$$

where d(v) is denoted as the degree of vertex v (the number of vertex adjacent to vertex v). Furtula and Gutman (2015) raised that the predictive ability of forgotten topological index is almost similar to that of first Zagreb index and for the acentric factor and entropy, and both of them obtain correlation coefficients larger than 0.95. This fact implies the reason why forgotten topological index is useful for testing the chemical and pharmacological properties of drug molecular structure [18]. In 2017, Naji et.al., [13], had introduced three new distance-degree-based topological indices depending on the second degrees of vertices (number of their second neighbours), and are so-called leap Zagreb indices of graph G and are respectively defined by

$$LM_1(G) = \sum_{v \in V(G)} d^2(v)$$

2. Pseudo-Regular Graphs.

Let $G = (V, E)$ be a simple, connected, undirected graph with n vertices and m edges. For any vertex $v_i \in V$ the degree of v_i is the number of edges incident on v_i. It is denoted by d_i or $d(v_i)$. A graph G is called regular if every vertex of G has equal degree. A bipartite graph is called semi regular if each vertex in the same part of a bipartition has the same degree. The 2 degree v_i[1] is the sum of the degree of the vertices adjacent to v_i and denoted by t_i [7]. The average degree of v_i is defined as $\frac{t_i}{d_i}$. For any vertex $v_i \in V$, the average degree of v_i is also denoted by $m(v_i) = \frac{t_i}{d_i}$.

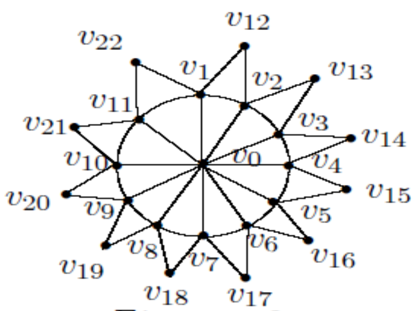


$Figure - 1$

A graph G is called pseudo-regular graph [1] if every vertex of G has equal average degree and $m(G) = (\frac{1}{n}) \sum_{v_i \in V(G)} m(u)$ is the average neighbor degree number of the graph G. A graph is said to be r-regular if all its vertices are of equal degree r. Every regular graph is a pseudo-regular graph, see [14, 12]. But the pseudo-regular graph need not be a regular graph. Pseudo-regular graph is shown in Figures 1 and 2.

In Figure 1, there are 14 vertices of degree 1, 7 vertices of degree 3, and 1 vertex of degree 7. So totally there are 22 vertices in the graph. Average degree of vertices of degree 1 is equal to. Average degree of vertices of degree 2 is equal to 9/3 = 3. Average degree of vertices of degree 7 is equal to 21/7 = 3. Therefore, average degree of each vertex is 3. Hence, it is a pseudo-regular graph. In Figure 2, average degree of each vertex is 5. Hence, the graph in Figure 2 is also a pseudo regular graph.

The relevance of pseudo-regular graph for the theory of nanomolecules and nanostructure should become evident from the following. There exist polyhedral (planar, 3-connected) graphs and infinite periodic planar graphs belonging to the family of the pseudo-regular graphs.



$Figure - 2$

The deltoidal hexecontahedron is a Catalan polyhedron with 60 deltoid faces, 120 edges, and 62 vertices with degrees 3, 4, and 5, and average degree of its vertices is 4.

In this paper, motivated by connectivity index, we introduce the Forgotten eccentricity connectivity topological index indices are equations (1) and (2) of Pseudo-Regular Graphs.

Definition 2.1. For a graph $G = (V, E)$, Forgotten eccentricity connectivity topological index of G is defined by

$$F\xi^c(G) = \sum_{v\in V(G)} d^3(v).e(v) \qquad (1)$$

where $d^3(v)$ be the degree of the vertices $v \in V(G)$ and $e(v)$ is a eccentricity of the vertices $v \in V(G)$, $e(v) = max\{d(u,v)$: for every u∈ V(G)}.

Definition 2.2. For a graph $G = (V, E)$, Forgotten eccentricity connectivity topological index of G is defined by

$$F\xi^c(G) = \sum_{v\in V(G)} d_2^3(v).e(v) \qquad (2)$$

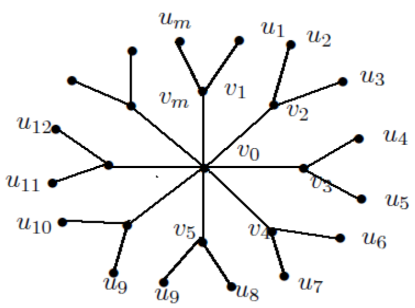
where $d_2^3(v)$ be the degree of the vertices $v \in V(G)$ and $e(v)$ is a eccentricity of the vertices $v \in V(G), (d_2(v) = |\{u \in V(G): d(u,v) = 2\}|$ and $e(v) = max\{d(u,v)$: for every u∈ V(G)}.

3. Computation forgotten eccentricity connectivity topological indices of Pseudo-Regular graphs.

Proposition 3.1. If $|p| \geq 3p + 4$, forgotten eccentricity connectivity index of Pseudo-Regular graph. Then

$$F\xi^c\,(G) = 2(2(p+1))^3 + 3p^3(p+1) + 32(2(p+1)), \quad \text{for } p \geq 1.$$

Proof: Let $V(G) = \{v_0, v_1, v_2, \ldots v_m, \; u_1, u_2, u_3, \ldots u_{m(p-1)}\}$, be the vertex set of G and v_0 be the central vertex of star graph $K_{1,m}$ $\{v_0, v_1, v_2, \ldots v_m\}$ are pendent vertices of $K_{1,m}$, where $m = p^2 - p + 1$ and the $(p-1)$ pendent vertices of $\{u_1, u_2, u_3, \ldots u_{m(p-1)}\}$ are attached with m pendent vertices v_0, $v_1, v_2, \ldots v_m$. Hence $d_2(v_0) = 2(p+1)$, $d_2(v_i) = p$, $d_2(u_j) = 2$, $e(v_0) = 2$, $e(v_i) = 3$, and $e(u_j) = 4$ (see Fig.3. for more details). Forgotten eccentricity connectivity topological index of G, by using the equation (2). Now



$Figure - 3$

$$F\xi^c\,(G) = {d_2}^3(v_0)e(v_0).1 + \sum_{i=1}^{p+1} {d_2}^3(v_i)e(v_i) + \sum_{j=1}^{2(p+1)} {d_2}^3(u_j)e(u_j)$$

$$= 2((p+1))^3.2.1 + \sum_{i=1}^{p+1} p^3.3 + \sum_{j=1}^{2(p+1)} 2^3.4$$

$= 2(2(p+1))^3 + 3p^3(p+1) + 32(2(p+1)),$ for $p \geq 1.$

Proposition 3.2. If $|p| \geq 3p+4$, forgotten eccentricity connectivity index of Pseudo-Regular graph. Then

$F\xi^c(G) = 2(p+1)^3 + 3^4(p+1) + 4(2p+2),$ for $p \geq 1.$

Proof: By Theorem 3.1., Hence, $d(v_0) = (p+1)$, $d_2(v_i) = 3$, $d(u_j) = 1$, $e(v_0) = 2$, $e(v_i) = 3$, and $e(u_j) = 4$ (see Fig.3), by using the equation (1). Now

$$F\xi^c(G) = d^3(v_0)e(v_0).1 + \sum_{i=1}^{p+1} d^3(v_i)e(v_i) + \sum_{j=1}^{2(p+1)} d^3(u_j)e(u_j)$$

$$= (p+1)^3.2.1 + \sum_{i=1}^{p+1} 3^3.3 + \sum_{j=1}^{2(p+1)} 1^3.4$$

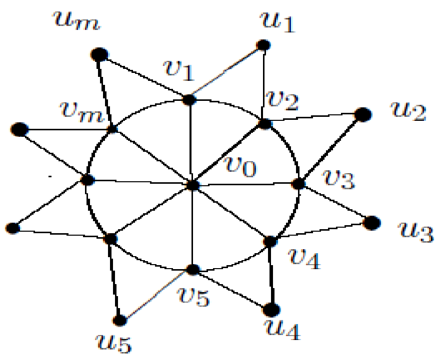
$= 2(p+1)^3 + 3^4(p+1) + 4(2p+2),$ for $p \geq 1.$

Proposition 3.3. If $|p| \geq 2p+11$, forgotten eccentricity connectivity topological index of Pseudo-Regular graph. Then

$$\xi F^c(G) = \begin{cases} 444, & if\ |p| = 7, \\ 1844, & if\ |p| = 9, \\ 3085, & if\ |p| = 11, \\ 2(p+5)^3 + 3(p+4)^3(p+5) + 500(p+5) & for\ p \geq 1. \end{cases}$$

Proof: Let $V(G) = \{v_0, v_1, v_2, \dots v_m, \quad u_1, u_2, u_3, \dots u_{m(p-3)}\}$, be the vertex set of G and v_0 be the central vertex of wheel graph W_m and $\{v_0, v_1, v_2, \dots v_m\}$ are vertex of cycle C_m in the clockwise direction and $\{u_1, u_2, u_3, \dots u_{m(p-3)}\}$ are the pendant vertices joined to every vertex in the cycle except the central vertex, where $m = p^2 - 3p + 3$.

Hence, $d_2(v_0) = (p+5)$, $d_2(v_i) = p+4$, $d_2(u_j) = 5$, $e(v_0) = 2$, $e(v_i) = 3$, and $e(u_j) = 4$ (see Fig. 4. for more details).



$Figure - 4$

Forgotten eccentricity connectivity topological index of G, by using the equation (2). Now

$$F\xi^c(G) = {d_2}^3(v_0)e(v_0).1 + \sum_{i=1}^{p+5} {d_2}^3(v_i)e(v_i) + \sum_{j=1}^{2(p+5)} {d_2}^3(u_j)e(u_j)$$

$$= (p+5)^3.2.1 + \sum_{i=1}^{p+5} (p+4)^3.3 + \sum_{j=1}^{2(p+5)} 5^3.4$$

$$= 2(p+5)^3 + 3(p+4)^3(p+5) + 500(p+5) \quad for\ p \geq 1.$$

Proposition 3.4. If $|p| \geq 2p + 11$, forgotten eccentricity connectivity topological index of Pseudo-Regular graph. Then

$$\xi F^c(G) = \begin{cases} 852, & if\ |p| = 7, \\ 1224, & if\ |p| = 9, \\ 2245, & if\ |p| = 11, \\ 2(p+5)^3 + 407(p+5), & for\ p \geq 1. \end{cases}$$

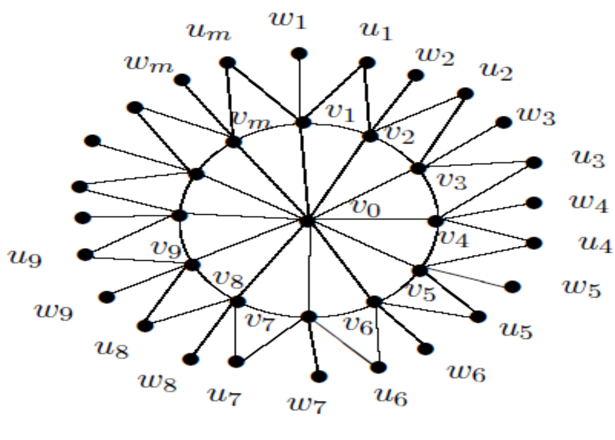
Proof: By the Theorem 3.3. Hence, $d(v_0) = (p+5)$, $d(v_i) = 5$, $d(u_j) = 2$, $e(v_0) = 2$, $e(v_i) = 3$, and $e(u_j) = 4$ (see Fig. 4. for more details). Forgotten eccentricity connectivity topological index of \$G\$, by using the equation (1). Now

$$F\xi^c(G) = d^3(v_0)e(v_0).1 + \sum_{i=1}^{p+5} d^3(v_i)e(v_i) + \sum_{j=1}^{p+5} d^3(u_j)e(u_j)$$
$$= (p+5)^3.2.1 + \sum_{i=1}^{p+5} 5^3.3 + \sum_{j=1}^{p+5} 2^3.4$$
$$= 2(p+5)^3 + 407(p+5), \quad for\ p \geq 1$$

Proposition 3.5. If $|p| \geq 3p + 13$, forgotten eccentricity connectivity topological index of Pseudo-Regular graph. Then

$$\xi F^c(G) = \begin{cases} 3663, & if\ |p| = 10, \\ 8140, & if\ |p| = 13, \\ 2(2p+8)^3 + 3(p+5)^3(p+4) + 1872(p+4), & for\ p \geq 1. \end{cases}$$

Proof: Let $V(G) = \{v_0, v_1, v_2, \dots v_m, u_1, u_2, u_3, \dots, u_{m(p-5)}, w_1, w, \dots, w_m\}$ be the vertex set of G and v_0 as the central vertex of wheel graph w_m, where $m = p^2 - 3p + 1$ and $\{u_1, u_2, u_3, \dots u_{m(p-5)}\}$ are the pendant vertices and $\{w_1, w, \dots, w_m\}$ are the vertices joined to the end vertices of each edge of a wheel graph except the central vertex (see Fig.5 for more details) [12]. Hence, $d_2(v_0) = 2p + 8$, $d_2(v_i) = p + 5$, $d_2(u_j) = 7$, $d_2(w_k) = 5$ $e(v_0) = 2$, $e(v_i) = 3$, $e(u_j) = 4$ and $e(w_k) = 4$, where $i = j = k = 1, 2, 3, . . ., m$.



$Figure - 5$

Forgotten eccentricity connectivity topological index of \$G\$, by using the equation (2). Now

$$F\xi^c\,(G) = {d_2}^3(v_0)e(v_0).1 + \sum_{i=1}^{p+4} {d_2}^3(v_i)e(v_i) + \sum_{j=1}^{p+4} {d_2}^3(u_j)e(u_j) + \sum_{k=1}^{p+4} {d_2}^3(w_k)e(w_k)$$
$$= (2p+8)^3.2.1 + \sum_{i=1}^{p+4}(p+5)^3.3 + \sum_{j=1}^{p+4} 7^3.4 + \sum_{k=1}^{p+4} 5^3.4$$
$$= 2(2p+8)^3 + 3(p+5)^3(p+4) + 1872(p+4)\,, \qquad for\ p \geq 1.$$

Proposition 3.5. If $|p| \geq 3p+13$, forgotten eccentricity connectivity topological index of Pseudo-Regular graph. Then

$$\xi F^c\,(G) = \begin{cases} 1431, & if\ |p| = 10, \\ 2828, & if\ |p| = 13, \\ 2(p+4)^3 + 684(p+4)\,, & for\ p \geq 1. \end{cases}$$

Proof: By the Theorem 3.5, Hence, $d_2(v_0) = p+4$, $d_2(v_i) = 6$, $d_2(u_j) = 2$, $d_2(w_k) = 1$, $e(v_0) = 2$, $e(v_i) = 3$, $e(u_j) = 4$ and $e(w_k) = 4$, where $i = j = k = 1, 2, 3, . . ., m$.

$$F\xi^c\,(G) = d^3(v_0)e(v_0).1 + \sum_{i=1}^{p+4} d^3(v_i)e(v_i) + \sum_{j=1}^{p+4} d^3(u_j)e(u_j) + \sum_{k=1}^{p+4} d^3(w_k)e(w_k)$$
$$= (p+4)^3.2.1 + \sum_{i=1}^{p+4} 6^3.3 + \sum_{j=1}^{p+4} 2^3.4 + \sum_{j=1}^{p+4} 1^3.4$$
$$= 2(p+4)^3 + 684(p+4)\,, \qquad for\ p \geq 1.$$

4. Conclusion

In this manuscript, we clearly determined the Forgotten eccentricity connectivity topological index of pseudo-regular graphs.

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