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The Different Kinds of Intuitionistic Multi L – Fuzzy Cosets with their Interactions in Normality Conditions

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ABSTRACT

In this paper, we introduce and investigate the properties of Intuitionistic Multi L - Fuzzy Cosets,Pseudo Intuitionistic Multi L – Fuzzy Cosets, and Pseudo Intuitionistic Multi L – Fuzzy Double Cosets of an Intuitionistic Multi L – Fuzzy Normal Subgroup of a group G. We introduce the concepts of these fuzzy cosets and examine their structural characteristics under various group operations. We establish necessary and sufficient conditions for an Intuitionistic Multi L – Fuzzy subset to form a normal subgroup and explore its implications in the coset structure. Furthermore, we analyze the relationships between different types of cosets, including their algebraic properties and interactions with normality conditions.

Keywords: Intuitionistic Multi L-Fuzzy Coset (IMLFC), Intuitionistic Multi L–Fuzzy Middle Coset (IMLFMC), Pseudo Intuitionistic Multi L – Fuzzy Coset (PIMLFC), Pseudo Intuitionistic Multi L–Fuzzy Double Coset (PIMLFDC), Intuitionistic Multi L-Fuzzy Normal Subgroup (IMLFNSG).

1. INTRODUCTION

Fuzzy set theory has been widely applied to various fields of mathematics, particularly in group theory, where the concept of fuzzy subgroups has gained significant attention. It has evolved significantly to accommodate various levels of uncertainty and vagueness in mathematical structures. One such extension is intuitionistic fuzzy sets, introduced by Atanassov, which incorporate both membership and non-membership functions. Further generalizations, such as L- fuzzy sets, multi L- fuzzy sets, and their intuitionistic variants, have provided a richer framework for studying algebraic structures in uncertain environments.

In group theory, the concept of fuzzy subgroups and their cosets plays a crucial role in understanding the structural properties of groups under fuzzification. In this context, we introduce and investigate the intuitionistic multi L- fuzzy coset, pseudo intuitionistic multi L- fuzzy coset, and pseudo intuitionistic multi L- fuzzy double coset associated with an intuitionistic multi L- fuzzy normal subgroup of a group G.

The study of these cosets is essential for extending classical normal subgroup properties to the fuzzy environment while maintaining consistency with intuitionistic and multi L- fuzzy frameworks. This paper explores fundamental properties, algebraic operations, and relationships between these cosets, providing insight into their role in generalized fuzzy group theory. The introduction of pseudo versions further refines the classification of elements, leading to potential applications in decision-making, pattern recognition, and algebraic structures under uncertainty.

This research aims to deepen the understanding of intuitionistic multi L- fuzzy algebraic structures by formalizing and analyzing these novel coset concepts in a group-theoretic setting.

2. PRELIMINARIES

This section lists the fundamental definitions used in the sequel.

2.1 Definition

Let X be a non-empty set. A fuzzy set A of X is defined by $A: X \to [0,1]$.

2.2 Definition

Let X be non-empty set. Let $A = \{\langle x, \mu_{A_i}(x), \gamma_{A_i}(x)\rangle : x\epsilon X\}$ in X is defined as a set of ordered sequences.

ie., $A = \{\langle x, \left(\mu_{A_1}(x), \mu_{A_2}(x), \dots, \mu_{A_i}(x), \dots\right), \left(\gamma_{A_1}(x), \gamma_{A_2}(x), \dots, \gamma_{A_i}(x), \dots\right)\rangle : x\epsilon X\}$.

Where $\mu_{A_i}: X \to [0,1], \gamma_{A_i}: X \to [0,1]$ and $0 \le \mu_{A_i}(x) + \gamma_{A_i}(x) \le 1$ for all i.

Here, $\mu_{A_1}(x) \ge \mu_{A_2}(x) \ge \dots \ge \mu_{A_i}(x) \ge \dots$, for all $x\epsilon X$ are decreasingly ordered sequence. Then the set A is said to be an Intuitionistic Multi L-fuzzy subset (IMLFS) of X.

Remark

Since we arrange the membership sequence in decreasing order, the corresponding non-membership sequence may not be in decreasing or increasing order.

2.3 Definition

The Intuitionistic Multi L-fuzzy subset $A = \{\langle x, \mu_{A_i}(x), \gamma_{A_i}(x)\rangle : x\epsilon X\}$ of a group G is said to be Intuitionistic Multi L-fuzzy subgroup of G (IMLFSG) if it satisfies the following: For all $x, y \in G$,

1. $\mu_{A_i}(xy) \ge min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$ and $\gamma_{A_i}(xy) \le max\{\gamma_{A_i}(x), \gamma_{A_i}(y)\}$,
2. $\mu_{A_i}(x^{-1}) = \mu_{A_i}(x)$ and $\gamma_{A_i}(x^{-1}) = \gamma_{A_i}(x)$.

Or Equivalently, if A is IMLFSG of G iff

$\mu_{A_i}(xy^{-1}) \ge min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$ and $\gamma_{A_i}(xy^{-1}) \le max\{\gamma_{A_i}(x), \gamma_{A_i}(y)\}$.

3. THE INTUITIONISTIC MULTI L-FUZZY COSETS WITH THEIR INTERACTIONS IN NORMALITY CONDITIONS

In this section, we discuss some properties of Intuitionistic Multi L- fuzzy cosets and Pseudo Intuitionistic Multi L – fuzzy cosets of an Intuitionistic Multi L- fuzzy normal subgroup of the group G.

3.1 Definition

Let A be an Intuitionistic Multi L-fuzzy subgroup of a group G. For any $a\epsilon G$, the Intuitionistic Multi L–fuzzy (left)coset (IMLFC)(aA) of G is defined by, $(aA) = \left\{\left(x, \mu_{(aA_i)}(x), \gamma_{(aA_i)}(x)\right) / x \in G\right\}$ where

1. $\mu_{(aA_i)}(x) = \mu_{A_i}(a^{-1}x)$
2. $\gamma_{(aA_i)}(x) = \gamma_{A_i}(a^{-1}x)$, for all $x\epsilon G$.

3.2 Definition

Let A be an Intuitionistic Multi L-fuzzy subgroup of a group G. For any $a, b\epsilon G$, the Intuitionistic Multi

L–fuzzy middle coset (IMLFMC) (aAb) of G is defined by,

$$(aAb) = \left\{\left(x, \mu_{(aA_ib)}(x), \gamma_{(aA_ib)}(x)\right)/x \in G\right\} \text{ where}$$

(i) $\mu_{(aA_ib)}(x) = \mu_{A_i}(a^{-1}xb^{-1})$

(ii) $\gamma_{(aA_ib)}(x) = \gamma_{A_i}(a^{-1}xb^{-1})$, for all $x\epsilon G$.

Remarks

1. If $a = e$ in G, then the Intuitionistic Multi L-fuzzy coset (aA) of G is defined by,
   (i) $\mu_{(aA_i)}(x) = \mu_{A_i}(x)$
   (ii) $\gamma_{(aA_i)}(x) = \gamma_{A_i}(x)$, where A is an IMLFSG of G.
2. The Intuitionistic Multi L-fuzzy middle coset of an Intuitionistic Multi L-fuzzy group A of the group G determined by the element $a, b \in G$ is an IMLFSG of G if $b = a^{-1}$.

3.3 Definition

Let A be an Intuitionistic Multi L-fuzzy subgroup of a group G and an element $a\epsilon G$. Then Pseudo Intuitionistic Multi L–fuzzy coset (PIMLFC) $(aA)^p$ of G is defined by,

$$(aA)^p = \left\{\left(x, \mu_{(aA_i)^p}(x), \gamma_{(aA_i)^p}(x)\right)/x \in G\right\} \text{ where}$$

(i) $\mu_{(aA_i)^p}(x) = p(a)\mu_{A_i}(x)$

(ii) $\gamma_{(aA_i)^p}(x) = p(a)\gamma_{A_i}(x)$, for all $x\epsilon G$ and $p\epsilon P$.

3.4 Definition

Let A and B be any two IMLFSG's of a group G, then for any element $a \in G$. Then the Pseudo Intuitionistic Multi L-fuzzy double coset (PIMLFDC) $(AaB)^p$ is defined by,

$$(AaB)^p = \left\{\left(x, \mu_{(A_iaB_i)^p}(x), \gamma_{(A_iaB_i)^p}(x)\right)/x \in G\right\} \text{ where}$$

(i) $\mu_{(A_iaB_i)^p}(x) = \mu_{((aA_i)^p \cap (aB_i)^p)}(x) = min\{(\mu_{(aA_i)^p}(x)), (\mu_{(aB_i)^p}(x))\}$ and

(ii) $\gamma_{(A_iaB_i)^p}(x) = \gamma_{((aA_i)^p \cup (aB_i)^p)}(x) = max\{(\gamma_{(aA_i)^p}(x)), (\gamma_{(aB_i)^p}(x))\}$,

for every $x \in G, p\epsilon P$, where $P = \{p(a)/p(a) \in [0,1]\}$.

3.5 Definition

The Intuitionistic Multi L-fuzzy subgroup of a group G is called an Intuitionistic Multi L-fuzzy normal subgroup (IMLFNSG) of G, if for every $x, y\epsilon G$then

1. $\mu_{A_i}(xy) = \mu_{A_i}(yx)$
2. $\gamma_{A_i}(xy) = \gamma_{A_i}(yx)$

   Where, $\mu_{A_i} = \max(\mu_{A_1}, \mu_{A_2}, \mu_{A_3}, \ldots\ldots)$and $\gamma_{A_i} = \min(\gamma_{A_1}, \gamma_{A_2}, \gamma_{A_3}, \ldots\ldots.)$.

3.6 Theorem

Let A be an Intuitionistic Multi L-fuzzy normal subgroup of a group G. The Intuitionistic Multi L- fuzzy coset (aA) is also an Intuitionistic Multi L- fuzzy normal subgroup of G.

Proof

We know that, the Intuitionistic Multi L-fuzzy coset (aA) of a Intuitionistic Multi L- fuzzy normal subgroup A of the group G determined by the element $a \in G$ is also an IMLFSG of G.

To prove that the Intuitionistic Multi L- fuzzy coset (aA) is an IMLFNSG of G.

For all$x, y \in G$.

(i) Now, $\mu_{(aA_i)}(xy) = \mu_{A_i}(a^{-1}xy)$

$= \mu_{A_i}(a^{-1}yx)$ [∵ by definition 3.5]

$= \mu_{(aA_i)}(yx)$

i.e., $\mu_{(aA_i)}(xy) = \mu_{(aA_i)}(yx)$

(ii) Similarly, $\gamma_{(aA_i)}(xy) = \gamma_{A_i}(a^{-1}xy)$

$= \gamma_{A_i}(a^{-1}yx)$ [$\because$ by definition 3.5]

$= \gamma_{(aA_i)}(yx)$

i.e., $\gamma_{(aA_i)}(xy) = \gamma_{(aA_i)}(yx)$

Hence, the Intuitionistic Multi L-fuzzy coset (aA) is also an INMLFNSG of the group G.

3.7 Theorem

If A is an Intuitionistic Multi L- fuzzy normal subgroup of a group G. Then for any $a \in G$, the Intuitionistic Multi L-fuzzy middle coset (aAa^{-1}) is also an IMLFNSG of a group G.

Proof

We know that, the Intuitionistic Multi L- fuzzy middle coset (aAa^{-1}) of G is also an IMLFSG of G.

To prove thatIntuitionistic Multi L- fuzzy middle coset $(aAa^{-1})$is also an Intuitionistic Multi L- fuzzy normal subgroup of G.

For every $x, y \in G$.

(i) Now,$\mu_{(aA_ia^{-1})}(xy) = \mu_{A_i}(a^{-1}xya)$

$= \mu_{A_i}(a^{-1}yxa)$ [$\because$ A is an IMLFNSG of G]

$= \mu_{(aA_ia^{-1})}(yx)$

i.e., $\mu_{(aA_ia^{-1})}(xy) = \mu_{(aA_ia^{-1})}(yx)$

(ii) Also,$\gamma_{(aA_ia^{-1})}(xy) = \gamma_{A_i}(a^{-1}xya)$

$= \gamma_{A_i}(a^{-1}yxa)$ [$\because$ A is an IMLFNSG of G]

$= \gamma_{(aA_ia^{-1})}(yx)$

i.e., $\gamma_{(aA_ia^{-1})}(xy) = \gamma_{(aA_ia^{-1})}(yx)$

Hence, the Intuitionistic Multi L-fuzzy middle coset (aAa^{-1}) of a Intuitionistic Multi L-fuzzy normal subgroup A of the group G determined by the element $a \in G$ is also an Intuitionistic Multi L-fuzzy normal subgroup (IMLFNSG) of a group G.

3.8 Theorem

If A is an Intuitionistic Multi L-fuzzy normal subgroup of a group G, then the set$G/A = \{xA/x \in G\}$ is a group with the operation $(xA)(yA) = (xy)A$.

Proof

Let A be an Intuitionistic Multi L-fuzzy normal subgroup of a group G.

Let $x, y \in G$.

Given the set $G/A = \{xA/x \in G\}$.

Then $xA, yA \in G/A$.

Clearly, $y^{-1} \in G$

Therefore, $y^{-1}A \in G/A$

Now, $(xA)(y^{-1}A) = (xy^{-1})A$ in G/A.

Hence, G/A is a group.

Hence the theorem.

3.9 Theorem

If A be an Intuitionistic Multi L- fuzzy normal subgroup of a group G.Then the Pseudo Intuitionistic Multi L-fuzzy coset$(aA)^p$ is also an Intuitionistic Multi L- fuzzy normal subgroup of a group G for every$a \in G$.

Proof

We know that, the Pseudo Intuitionistic Multi L-fuzzy coset $(aA)^p$ of a Intuitionistic Multi L- fuzzy normal subgroup A of the group G determined by the element $a \in G$ is also an IMLFSG of a group G.

To prove that the Pseudo Intuitionistic Multi L-fuzzy coset $(aA)^p$ is an Intuitionistic Multi L- fuzzy normal subgroup of a group G.

For every $x, y \in G$, we have

(i) $\mu_{(aA_i)^p}(xy) = p(a)\mu_{A_i}(xy)$ [∵ by definition 3.3]

$= p(a)\mu_{A_i}(yx)$ [∵ A is an IMLFNSG of G]

$= \mu_{(aA_i)^p}(yx)$

i.e.,$\mu_{(aA_i)^p}(xy) = \mu_{(aA_i)^p}(yx)$

(ii) Also, $\gamma_{(aA_i)^p}(xy) = p(a)\gamma_{A_i}(xy)$ [∵ by definition 3.3]

$= p(a)\gamma_{A_i}(yx)$ [∵ A is an IMLFNSG of G]

$= \gamma_{(aA_i)^p}(yx)$

i.e., $\gamma_{(aA_i)^p}(xy) = \gamma_{(aA_i)^p}(yx)$

Hence $(aA)^p$ is an IMLFNSG of a group G, for every $a \in G$.

3.10 Theorem

Let A and B be any two Intuitionistic Multi L- fuzzy normal subgroups of a group G. Then the Pseudo Intuitionistic Multi L-fuzzy double coset $(AaB)^p$ of a Intuitionistic Multi L-fuzzy normal subgroups A and B of the group G determined by the element $a \in G$ is also an Intuitionistic Multi L-fuzzy normal subgroup of a group G.

Proof

We know that, the Pseudo Intuitionistic Multi L-fuzzy double coset $(AaB)^p$ of a Intuitionistic Multi L-fuzzy normal subgroups A and B of the group G determined by the element $a \in G$ is also an IMLFSG of a group G.

To prove that the Pseudo Intuitionistic Multi L-fuzzy double coset $(AaB)^p$ is an Intuitionistic Multi L-fuzzy normal subgroup of a group G.

For all$x, y \in G$.

(i) Now, $\mu_{(A_iaB_i)^p}(xy) = \mu_{((aA_i)^p \cap (aB_i)^p)}(xy)$ [∵ by definition 3.4]

$= min\{\mu_{(aA_i)^p}(xy), \mu_{(aB_i)^p}(xy)\}$

$= min\{\mu_{(aA_i)^p}(yx), \mu_{(aB_i)^p}(yx)\}$ [∵ A & B are IMLFNSG of G]

$= \mu_{((aA_i)^p \cap (aB_i)^p)}(yx)$

i.e., $\mu_{(A_iaB_i)^p}(xy) = \mu_{(A_iaB_i)^p}(yx)$

(ii) Similarly, $\gamma_{(A_iaB_i)^p}(xy) = \gamma_{((aA_i)^p \cup (aB_i)^p)}(xy)$ [∵ by definition 3.4]

$= max\{\gamma_{(aA_i)^p}(xy), \gamma_{(aB_i)^p}(xy)\}$

$= max\{\gamma_{(aA_i)^p}(yx), \gamma_{(aB_i)^p}(yx)\}$ [∵ A & B are IMLFNSG of G]

$= \gamma_{((aA_i)^p \cup (aB_i)^p)}(yx)$

i.e., $\gamma_{(A_iaB_i)^p}(xy) = \gamma_{(A_iaB_i)^p}(yx)$

Hence, the Pseudo Intuitionistic Multi L-fuzzy double coset $(AaB)^p$ is an IMLFNSG of a group G.

3.11 Theorem

Let A be an Intuitionistic Multi L-fuzzy subgroup of G. Then A is an Intuitionistic Multi L- fuzzy normal subgroup of G iff $xyA = yxA$ for al l$x, y \in G$.

Proof

Given A is an IMLFSG of G.

(i) To prove that $xyA = yxA$ for all $x, y \in G$.

Let A is an IMLFNSG of a group G and for all $x, y \in G$.

Then,$\mu_{A_i}(xy) = \mu_{A_i}(yx)$ and $\gamma_{A_i}(xy) = \gamma_{A_i}(yx)$

$\mu_{A_i}((xy)^{-1}) = \mu_{A_i}((yx)^{-1})$ and $\gamma_{A_i}((xy)^{-1}) = \gamma_{A_i}((yx)^{-1})$ [∵ A is an IMLFSG of G]

$\mu_{A_i}((xy)^{-1}e) = \mu_{A_i}((yx)^{-1}e)$ and $\gamma_{A_i}((xy)^{-1}e) = \gamma_{A_i}((yx)^{-1}e)$

$\mu_{(xyA_i)}(e) = \mu_{(yxA_i)}(e)$ and $\gamma_{(xyA_i)}(e) = \gamma_{(yxA_i)}(e)$

Hence, $xyA = yxA$ for all $x, y \in G$.

(ii) Conversely, assume that $xyA = yxA$ for all $x, y \in G$.

To prove that A is an IMLFNSG of G.

Let, $xyA = yxA$ for all $x, y \in G$.

Then, $\mu_{A_i}\left(((xy)^{-1})z\right) = \mu_{A_i}\left(((yx)^{-1})z\right)$ and $\gamma_{A_i}\left(((xy)^{-1})z\right) = \gamma_{A_i}\left(((yx)^{-1})z\right)$, for all$z \in G$.

Let, $z = e$ then

$\mu_{A_i}\left(((xy)^{-1})e\right) = \mu_{A_i}\left(((yx)^{-1})e\right)$ and $\gamma_{A_i}\left(((xy)^{-1})e\right) = \gamma_{A_i}\left(((yx)^{-1})e\right)$

$\mu_{A_i}((xy)^{-1}) = \mu_{A_i}((yx)^{-1})$ and $\gamma_{A_i}((xy)^{-1}) = \gamma_{A_i}((yx)^{-1})$

$\mu_{A_i}(xy) = \mu_{A_i}(yx)$ and $\gamma_{A_i}(xy) = \gamma_{A_i}(yx)$ [∵ A is an IMLFSG of G]

Hence, A is an IMLFNSG of G.

Hence the theorem.

CONCLUSION

The extension of classical group theoretical concepts into Intuitionistic Multi L- fuzzy settings leads to richer algebraic structures with broader applications in fuzzy logic, decision-making, and uncertainty modeling. The interplay between normal subgroups and fuzzy cosets offers new perspectives on subgroup relationships and structural decomposition within multi L- fuzzy group theory. These findings provide a foundation for further exploration in fuzzy algebraic topology, fuzzy group homomorphism, and applications in computational intelligence.

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