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A Newer Approach to Aerodynamic Design of Wings for Wide Range of Mach number Application

S C Gupta

Department of Aeronautical Engineering, MVJ College of Engineering, Bangalore

Abstract

A flat surface wing is optimized for minimum induced drag for a high subsonic Mach number. Wing for transport airplane role is considered.Optimal warp is separated into twist and camber. Angle-of-attack is alleviated by the value of twist at root, and the optimal warp is reduced by this value. Now aerofoil thickness is superimposed and the 3-D aerofoils so generated are analysed for transonic flow through finite difference methodology.

Nomenclature

A = Panel Area
$a_{i,j}$ = Influence coefficient of j_{th}panel on i_{th} control point
b = wing span
c = local chord
Cr = Root chord
Cp = Pressure coefficient
ΔCp =Pressure difference coefficient
C_D =Induced drag coefficient
C_L =Lift coefficient
D = Induced drag
L = Lift
M = local Mach number
M_∞ = Mach number of freestream
N = Number of panels
U = Freestream velocity
u,v,w = Three components of velocities in the x, y, and z directions respectively.
x,y,z = Chordwise, spanwise and vertical coordinates respectively
ρ =Density
α = Angle-of-attack (Alpha)
γ = Circulation strength
φ =Perturbation velocity potential

Suffix

i = Control point index

j = Panel index

Introduction

Spanwise lift distribution on a 3-D wing can be tailored to produce minimum induced drag through washout effects. Resulting fall in lift can be recovered through cambering of aerofoils. Washout can be created in a controlled manner to retain prescribed lift and reduce the induced drag. The process requiresaninitial representation of aerofoil to start with an objectivewhich is to reduce lift dependent drag without affecting the prescribed Lift. Potential flow is considered in this work. Minimum drag due to lift is considered as the objective for the optimization. Lift is determined through panel methodology. Matrix of optimization is formed through principles of calculus of variations. High subsonic Mach number is considered, a wing of aspect ratio of seven is considered with a moderate leading and trailing edge sweeps.Such a wing has application on a transport aircraft. Higher aspect ratio means more span and lesser lift dependent drag, but the wing root bending moment becomes large and torsion effects are undesirable. Higher aspect ratio wing also has smaller root chord. Because of these considerations a little lower side of aspect ratio is preferred.Initially a flat surface is considered for optimization. The resulting optimal warp is split into twist and camber. Angle-of-attack is alleviated by the value of twist at root, and camber is also reduced by this value. Thus, maintaining the same value of lift coefficient and aerodynamic efficiency at a reduced value of angle-of-attack, to which the wing is optimized.

The remaining camber is combined with aerofoil thickness and residual twist is superimposed. NACA 0008 aerofoil is taken for this purpose. This combination is subjected to transonic flow analysis. Finite difference methodology is formulated for developing the computer program. When the normal component of flow to local panel sweep is less thanunity,then the partial derivatives in difference equations are approximated by central difference.When the normal component of flow to local panel sweep is greater than unity, then the partial derivatives in difference equations are approximated by upwind difference. This allows the numeric scheme to mimic physical behavior of flow field. The changeover of schemes is equivalent to injection of artificial viscosity.

Mathematical Formulation

In the approach made herein, wing is represented by a large number of constant pressure panels to model circulation. These panels are used for estimation of pressure difference coefficients. Only half of the wing is considered. Other half of wing is imaged. The program uses vortex panel method to determine singularity strength of each of panels. Panelcirculation (γ) is determined through tangential flow boundary condition and pressure difference coefficient is given by $\Delta Cp = 2\gamma / U$. Lift is determined through integration of ΔCp over the chord and span. Program developed herein generates the output matrix of pressure difference coefficients from where lift, induced drag and moments are determined. Thereafter optimization constraint of lift is introduced i.e., lift before and after the optimization is maintained same. Due to the effort of optimization, the initial flat wing becomes cambered and gets twisted thereby creating washout. Chordwise paneling is taken for geometry discretization.

Objective function (F) for the drag minima and specified value of lift $\bar{L}$ is written in

Lagrange form using Lagrange multiplier λ_0 as below:

$$F = D + \lambda_0 (L - \bar{L}) \quad (1)$$

Lift is obtained from the following expression:

$$L = \rho U\left[A_1\gamma_1 + \cdots + A_N\gamma_N\right] \tag{2}$$

Here $\gamma_1, \ldots\ldots \gamma_N$ are circulation strength of N number of panels & $A_1, .. A_N$ are related panel areas. Matrix of optimization Eq.(3) is obtained through differentiation of objective function (F) w.r.t circulation γ [3,4].

$$\begin{bmatrix} 2A_1a_{1,1} & . & . & . & (A_1a_{1,N} + A_Na_{N,1}) & A_1 \\ . & & & & . & . \\ . & & & & . & . \\ . & & & & . & . \\ (A_Na_{N,1} + A_1a_{1,N}) & . & . & . & 2A_Na_{N,N} & A_N \\ A_1 & . & . & . & A_N & 0 \end{bmatrix} \times \begin{bmatrix} \gamma_1 \\ . \\ . \\ . \\ \gamma_N \\ \lambda_0 \end{bmatrix} = \begin{bmatrix} 0 \\ . \\ . \\ . \\ 0 \\ \bar{L} \end{bmatrix} \tag{3}$$

Once the circulation is found, the optimal ($dz/dx = ZX_i$) is to support the minimum drag producing lift is determined from the following equation:

$$ZX_i = (a_{i,1}\gamma_1 + \cdots + a_{i,n}\gamma_n)A_i \tag{4}$$

Drag is given by Eq. (6)

$$D = \rho U\begin{bmatrix} (a_{1,1}\gamma_1 + \ldots + a_{1,N}\gamma_N)\gamma_1A_1 + (a_{2,1}\gamma_1 + \ldots. + a_{2,N}\gamma_N)\gamma_2A_2 \\ + \ldots\ldots\ldots\ldots\ldots\ldots\ldots. + (a_{N,1}\gamma_1 + \ldots. + a_{N,N}\gamma_N)\gamma_NA_N \end{bmatrix} \tag{5}$$

Resulting optimal warp ZX_i is separated into spanwise twist and camber.

Non-linear Eq.(6) is taken for the development of full potential flow[4,5]. This equation is of mixed type representing both elliptic as well as hyperbolic nature. It valid for subsonic, transonic and supersonic regimes.

$$\Bigg[\underbrace{1 - M_\infty^2 - \frac{2}{U}M_\infty^2\varphi_x\left(1 + \frac{\gamma - 1}{2}M_\infty^2\right)}\Bigg]\varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0 \tag{6}$$

>0, elliptic equation

<0, hyperbolic equation

Local Mach number is given by expression below [2]:

$$M^2 = \left[1 + 2\frac{u}{U}\left(1 + \frac{\gamma - 1}{2}M_\infty^2\right)\right]M_\infty^2$$

Thus Eq.(6) becomes:

$$\left[1 - M^2\right]\varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0$$

Equation is solved with finite difference methodology with point relaxation process. Marching is first done in chordwise direction, and then in spanwise direction, and lastly in vertical direction.

The computational plane from the geometric plane is developed in the following manner as shown in Figure 1.

$$\xi = \frac{x - x_{1e}}{x_{te} - x_{le}}, \ \eta = y, \ \zeta = z \qquad (7)$$

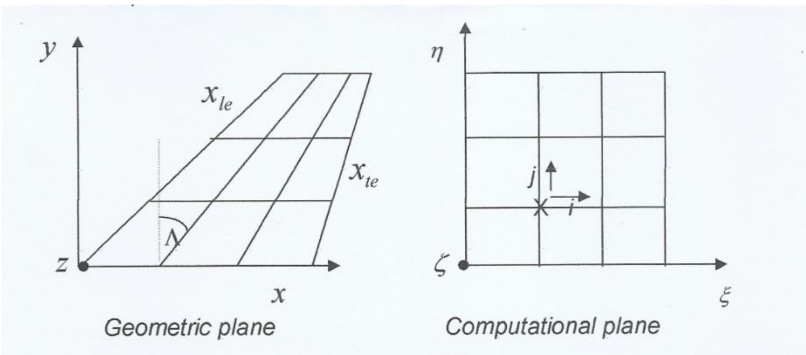

Figure 1. Geometric and computational planes

Using this transformation, the values of ϕ derivatives become as below:

$$\phi_x = \phi_\xi \, \xi_x$$

$$\phi_y = \phi_\xi \, \xi_y + \phi_\eta$$

$$\phi_z = \phi_\zeta$$

$$\phi_{xx} = \left(\phi_\xi \quad \xi_x\right)_\xi \xi_x$$

$$\phi_{yy} = \left(\phi_\xi \quad \xi_y + \phi_\eta\right)_\xi \xi_y + \left(\phi_\xi \quad \xi_y + \phi_\eta\right)_\eta$$

$$\phi_{zz} = \phi_{\xi\xi}$$

Through these expressions, transonic small perturbation Eq.(1) is rewritten below.

$$\left(1 - M^2\right)\left(\phi_\xi \, \xi_x\right)_\xi + \frac{\xi_y}{\xi_x}\left(\phi_\xi \, \xi_y + \phi_\eta\right)_\xi + \frac{1}{\xi_x}\left(\phi_\xi \, \xi_y + \phi_\eta\right)_\eta + \frac{1}{\xi_x}\phi_{\zeta\zeta} = 0 \qquad (8)$$

where $\xi_x = 1/c$,and $\xi_y = \tan \wedge / c$

or $\left[\left(1 - M^2\right)\xi_x^2 + \xi_y^2\right] \quad \varphi_{\xi\xi} + \xi_y \left(\varphi_{\eta\xi} + \varphi_{\xi\eta}\right) + \varphi_{\eta\eta} + \varphi_{\varsigma\varsigma} = 0$ (9)

Condition for elliptic nature of this equation that represents subsonic leading edges of panels in a locally supersonic flow is $\left[\left(1 - M^2\right) \xi_x^2 + \xi_y^2\right] > 0$.

Condition for hyperbolic nature of this equation representing supersonic leading edges of panels is $\left[\left(1 - M^2\right) \xi_x^2 + \xi_y^2\right] < 0$.

The expressions for $\left(\phi_\eta\right)_\xi$ and $\left(\phi_\xi\right)_\eta$ for elliptic region are given by Eqs. (10,11), and expressions for hyperbolic region are given by Eqs. (11,12).

The derivatives of φ are determined from following equations.

$$\left(\varphi_\eta\right)_\xi = \frac{\varphi_{\eta_{i+1,j,k}} - \varphi_{\eta_{i-1,j,k}}}{\xi_{i+1,j,k} - \xi_{i-1,j\,k}}$$

$$= \frac{\left(\varphi_{i+1,\,j+1,\,k} - \varphi_{i+1,\,j-1,\,k}\right)}{\left(\xi_{i+1,\,j,\,k} - \xi_{i-1,\,j\,k}\right)\left(\eta_{i+1,\,j+1,\,k} - \eta_{i+1,\,j-1,\,k}\right)} - \frac{\left(\varphi_{i-1,\,j+1,\,k} - \varphi_{i-1,\,j-1,\,k}\right)}{\left(\xi_{i+1,\,j,\,k} - \xi_{i-1,\,j,\,k}\right)\left(\eta_{i-1,\,j+1,\,k} - \eta_{i-1,\,j-1,\,k}\right)} \quad (10)$$

$$\left(\varphi_{\xi}\right)_{\eta} = \frac{\varphi_{\xi_{i,\,j+1,\,k}} - \varphi_{\xi_{i,\,j-1,\,k}}}{\eta_{i,\,j+1,\,k} - \eta_{i,\,j-1,\,k}}$$

$$= \frac{\left(\varphi_{i+1,\,j+1,\,k} - \varphi_{i-1,\,j+1,\,k}\right)}{\left(\eta_{i,\,j+1,\,k} - \eta_{i,\,j-1,\,k}\right)\left(\xi_{i+1,\,j+1,\,k} - \xi_{i-1,\,j+1,\,k}\right)} - \frac{\left(\varphi_{i+1,\,j-1,\,k} - \varphi_{i-1,\,j-1,\,k}\right)}{\left(\eta_{i,\,j+1,\,k} - \eta_{i,\,j-1,\,k}\right)\left(\xi_{i+1,\,j-1,\,k} - \xi_{i-1,\,j-1,\,k}\right)} \quad (11)$$

$$\left(\varphi_{\eta}\right)_{\xi} = \frac{\varphi_{\eta_{i,\,j,\,k}} - \varphi_{\eta_{i-2,\,j,\,k}}}{\xi_{i,\,j,\,k} - \xi_{i-2,\,j,\,k}}$$

$$= \frac{\left(\varphi_{i,\,j+1,\,k} - \varphi_{i,\,j-1,\,k}\right)}{\left(\xi_{i,\,j,\,k} - \xi_{i-2,\,j,\,k}\right)\left(\eta_{i,\,j+1,\,k} - \eta_{i,\,j-1,\,k}\right)} - \frac{\left(\varphi_{i-2,\,j+1,\,k} - \varphi_{i-2,\,j-1,\,k}\right)}{\left(\xi_{i,\,j,\,k} - \xi_{i-2,\,j,\,k}\right)\left(\eta_{i-2,\,j+1,\,k} - \eta_{i-2,\,j-1,\,k}\right)} \quad (12)$$

$$\left(\varphi_{\xi}\right)_{\eta} = \frac{\varphi_{\xi_{i,\,j+1,\,k}} - \varphi_{\xi_{i,\,j-1,\,k}}}{\eta_{i,\,j+1,\,k} - \eta_{i,\,j-1,\,k}}$$

$$= \frac{\left(\varphi_{i,\,j+1,\,k} - \varphi_{i-2,\,j+1,\,k}\right)}{\left(\eta_{i,\,j+1,\,k} - \eta_{i,\,j-1,\,k}\right)\left(\xi_{i,\,j+1,\,k} - \xi_{i-2,\,j+1,\,k}\right)} - \frac{\left(\varphi_{i,\,j-1,\,k} - \varphi_{i-2,\,j-1,\,k}\right)}{\left(\eta_{i,\,j+1,\,k} - \eta_{i,\,j-1,\,k}\right)\left(\xi_{i,\,j-1,\,k} - \xi_{i-2,\,j-1,\,k}\right)} \quad (13)$$

Equation (9) is transformed into difference equation through above relations and solved by point relaxation process. In the case of supersonic leading edges of panels $\left[\left(1 - M^2\right)\xi_x^2 + \xi_y^2\right] < 0$ which indicates requirement of upwind bias.

Boundary Conditions. Tangential flow boundary conditions are used for solution process i.e. in potential flow the flow velocity vector $\vec{q}$ immediately adjacent to the wall must be tangent to the wall. If $\hat{n}$ is a unit normal vector at a point on the surface, the wall boundary condition can be given by $\vec{q}\hat{n} = 0$ (at the surface) i.e., the velocity perpendicular to the wall is zero thus;.

$$\frac{w(x,0)}{U + u(x,0)} = \left(\frac{\partial z}{\partial x}\right)_c \quad (14)$$

Pressure coefficient is given by Eq. (15).

$$C_P = -\left(2\frac{u}{U} + \left(1 - M_{\infty}^2\right)\frac{u^2}{U^2} + \frac{v^2 + w^2}{U^2}\right) \quad (15)$$

Critical pressure coefficient is given by Eq. (16).

$$C_{Pcr} - \frac{2}{\gamma M_{\infty}^{2}}\left[1-\left(\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}M_{\infty}^{2}\right)^{\frac{\gamma}{\gamma-1}}\right]$$

Results and Discussions

Wing as shown by template belowis taken for our studies:

Leading edge sweep = 28.0^0 Trailing edge sweep = 14.6^0
Aspect ratio = 7 Taper ratio = 0.35

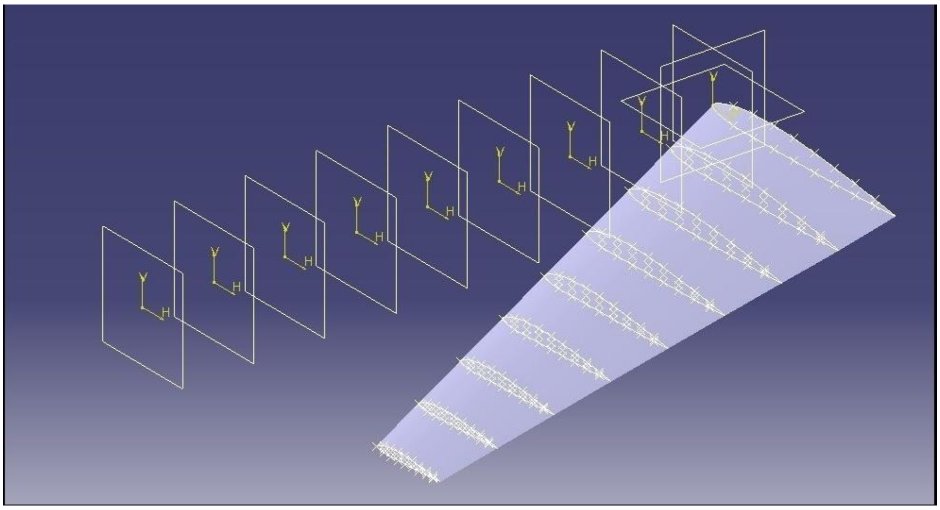

Figure-2.Template showing configuration of the wing

Tables 1 and 2 show the aerodynamic coefficients before and after the optimisation. There is substantial drag reduction by this otimisation process. Chordwise variation of vertical ordinates of optimal warp are shown in Figure 3. These are split into spanwise twist and camber (Figure 4 and 5 to refer). Wing root value of twist is 1.35 degree, and this is used to alleviate angle-of-attack. Also, the camber is altered by subtracting it from this value. Resulting camber is superimposed with remaining twist and angle-of-attack now is 2.65 degrees. This combination does not alter the optimal aerodynamic coefficients that are obtained for 4 degree of angle-of-attack given by Table-2. The aerofoils with newer data are now operated at original alpha value of 4^0. Table-3 shows the resulting data.

Table-1 Data before optimization, Alpha= 4^0

Mach Number	C_L	C_D	C_L/C_D
0.75	.376	.0262	14.37

Table-2 Data after optimization, Alpha= 4^0

Mach Number	C_L	C_D	C_L/C_D
0.75	.376	.0093	40.5

Table-2 Data after alleviation of Alpha, at Alpha=4⁰

Mach Number	C_L	C_D	C_L/C_D
0.75	.510	.0195	26.1

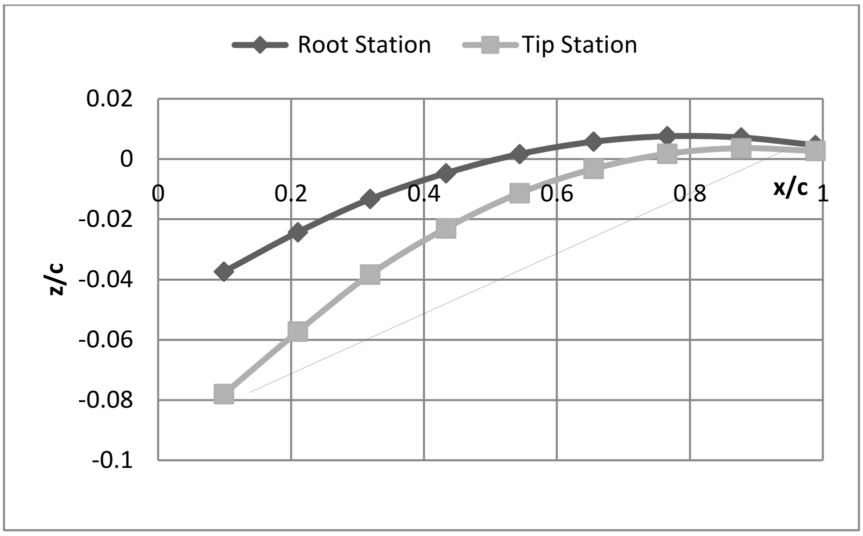

Figure 3. Chordwise variation of vertical ordinates of optimal wing, M_∞=0.75 and alpha = 4^0

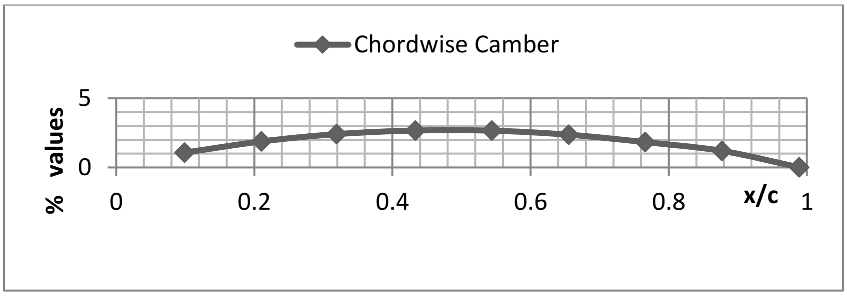

Figure 5.Chordwise camber variation

Now the thickness of NACA 0008 aerofoil is superimposed on it. Finite difference method is now used and Mach number is varied from 0.75 onwards. Figure 6 shows the chordwise pressure coefficients on upper surface for three different Mach numbers at a tip station. A developed shock is seen to appear at freestream Mach number of 0.875. Now for this value of Mach number the spanwise pressure coefficients are plotted for upper surface. Figure 7 shows these plots which signify shocks with larger pressure gradients occurring towards wing outboards.

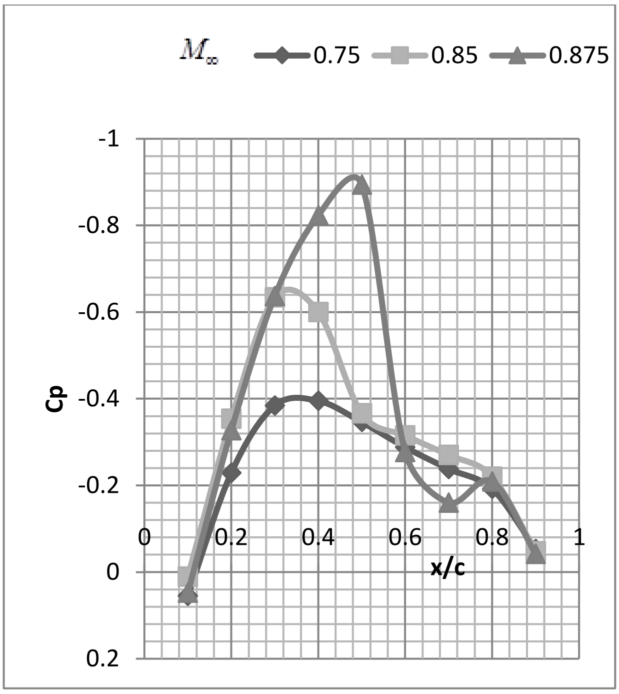

Figure 6.Chordwise pressure coefficients

Upper surface -Tip Station

Figure 8 shows the field value plots for this Mach number value on upper surface. Figure 9 shows the effect of further increasing the Mach number. Though it is not a practical situation, it is only done to see as to what happens to pressure distribution. When freestream Mach number is 0.95, there is continuous increase in suction and flow continues to remain supersonic. This happens at tip and root stations, and thus all along the trailing edge. Figure 10 & 11 to refer.

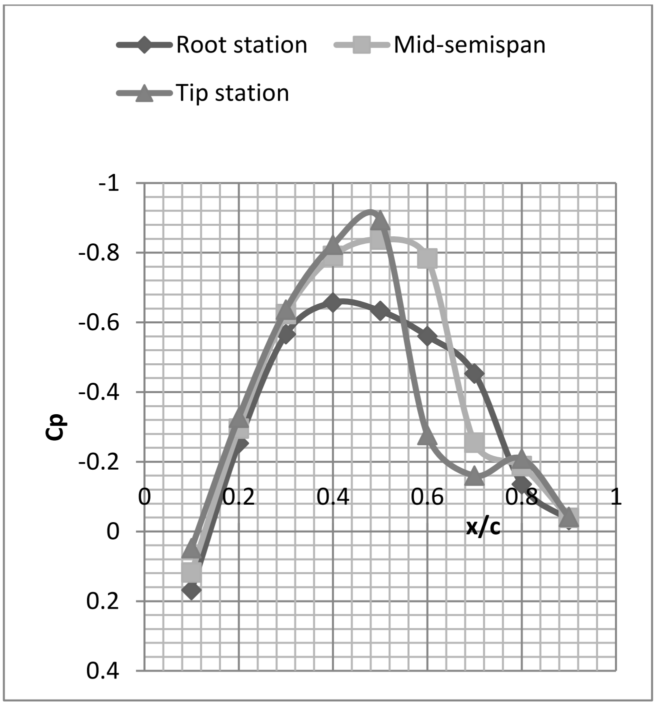

Figure 7. Pressure coefficients at M_∞ =.875

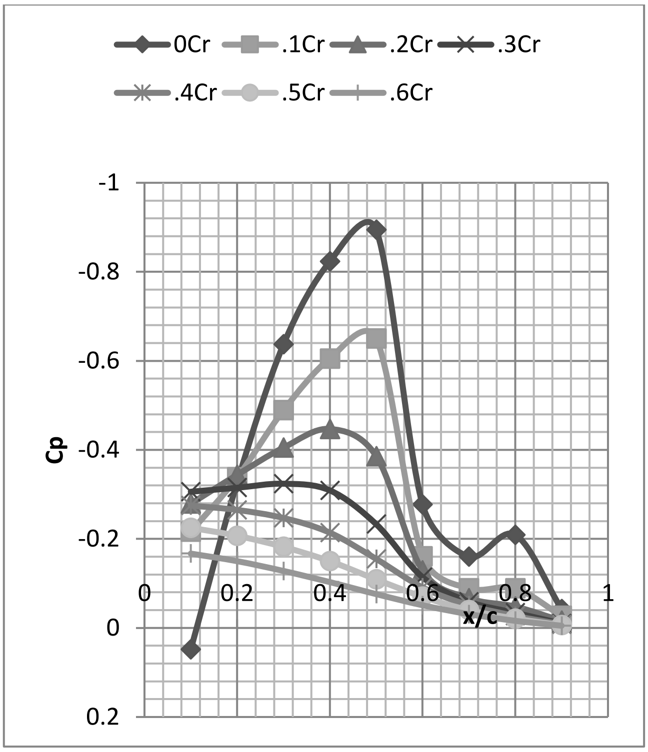

Figure 8. Field values at M_∞ **=0.875**

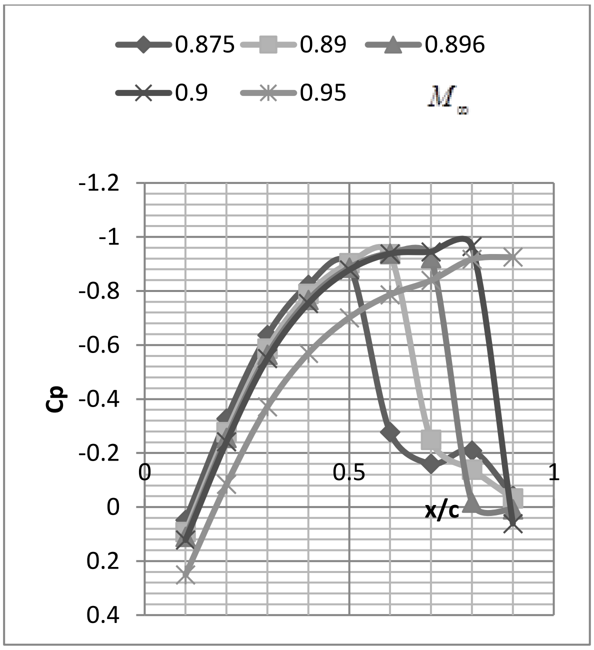

Figure 9.Pressure coefficients on upper surface at Tip station-Effect of increasing Mach number

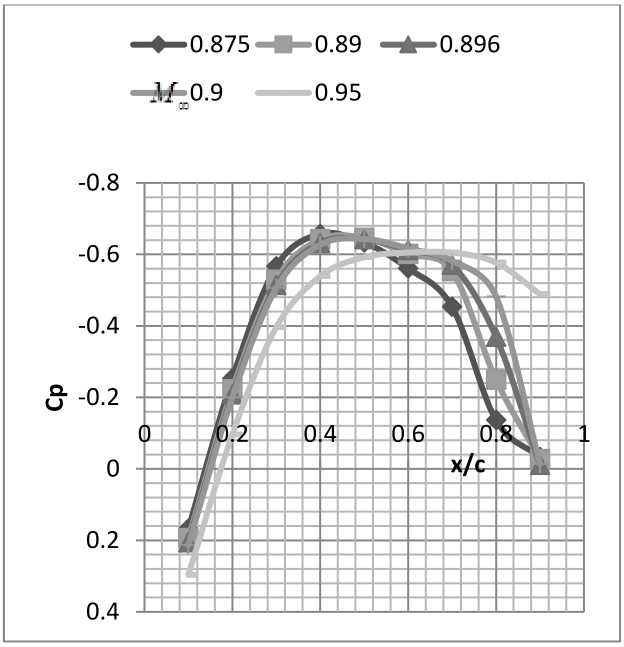

Figure 10.Pressure coefficients on upper surface at
Root station-Effect of increasing Mach number

Conclusions

Combination of camber and washout is seen to result in substantial reduction in lift dependent drag. Optimisation process is attempted for two different Mach numbers. While the spanwise twist is seen to vary with compressibility effects the camber remains invariant, **which is not explainable**. Desired chordwise peak loading can be obtained by imposing specific pitching moment constraint. However it is accompanied by penalty in drag reduction. It is because of reflex that is caused in camber. Loading resulting from decreased pitching moment is favorable from structural layup point of view. Angle-of-attack can be alleviated by the value of twist at root, and optimal warp equally altered by same amount. Thus an optimized wing can be made to generate same lift at a lower angle-of –attack.

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