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Hydromagnet Free Convection with Combined Heat and Mass Transfer of Viscouss Fluid in Rotating System

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Abstract

In the present investigation examine unsteady free convection flow due to heat and mass transfer of Viscous fluid bounded by an infinite vertical porous plate in rotating system under the influence of a uniform transverse magnetic field of constant strength is studied. The fluid and the plate are in a state of solid body rotation with uniform angular velocity. The temperature and concentration at the plate are assumed oscillatory with time about a constant non-zero mean. The problem is solved by using a regular expansion technique for small value of frequency parameter and perturbation techniques . Approximate solutions for the primary and secondary velocity, temperature and concentration fields are obtained. Expressions for skin-friction at the plate due to primary velocity and secondary velocity, rate of heat and mass transfer are also derived. Numerical calculations are carried out for different values of parameters occurring into the equations of the problem under study and shown through tables and graphs.

Keyword: Hydromagnetic, Rotating system , Convection , Mass Transfer, Rotating System, Primary and Secondary velocity, MHD Flow.

1. INTRODUCTION

Free convection flow is in fact the transport of energy resulting from a distributed force arising from variation of density. Such a flow, specially in porous medium, is one of the most interesting subject matter because of its wide applications in many industrial problems particularly in nuclear industry, petroleum, and chemical industry. Such type of applications includes natural circulation in isothermal reservoir, heat storage beds, grain storage extraction of geothermal energy and thermal insulation design. The magnetic effects can also be vitally used in power generation.

Several buoyancy driven boundary layer flow have been studied by Cheng and Minkowycz [1977], Cheng [1977, 1977], Debnath [1979], Raptis et. al. [1981, 82] etc. Oscillatory flow through porous medium has been analyzed by Raptis [1983] for small amplitude of oscillations only. To overcome this restriction Singh et. al. [1989] have analysed MHD flow through a porous medium by the presence of free stream velocity. Hossian et. al. [1998] have studied the effects of dust particles on the flow of incompressible fluid in a rotating system. Singh et. al. [2001] have studied effects of Hall current on unsteady hydromagnetic boundary layer flow in rotating dusty viscous liquid while Sharma et. al. [2001] have studied thermos solute convection in Rivlin-Ericksen rotating flow in porous medium under transverse magnetic field. Recently, Takhar et. al. [2003] have studied unsteady mixed convection flow from a rotating vertical cone with a magnetic field, heat and mass transfer. More recently, Gupta et. al. [2005] have analysed free convection heat and mass transfer flow through a porous medium with heat source / sink under the influence of magnetic field.

The object of the present investigation is to study the MHD unsteady free convection flow due to heat and mass transfer bounded by an infinite vertical porous plate under rotating system. Considering the fluid and the plate are in a state of solid body rotation with constant angular velocity, the problem of Gupta et. al. [2005] and Hydromagnetic free convection with combined heat and mass transfer in rotating system [2007] Kumar Ashish et.al is extended. Approximate solutions for the primary and secondary velocity, temperature and concentration fields are obtained. Expressions for skin-friction at the plate due to primary velocity and secondary velocity, rate of heat and mass transfer are also derived. The results obtained are discussed through tables and graphs.

2. FORMULATION OF THE PROBLEM

Consider oscillatory flow of an incompressible, electrically conducting, viscous liquid past an infinite, hot, vertical porous plate with constant heat source. In cartesian coordinate (x, y, z) system in presence of uniform magnetic field. We assume x-axis and y-axis in the plane of the porous plate and z-axis normal to the plate with velocities (u, v, w) in these directions respectively. Both the liquid and the plate are considered in a state of rigid body rotation about z-axis with uniform angular velocity Ω. Further we assume that the uniform magnetic field $\overrightarrow{B_0} = \mu_e \overrightarrow{H}$ where $\overrightarrow{H} = (0, 0, H_0)$ is applied in the z direction and the magnetic Reynold number is assume small. The constant heat source Q is assumed at $z = 0$ and the suction velocity at the plate is $w = -w_0\left(1+\varepsilon e^{\text{int}}\right)$ where w_0 and n are positive real numbers. In this analysis buoyancy force, Hall effect, effect due to perturbation of the field, induced magnetic field and polarization effect are ignored. Since the plate is infinite in extant, all physical variables depends on z and t only. Initially, when $t \le 0$ the plate and the fluid are assumed to be at the same temperature T_∞ and the foreign mass is assumed to be uniformly distributed in the flow region such that it is everywhere C_∞. At $t > 0$, the temperature of the plate is instantaneously raised to T_w and the concentration of species is raised to C_w and temperature maintained constant. Under the above stated assumptions and usual Boussinesq's approximation the equations of motion are :

$$\frac{\partial u}{\partial t} - w\frac{\partial u}{\partial z} - 2\Omega v = \vartheta\frac{\partial^2 u}{\partial z^2} + g\beta\left(T - T_\infty\right) + g\beta^*\left(C - C_\infty\right) - \frac{\sigma\mu_e^2 H_0^2}{\rho}u \qquad (1)$$

$$\frac{\partial v}{\partial t} - w\frac{\partial v}{\partial z} + 2\Omega u = \vartheta\frac{\partial^2 v}{\partial z^2} - \frac{\sigma\mu_e^2 H_0^2}{\rho}v \qquad (2)$$

$$\frac{\partial T}{\partial t} - w\frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial z^2} + Q\left(T - T_\infty\right) \qquad (3)$$

$$\frac{\partial C}{\partial t} - w\frac{\partial C}{\partial z} = D_M\frac{\partial^2 C}{\partial z^2} \qquad (4)$$

For suction velocity, we have

$$w = -w_0\left(1+\varepsilon e^{\text{int}}\right) \tag{5}$$

where g is the acceleration due to gravity, β is the volumetric coefficient of the thermal expansion, $\beta *$ is the volumetric coefficient with concentration, σ is the electrical conductivity of the liquid, ρ is the density of the liquid, μ_e is the magnetic permeability, H_0 is the constant magnetic field, T is the temperature, C is the concentration, D_M is the molecular diffusivity and the other symbols have their usual meaning.

The boundary conditions relevant to the problem are :

$$u=0,\ v=0,\ T=T_w+\varepsilon\left(T_w-T_\infty\right)e^{int},$$
$$C=C_w+\varepsilon\left(C_w-C`_\infty\right)e^{int} \quad \text{at} \quad z=0$$
$$u\to 0, \quad v\to 0, \quad T\to T_\infty, \quad C\to C_\infty \quad \text{as} \quad z\to\infty \tag{6}$$

We introduce the following non-dimensional quantities :

$$u^* = \frac{u}{w_0 FG},\ v^* = \frac{v}{w_0 FG},\ T^* = \frac{T-T_\infty}{\left(T_w-T_\infty\right)F},\ C^* = \frac{C-C_\infty}{\left(C_w-C_\infty\right)F},$$
$$z^* = \frac{w_0 z}{w}, \quad t^* = nt, \quad n^* = \frac{nW}{w_0^2}$$

Introducing above non-dimensional variables, the equations (1)-(4), after ignoring the stars over them, reduce to :

$$\frac{\partial^2 u}{\partial z^2}+\frac{\partial u}{\partial z}-n\left(\frac{\partial u}{\partial t}+u\frac{F'}{F}\right)+T+CN-M^2u+2\Omega v=0 \tag{7}$$

$$\frac{\partial^2 v}{\partial z^2}+\frac{\partial v}{\partial z}-n\left(\frac{\partial v}{\partial t}+v\frac{F'}{F}\right)-M^2v-2\Omega u=0 \tag{8}$$

$$\frac{1}{P_r}\frac{\partial^2 T}{\partial z^2}+\frac{\partial T}{\partial z}-n\left(\frac{\partial T}{\partial t}+T\frac{F'}{F}\right)+ST=0 \tag{9}$$

$$\frac{1}{S_c}\frac{\partial^2 C}{\partial z^2}+\frac{\partial C}{\partial z}-n\left(\frac{\partial C}{\partial t}+C\frac{F'}{F}\right)=0 \tag{10}$$

Now using $q=u+iv$ in the equations (7)-(8), we get :

$$\frac{\partial^2 q}{\partial z^2}+\frac{\partial q}{\partial z}-n\left(\frac{\partial q}{\partial t}+q\frac{F'}{F}\right)-M^2q+2i\Omega q+T+CN=0 \tag{11}$$

The boundary conditions (6) are transformed to :

$$q=0, \quad T=1, \quad C=1 \quad \text{at} \quad z=0$$
$$q\to 0, \quad T\to 0, \quad C\to 0 \quad \text{as} \quad z\to\infty \tag{12}$$

3. SOLUTION OF THE PROBLEM

To solve the problem, we can express $q(z,t)$, $T(z,t)$ and $C(z,t)$ in the power series of n in the form :

$$q(z,t) = q_0(z) + \sum_{r=1}^{\infty} n^r q_r(z,t)$$

$$T(z,t) = T_0(z) + \sum_{r=1}^{\infty} n^r T_r(z,t)$$

$$C(z,t) = C_0(z) + \sum_{r=1}^{\infty} n^r C_r(z,t) \quad (13)$$

Using the expressions of (13) into (9)-(11), we get the following set of equations:

$$\frac{1}{P_r}\frac{\partial^2 T_0}{\partial z^2} + \frac{\partial T_0}{\partial z} + ST_0 = 0 \quad (14)$$

$$\frac{1}{P_r}\frac{\partial^2 T_1}{\partial z^2} + \frac{\partial T_1}{\partial z} + ST_1 = \frac{F'}{F}T_0(z) \quad (15)$$

$$\frac{1}{P_r}\frac{\partial^2 T_2}{\partial z^2} + \frac{\partial T_2}{\partial z} + ST_2 = \frac{\partial T_1(z,t)}{\partial t} + \frac{F'}{F}T_1(z,t) \quad (16)$$

$$\frac{1}{S_c}\frac{\partial^2 C_0}{\partial z^2} + \frac{\partial C_0}{\partial z} = 0 \quad (17)$$

$$\frac{1}{S_c}\frac{\partial^2 C_1}{\partial z^2} + \frac{\partial C_1}{\partial z} = \frac{F'}{F}C_0(z) \quad (18)$$

$$\frac{1}{S_c}\frac{\partial^2 C_2}{\partial z^2} + \frac{\partial C_2}{\partial z} = \frac{\partial C_1(z,t)}{\partial t} + \frac{F'}{F}C_1(z,t) \quad (19)$$

$$\frac{\partial^2 q_0}{\partial z^2} + \frac{\partial q_0}{\partial z} - M^2 q_0 - 2i\Omega q_0 = -T_0(z) - NC_0(z) \quad (20)$$

$$\frac{\partial^2 q_1}{\partial z^2} + \frac{\partial q_1}{\partial z} - M^2 q_1 - 2i\Omega q_1 = \frac{F'}{F}q_0(z) - T_1(z,t) - NC_1(z,t) \quad (21)$$

$$\frac{\partial^2 q_2}{\partial z^2} + \frac{\partial q_2}{\partial z} - M^2 q_2 - 2i\Omega q_2 = \frac{\partial q_1(z,t)}{\partial t} + \frac{F'}{F}q_1(z,t) - T_2(z,t) - NC_2(z,t) \quad (22)$$

The boundary conditions (12) are transformed to:

$$q_0 = q_1 = q_2 = 0, \quad T_0 = 1, T_1 = T_2 = 0,$$
$$C_0 = 1, C_1 = C_2 = 0 \quad \text{at} \quad z = 0$$
$$q_0 = q_1 = q_2 \to 0, \quad T_0 = T_1 = T_2 \to 0,$$
$$C_0 = C_1 = C_2 \to 0 \quad \text{as} \quad z \to \infty \quad (23)$$

The solutions of (14)-(22), under the transformed boundary conditions (23) are :

$$T_0(z) = e^{-R_1 z} \quad (24)$$

$$C_0(z) = e^{-S_c z} \quad (25)$$

$$q_0(z) = \left(\frac{1}{D_3} + \frac{N}{D_2}\right)e^{-D_1 z} - \frac{1}{D_3}e^{-R_1 z} - \frac{N}{D_2}e^{-S_c z} \tag{26}$$

$$T_1(z,t) = -\frac{P_r F'}{F R_2} z e^{-R_1 z} \tag{27}$$

$$C_1(z,t) = -\frac{F'}{F} z e^{-S_c z} \tag{28}$$

$$q_1(z,t) = \frac{F'}{F}\left[\left(D_4 + D_5 + D_6 z\right)e^{-D_1 z} - \left(D_5 + \frac{P_r z}{R_2 D_3}\right)e^{-R_1 z} - \left(D_4 - \frac{Nz}{D_2}\right)e^{-S_c z}\right] \tag{29}$$

$$T_2(z,t) = \frac{P_r^2 F''}{F R_2^2}\left(\frac{z}{2} + \frac{1}{R_2}\right) z e^{-R_1 z} \tag{30}$$

$$C_2(z,t) = \frac{F''}{F}\left(\frac{z}{2} + \frac{1}{S_c}\right) z e^{-S_c z} \tag{31}$$

$$q_2(z,t) = \frac{F'}{F}\left[\left(D_9 z - D_{10} - D_{11} + \frac{D_6 z^2}{2(1-2D_1)}\right)e^{-D_1 z} - \left(\frac{P_r^2 z^2}{2R_2^2 D_3} - D_7 z - D_{10}\right)e^{-R_1 z} - \left(\frac{Nz^2}{2D_2} - D_8 z - D_{11}\right)e^{-S_c z}\right] \tag{32}$$

Hence primary velocity $u(z,t)$ and secondary velocity $v(z,t)$ are :

$$u(z,t) = u_0(z) + n u_1(z,t) + n^2 u_2(z,t)$$

$$v(z,t) = v_0(z) + n v_1(z,t) + n^2 v_2(z,t)$$

4. SKIN-FRICTION AND RATE OF HEAT

The skin-friction (τ_p) due to primary velocity and skin-friction (τ_s) due to secondary velocity at the plate are obtained as follows :

$$\tau_p = \left(\frac{\partial u}{\partial z}\right)_{z=0} = R_{24} - \varepsilon \frac{n^2}{\left(1 + 2\varepsilon\cos nt + \varepsilon^2\right)}\left\{R_{26}\sin nt + R_{27}\left(\cos nt + \varepsilon\right)\right\} \tag{33}$$

$$\tau_s = \left(\frac{\partial v}{\partial z}\right)_{z=0} = R_{25} + \varepsilon \frac{n^2}{\left(1 + 2\varepsilon\cos nt + \varepsilon^2\right)}\left\{R_{26}\left(\cos nt + \varepsilon\right) - R_{27}\sin nt\right\} \tag{34}$$

The rate of heat transfer in terms of Nusselt number (N_u) at the plate is :

$$N_u = \left(\frac{\partial T}{\partial z}\right)_{z=0} = -R_1 + \varepsilon \frac{n^2}{\left(1+2\varepsilon\cos nt+\varepsilon^2\right)} \left\{ \frac{P_r^2 \sin nt}{R_2} - \frac{nP_r^2}{R_2^3}\left(\cos nt + \varepsilon\right) \right\} \quad (34)$$

The rate of mass transfer in terms of Sherwood number (S_h) at the plate is :

$$S_h = \left(\frac{\partial C}{\partial z}\right)_{z=0} = -S_c + \varepsilon \frac{n^2}{\left(1+2\varepsilon\cos nt+\varepsilon^2\right)} \left\{ \sin nt - \frac{n}{S_c}\left(\cos nt + \varepsilon\right) \right\} \quad (36)$$

TABLE-1

EFFECTS OF VARIOUS PARAMETERS ON SKIN-FRICTION (τ_p)

DUE TO PRIMARY VELOCITY

($S_c = 0.30$, $n = 0.2$, $t = 1.0$ and $\varepsilon = 0.1$)

P_r	M	G_r	G_m	S	τ_p
0.71	0.5	12.0	14.0	0.4	2.62412
7.00	0.5	12.0	14.0	0.4	0.68527
0.71	1.0	12.0	14.0	0.4	2.18758
0.71	0.5	15.0	14.0	0.4	4.32647
0.71	0.5	12.0	18.0	0.4	5.18268
0.71	0.5	12.0	14.0	0.6	2.48715

TABLE-2

EFFECTS OF VARIOUS PARAMETERS ON SKIN-FRICTION (τ_s)

DUE TO SECONDARY VELOCITY

($S_c = 0.30$, $n = 0.2$, $t = 1.0$ and $\varepsilon = 0.1$)

P_r	M	G_r	G_m	S	τ_s
0.71	0.5	12.0	14.0	0.4	-1.74785
7.00	0.5	12.0	14.0	0.4	-0.79412
0.71	1.0	12.0	14.0	0.4	-1.56712
0.71	0.5	15.0	14.0	0.4	-3.49021
0.71	0.5	12.0	18.0	0.4	-4.56514
0.71	0.5	12.0	14.0	0.6	-1.67587

TABLE-3

EFFECTS OF P_r AND S ON RATE OF HEAT TRANSFER

IN TERMS OF NUSSELT NUMBER (N_u)

(at $n = 0.2$, $t = 1.0$ and $\varepsilon = 0.1$)

P_r	S	N_u

0.71	0.5	-1.88627
7.00	0.5	-7.05724
11.4	0.5	-11.64986
0.71	1.0	-1.84717
0.71	0.0	-1.34108
0.71	-0.5	-4.13587

TABLE-4
EFFECTS OF S_c ON RATE OF MASS TRANSFER
IN TERMS OF SHERWOOD NUMBER (S_h)
(at $n = 0.2$, $t = 1.0$ and $\varepsilon = 0.1$)

S_c	S_h
0.22	-0.22743
0.30	-0.30467
0.60	-0.60582
0.66	-0.66594
0.78	-0.78493

5. DISCUSSION AND CONCLUSIONS

The equations of momentum and energy are solved and distributions plotted. An intensive study of the equations encountered into the problem shows that the primary velocity field is governed by Prandtl number (P_r), magnetic parameter (M), Grashof number (G_r), modified Grashof number (G_m) and heat source parameter (S). In order to get physical depth of the problem, the primary velocity, secondary velocity, temperature field and concentration field are discussed choosing different numerical values of the parameters encountered in the expressions of velocities fields, temperature field and concentration fields. To be realistic, the numerical values of Prandtl number are chosen to be $P_r = 0.71$, $P_r = 7.0$ and $P_r = 11.4$, which respectively correspond to air, water at 20^0C and water at 4^0C . The numerical values of Schmidt number are chosen to be $S_c = 0.22$, $S_c = 0.30$, $S_c = 0.60$, $S_c = 0.66$, $S_c = 0.78$, which respectively corresponds to hydrogen, helium, oxygen, water-vapor and ammonia. The numerical values of Grashof number are chosen for $G_r > 0$, which corresponds to cooling case of the plate. The numerical values of the remaining parameters, although chosen arbitrarily, are in agreement with those of researchers of the field.

The primary and secondary velocities fields are numerically observed to discuss the effects of Prandtl number (P_r), magnetic parameter (M), Grashof number (G_r), modified Grashof number (G_m) and heat source parameter (S) with the help of Fig.1 and Fig.2 while the skin-friction is numerically observed in Table-1 and Table-2 to discuss the effects of the said parameters. The temperature field is numerically observed to discuss the effects of Prandtl number (P_r) and heat source parameter (S) with the help of Fig.-3 while the rate of heat transfer is numerically presented in the Table-3. The concentration field is numerically observed to discuss the effects of Schmidt number (S_c) with the help of Fig.-4 while the rate of mass transfer is numerically presented in the Table-4 to discuss the effects of above stated parameters. In the case of velocities field, the effects are observed at $S_c = 0.30$, $n = 0.2$, $t = 1.0$ and $\varepsilon = 0.1$ while in the case of temperature field and concentration field the effects are observed at $n = 0.2$, $t = 1.0$ and $\varepsilon = 0.1$ for the purpose of discussion and conclusions. The conclusions of the study are drawn on the basis of these fixed numerical values and variation in values of the remaining parameters. The conclusions of the study are as follows :

1. The primary velocity increases near the plate and after attaining a maximum value it decreases as z increases.
2. An increase in G_r, G_m or S increases the primary velocity while an increase in P_r or M decreases the primary velocity.
3. The secondary velocity decreases near the plate and after attaining a minimum value it increases as z increases.
4. An increase in G_r, G_m or S decreases the secondary velocity while an increase in P_r or M increases the secondary velocity.
5. An increase in P_r or S decreases the temperature field.
6. It is interesting to note that for constant heat sink the temperature field increases while for constant heat source the temperature field decreases.
7. An increase in S_c decreases the concentration filed.
8. An increase in G_r or G_m increases the skin-friction (τ_p) due to primary velocity while an increase in P_r, M or S decreases the skin-friction (τ_p) due to primary velocity.
9. An increase in P_r, M or S increases the skin-friction (τ_s)due to secondary velocity while an increase in G_r or G_m decreases the skin-friction (τ_s) due to secondary velocity.
10. An increase in P_r decreases the rate of heat transfer in terms of Nusselt number (N_u) while an increase in S increases the rate of heat transfer in terms of Nusselt number (N_u)

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