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Lucasiewicz's Three-Valued Logic and Imperative Logic: A Comparative Study

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Abstract
Imperative sentences are neither true nor false. Three non-truth-functional values of satisfaction, violation and avoidance have been introduced for them. In Lucasiewicz's logic also has three values, namely, truth, falsity and indeterminate. This paper attempts to make a comparative study between two systems by mentioning a brief history of imperative logic developed by logicians based on the distinction between extended and deviant logic.

Keywords: Imperatives, truth-functional values, non-truth-functional values, Lucasiewitz's logic

1. INTRODUCTION

In recent times imperative logic has been developed in an effort to deal with arguments involving imperative sentences. An imperative sentence is that which expresses a command or an entreaty. For example, "Close the door!" An imperative sentence is neither true nor false and has a performative dimension. The necessity and usefulness of imperative logic lies in the fact that we meet imperative inferences in our day-to-day life. Imperative arguments have imperative sentences either as their premises or as their conclusions or as both. For example,
If you love him, respect him.
You love him.
Therefore, Respect him.
It has been found that the future contingent sentences have been one of the motivations for logicians to articulate some non-Aristotelian logic because these propositions cannot be handled with the scope of classical logic. A distinction is made between the two kinds of logic, namely, deviant and extended logics. In this paper attempt is to undertake few systems of imperative logic to consider it as either as an extension of classical logic i.e., a variety of extended logic or deviant logic. This paper has four sections. The first section dwells upon the motivation behind the appearance of non-Aristotelian logic. The second section deals with the distinction between the two kinds of logic, namely, extended and deviant logic. In the third section, few systems of imperative logic have been undertaken. In the concluding section, a view has been recommended that under which category imperative logic may fall – either deviant or extended logic.

2. Motivation behind the appearance of non-Aristotelian logic

In the Aristotelian logic which is commonly known as the classical two-valued logic, the common doctrine is that every proposition is either true or false.
But some philosophical problems arise questioning the acceptability of classical two-valued logic as

"the" logic. Nicholas Rescher makes the following comment in his Many-Valued Logic.

> Throughout the orthodox mainstream of the development of logic in the West, the prevailing view was that every proposition is either true or else false…This thesis, now commonly called the "Law of bivalence", closely bound up with the historic "Law of the Excluded Middle", was, however, already questioned in antiquity.[1]

One such problem is the problem of future contingent statements. An example of a future contingent statement is 'There will be a sea-battle tomorrow.' This statement is neither true nor false. It cannot be said that the statement is true because the sea-battle might happen or not. Nor can it be considered as false that there will not be a sea-battle tomorrow, because it is to admit that there will not be a sea-battle tomorrow. And that is not possible to predetermine the occurrence of a sea-battle one day before. It can be said that to find the truth value of this kind of statement i.e. either true or false, is to subscribe to fatalism.

In the chapter 9, of his treatise On Interpretation (De Interpretation) Aristotle discusses the problem of the logical status of the future contingent propositions.

The fact that Aristotle was aware of the problem of future contingency is clear from the writings of Susan Haack.[2] She shows that Aristotle intends to hold that if a general sentence or sentences, predicating something of an individual is true, then the denial of the original sentence is false. Aristotle holds that sentences about past and present tense are bivalent but he is not confident about the bivalency of the future tense sentences. She also says that according to Aristotle, if future tense sentences are bivalent, then they are either necessarily true or necessarily false. But everything does not happen necessarily. On the other hand some events are contingent. So future tense sentences are neither true nor false.

3. Distinction between deviant and extended logic

Susan Haack in Deviant Logic holds that several non-standard logical systems other than classical two-valued logic (C_2) need to be formulated. To quote Susan Haack:

> There are many systems of logic – many-valued systems and modal systems for instance – which are non-standard; that is, which differ in one way or another from classical logic. Because of this plurality of logics, the question whether, or in what way, non-standard systems are 'alternatives' to classical logic, naturally arises.[3]

Susan Haack distinguishes between a stronger sense and a weaker sense in which non-standard systems may be alternatives to classical logic. According to the strong sense, a system is alternative to classical logic in the sense that it may be employed instead of classical two-valued logic. On the other hand, according to the weaker sense, a system is alternative in the sense that they may be employed as well as the classical two-valued logic. The reason behind this view is that "the former tends to regard classical logic as mistaken, …. whereas the latter tends to regard classical logic as inadequate." [4]

Hence, non-classical logical systems may be classified into two broad categories, namely, extended logic and deviant logic. Extended logic is the supplement to the classical bivalent logic. It holds that C_2 is inadequate, so it has to be extended to remove its adequacy. In this system, the vocabulary of classical two valued logic is retained, but some axioms have added to it.

On the other hand, deviant logics are alternative logics to classical logic. The upholders of deviant logics consider classical bivalent logic as mistaken, and the mistake is to be rectified. To do so, it is better to set aside or throw away the whole system of classical two valued logic and another system of logic

should be built up. "…a Deviant system should have as a theorem the contradictory of a wff which classical logic has as a theorem... However, in accepting, say, 'p or not-p' as a theorem the classical logician is asserting something implicitly general (that, whatever p may be, 'p or not p' is true) and when intuitionist refuses to accept 'p or not p' as a theorem he does so because he thinks that in certain instances 'p or not p' is not true… there is … conflict – something, that is, which the classical logician asserts and the Deviant logicians denies." [5]

Extended logic is compatible with classical logic whereas a rival system is incompatible with C_2. "A rival system is, then, one the use of which is incompatible, and a supplementary system one the use of which is compatible, with the use of the standard system."[6]

Next, Susan Haack explains three relevant possibilities by which a system may be determined as either extended or deviant logic.

a) The class of wffs of L_1 properly includes the class of wffs of L_2 and the class of theorems/ valid inferences of L_2 are a proper subset of class of wffs and theorems/ valid formulas of L_1. The additional theorems/ valid occurrence L_1's additional vocabulary.

Then L_1 is an extension of L_2. It is an extended logic. For example, C_2 is an extension of the implicational fragments such as T or the Lewis systems are extensions of classical propositional calculus.

b) The class of wffs of L_1 and the class of wffs of L_2 coincide but the class of theorems/ valid inferences of L_1 differ from those of L_2. In that case, L_1 and L_2 are derivations of each other and if L_2 is classical logic, L_1 is a deviant logic.

c) The class of wffs of L_2 is a proper subset of those of L_1 and theorems/ valid inferences of L_1 and L_2 differ from each other both in the overlapping and non-overlapping areas then L_1 is a quasi- derivation of L_2 and L_1 is a quasi-deviant logic.

In order to address the issue whether imperative logic is a kind of extended or deviant logic it is necessary to see the evolution of this logic. As it has been seen that imperative logic lacks truth preservation logicians initially tend to neglect this type of propositions and deny the possibility of the logic of it.

Two trends have been found regarding the possibility of imperative logic. The first trend is the reductionism which holds that imperative sentences may be translated into indicative sentences and thus can be included within the scope of classical two-valued logic. So the question of considering imperative logic as either extended or deviant logic does not arise in this case. But the logicians who are non-reductionists hold that due to the performative feature an imperative sentence cannot be translated into indicative sentences. So the non-reductionist approach of imperative logic may be taken into account for the categorization as either extended or deviant logic. a separate system of logic is needed for imperative logic. It is the time to enter into the discussion of few systems of imperative logic.

4. Few systems of Imperative Logic

4.1. Jorgen Jorgensen (1938)

Jorgen Jorgensen, a Danish philosopher, proposes a dilemma which holds that only sentences which are capable of being true or false can function as premises or conclusions in an inference; but it seems evident that there are good imperative inferences. Though Jorgensen admits the possibility of imperative inferences he does not admit the possibility of imperative logic. To quote Jorgensen: … the derived indicative sentence can be dealt with … according to the rules of ordinary logic, which do not apply to

the imperative sentences.[7]

4.2. R.M.Hare (1952)

In The language of Morals[8] Hare claims that imperatives can be inferred from other imperatives and thus imperative logic is possible. In his another article, "Imperative Sentences"[9] he criticizes the reductionists for developing imperative arguments in terms of indicative characters of a sentence. He attempts to construct a logic of imperatives completely parallel to the logic of indicatives.

Hare distinguishes between imperative and indicative sentences with two examples.

1. Shut the door.
2. You are going to shut the door.

He holds that these two sentences have one part common, i.e. your shutting the door in immediate future.

This phrase can be converted in either into an indicative or imperative by adding two words, one each in both.

(1.1) Your shutting the door in the immediate future, yes.

(2.1) Your shutting the door in the immediate future, please.

In both (1) and (2) there is a reference of a 'descriptive function' of your shutting the door in immediate future which is called "descriptor". Now, the same descriptor of a sentence may be changed due to the presence of two different terms in it which he calls 'dictors', namely, 'please' and 'yes' which indicate the mood of the sentence. He calls it the 'dictive function' of the sentence. Thus, (1.1) becomes an imperative by adding the word 'yes' and (2.1) is an indicative sentence by adding 'please' to it.

Hare also sets down the following two rules, for validity:

"(1) No indicative conclusion can be validly drawn from a set of premises which cannot be validly drawn from the indicatives among them alone.

(2) No imperative conclusion can be validly drawn from a set of premises which does not contain at least one imperative premise."[10]

According to Hare, logic of imperative is bivalent in the same way as indicative sentences are. Here is one of reasons.

It is quite clear that if I do not say "Shut the door!" this does not compel me, logically, to say "Do not shut the door". I can say "You may either shut the door or not shut the door"; or I can say nothing at all. But similarly, if I do not say "You are going to shut the door" this does not compel me logically to say "You are not going to shut the door". I can say "You may be going to shut the door, and you may be going not to shut the door" or I can say nothing at all.(…) The truth is that our language possesses ways of speaking in a three-valued way and ways of speaking in a two-valued way… simple imperatives are normally two-valued.[11]

4.3. Nicholas Rescher (1966)

In the introduction of The Logic of Commands Nicholas Rescher considers the central problem of formulating logic of imperatives within the framework of two valued classical logic. Rescher does not admit truth and falsity as the values of a command. According to him, instead a command being either true or false, it is either satisfied or violated or bypassed. a command is bypassed if and only if it is bypassed every time. It is satisfied if and only if it is not violated by any time and it is satisfied by some times; violated if and only if it is violated by some time.

His formulation of the validity of a command inference on the basis of the analogous nature of it to an indicate inference amounts to the reduction of it to indicative inference. He defines a command inference as "an argument whose conclusion is a command, and whose premises include commands and possibly also assertoric statements".[12]

Rescher proceeds to discuss the validity of an imperative argument in the following way;

All felines are mammals

All lions are felines.

Therefore, All lions are mammals.

In relation to the validity of an imperative argument, he holds that just as the conclusion of the above inference is implicitly asserted or accepted by anyone overtly asserting the premises of this inference, so also where an inference involving commands is the case, the conclusion follows as above.

Always say 'please' to John when you ask him for the bread!

Ask John for the bread now.

Therefore, Say 'please' to John now!

In this argument, it is found that the conclusion follows validly from the premises in the sense that the "conclusion is tacitly or implicitly contained in its premises."[13] He states following three conditions for the validity of commands;

"(i) Anyone who overtly gives the premiss commands may legitimately claim (or be claimed) to have implicitly given the command conclusion.

(ii) Anyone who overtly receives the premiss commands may legitimately claim (or to be claimed) to have implicitly received the command conclusion.

(iii) Any course of action on the part of their common recipient which terminates the premiss commands cannot fail to terminate the command conclusion."[14]

4.4. Peter B.M.Vranas (2009)

The most recent developed system of logic for imperative logic is the system developed by Peter B. M. Vranas. He prefers to call an imperative sentence as prescription. A prescription is satisfied and violated. The prescription "Love him" is satisfied if you love him and violated if you do not love him. The proposition "You help him" is the satisfaction proposition and the proposition "You do not help him" is the violation proposition of the prescription "Help him". A prescription is an ordered pair of satisfaction proposition and violation proposition. These two propositions are logically incompatible propositions as both cannot be fulfilled at the same time. It is impossible for a prescription to be both satisfied and violated. The prescription 'I' may be defined as an ordered pair of sets S and V. Vranas defines a prescription as follows:

"A prescription is any ordered pair of logically incompatible propositions".[15]

The letter S stands for the satisfaction proposition and V stands for the violation proposition which may be symbolized in the following manner.

I= < S, V >

Prescriptions may be defined in the following way:

i) Unconditional prescription :

Help him = < you help him, you do not help him >.

ii) conditional prescription:

If you love him, help him=< you love him and help him, you love him but do not help him >.

It may be argued that the imperative logic would be isomorphic to classical logic, as the key concept of imperative logic which is a prescription has two values, namely satisfaction and violation. For this reason it is argued that imperative logic is nothing new but isomorphic to the classical logic in which a proposition is of two values- truth and falsity. In that case, imperative logic would be of no use and become very uninteresting. But the uniqueness and novelty of the imperative logic has been retained by the distinction between unconditional and conditional prescriptions. A conditional prescription is different from an unconditional prescription as the conditional prescription is conditional if it is prefixed or suffixed with a condition. It has been already mentioned that a prescription is defined in terms of two incompatible propositions which cannot be true at the same time, it may be neither satisfied nor violated, i.e., to be avoided. Vranas attributes this third value of avoidance to a conditional prescription. For this reason, he claims that though an unconditional prescription has two values of satisfaction and violation but three values, namely, satisfaction, violation and avoidance. For example,

If you love him, trust him.

This prescription is

a) satisfied if you love him and help him
b) violated if you love him but do not help him
c) avoided if you do not love him, no matter you help him or not.

Vranas justifies the reason for admitting the third value of avoidance for a conditional prescription. According to him, a material conditional in traditional logic, "If he proposes, you will marry him" is true, even if the antecedent does not hold, or if he does not propose because a false antecedent can entail a true proposition. But in the conditional prescription, "If he proposes, marry him", it cannot be considered as either satisfied or violated, if the antecedent does not hold or he does not propose. Hence, a third criterion is required to tackle this situation. If the antecedent does not hold, i.e. he does not propose, then the conditional prescription is neither satisfied nor violated, but avoided.

According to Vranas, prescriptions have logical connectives like negation, conjunction, disjunction, conditional and bi-conditional. They can be defined in terms of non-truth-functional connective i.e., satisfaction criterion and may be represented in tabular form in the following way:

4.4.1. Negation:

P	~p
S	V
V	S

4.4.2. Conjunction and Disjunction:

I / I′	I & I′ S A V	I ∨ I′ S A V
S	S S V	S S S
A	S A V	S A V
V	V V V	S V V

4.4.3. Conditional:

p → I	S	A	V

T/F	S/A/V			
T		S	A	V
F		V	A	S

4.4.4. Bi-conditional:

P ↔ I	S	A	V
T	S	A	V
F	A	A	A

Conclusion

It is found from the above discussion that there is no consensus among the logicians regarding the matter that imperative is either a kind of deviant or extended logic. Even a few years back imperative logic was not an established separate logical system. But the survey which I have undertaken in this paper shows that it is very difficult to categorize imperative logic as deviant or extended logic in a straight way. It has been admitted by the logicians that imperative sentences are different from indicative sentences.

In conclusion the following remarks may be made.

i) Jorgensen admits the possibility of imperative inference but he is not of opinion of the admittance of a separate system of imperative logic. Being a reductionist Jorgensen is of opinion that imperative sentences are reducible to indicative sentences. So, imperative logic, in his system, is same as the classical two-valued logic.

ii) In the system of R.M Hare he proposes that imperatives have two values, namely, 'assenting to' and 'dissenting from' but they are bivalent just as classical two-valued logic.

iii) Nicholas Rescher, develops a system for commands having three values; satisfaction, violation and bypass. But in proving the validity of an imperative argument Rescher takes the technique of the classical logic.

So the systems of Hare and Rescher are no better than classical two-valued logic and thus the question of determining imperative logic in their systems either as extended or deviant does not arise.

iv) Peter B. M Vranas admits three values of a prescription, namely, satisfaction, violation and avoidance and attempts to develop the system formally by introducing the logical connectives of imperative sentences. He is influenced by J. Lucasiewicz (1920). Lucasiewicz represents the truth tables for the three-valued logic in the following way:

P	¬ p
T	F
I	I
F	T

p\q	p∧q T I F	p∨q T I F	p→q T I F	P ↔ q T I F
T	T I F	T T T	T I F	T I F
I	I I F	T I I	T T I	I T I

F	F F F	T I F	T T T	F I T

Lucasiewicz's three valued system falls under the scope of deviant logic. So there may be a tendency to consider Vranas' system to consider a variety of deviant logic. But on a closer scrutiny it is not found that it also suffers from some inadequacy and needs some modifications.

a) Vranas does not admit the avoidance value for bi-conditional prescriptions. As the avoidance proposition is the negation of the context of a prescription, the context of a bi-conditional is the condition of two prescriptions. For example, "Trust him if and only if you love him". It is the conjunction of the conditionals "Trust him if you love him" and "Trust him only if you love him" which means that it is the conjunction of two conditionals " If you love him, trust him" and "If you do not love him, do not trust him". The context of the bi-conditional is 'you love him or you do not love him'. It cannot be avoided. But a bi-conditional, being a conjunction of two conditional prescriptions should have an avoidance value as a conjunction in his system has an avoidance set.

b) In Vranas' system all prescriptions irrespective of conditional and unconditional do not have three values of satisfaction, violation and avoidance. He attributes two values, namely, satisfaction and violation to an unconditional prescription but three values, satisfaction, violation and avoidance to a conditional one.

c) In order to consider a logical system as a deviant logic two possibilities may be considered.
A deviant system may have as a theorem the contradictory of a wff which classical logic has as a theorem.

2. A deviant logic should merely lack as a theorem a wff which classical logic has as a theorem.

For example:

'p or not p' is a theorem in C_2.

To see this theorem of C_2 by putting the values of imperative logic we find the following result:

p∨~ p

S∨~S

In imperative logic '~S' means either 'V' or 'A'.

So,

a) S∨V

S

b) SvA

S

In both cases it amounts to satisfaction.

Now if we can consider the value of satisfaction preservation criterion equivalent to truth preservation criterion then imperative logic may be considered as an extension of classical two-valued logic. But the question which still remains that as imperative sentences lack truth value and are defined in terms of non-truth-functional criterion of satisfaction can it be considered as either extended or deviant logic? Because an extended logic is an extension of classical two-valued logic which has two values, namely, truth and falsity. Nor it can be a deviant logic as a sentence in deviant logic has three values and the first of which is truth. Hence it may be concluded that it may be difficult to consider imperative logic as either a deviant logic or an extended logic in a straight way.

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